ANALYSING DYNAMICS OF NON-DIRECTED SOCIAL NETWORKS

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Statistical inference for network dynamics

Data:

2 or more repeated observations
on a network between a fixed set of n actors
where ties are undirected,
and refer to a longer-lasting state
of the relationship between the two actors.

e.g.: friendship; being a regular sex partner;
collaboration; strategic alliance.

Compared to existing models for analyzing
dynamics of *directed* networks
(Snijders, *Soc. Meth. 2001*; SIENA program),

for *non-directed* networks, statistics are simpler
but the dynamics is more complicated to model
because two actors are involved in tie creation / break up.

It is assumed that between the observation moments,
time runs on continuously;
changes could be made at any moment,
but at each single moment only a single tie variable may change.

The current state of the changing network acts as a
dynamic constraint on each actor's behavior.

Thus, network dynamics treated
as an *endogenous dynamic process* with an inbuilt inertia.

Relation is denoted by the adjacency matrix X,
where tie variable $X_{ij} = X_{ji}$ indicates by values 1 and 0,
respectively, that actors i and j are tied / are not tied.

Assumption: two-step process:

1. Opportunity for changing one tie variable X_{ij} ;
   these opportunities can occur continuously between observations

2. Tie probabilities depend on $\left\{ \begin{array}{c} \text{actor } i \\ \text{actor } j \end{array} \right\}$ in some interdependence.

1. Opportunities for change

This first model component specifies the stochastic time moments where a tie variable *could* be changed.

Two options:

A. *One-sided initiative*:
one actor is chosen – denoted i
for whom one tie variable X_{ij} may change,
where j still is to be determined.

B. *Two-sided opportunity*:
a pair of actors meet – denoted (i, j)
for who may change their tie variable X_{ij}.

The moments where this happens constitute
a stochastic process in continuous time:
A. for actor i, opportunities occur at a rate λ_i
B. for pairs (i, j), meetings occur at a rate $\lambda_{ij}\,\lambda_j$,
e.g. $\lambda_{ij} = \lambda_i\,\lambda_j$.

('At a rate r' means that in short time intervals of length dt,
the probability of occurrence is approximately $r\,dt$.)

The rate functions λ_i may be constant,
but can also depend
on covariates and network position (degree, etc.).

Parameter interpretation:

In Models A, where the initiative is one-sided, the rate function is comparable to the rate function in directed models.

In Models B, however, the pair of actors is chosen at a rate which is the *product* of the rate functions
λ_i and λ_j for the two actors.
This means that opportunities for change of
the single tie variable X_{ij} occur at the rate $\lambda_i \times \lambda_j$.

The numerical interpretation of the rates obtained is different from that in Models A.

2. Decisions about changing ties

When there is an opportunity for change,
actors decide on changes in their ties depending on
preferences – costs – constraints,
all subsumed in one *objective function* $f_i(\beta, x)$
(i is the actor, x is the state of the network)
plus unknown (random) influences.

β represents the unknown parameters
that will have to be estimated statistically.

The actors would like to obtain
a high value of their objective function.

This purposive decision interpretation can be used for explanation, but a decision interpretation is not necessary.
The objective function defines
the probabilities for change, which may be regarded as
what the actors seem to optimize,
the direction into which changes tend to take place.

Differences in objectives between actors are allowed
only when these can be captured by measured covariates.

There may be asymmetries
between the creation and dissolution of ties:
complement utility function by *endowment function*
which contributes only to the value of changes $1 \Rightarrow 0$ (dissolution)
and not to changes $0 \Rightarrow 1$ (creation).

Different ways for combining actors' objectives

1. Unilateral imposition of a tie (disjunctive).
2. Mutual agreement required for a tie to exist (conjunctive).
3. Gain for one may outweigh loss for the other (compensatory).

This is to be combined with
A: unilateral initiative; B: two-sided opportunity.

The combination A-3 seems less likely (but is very well possible) and provisionally not worked out. This gives five combinations:

A1. Forcing model:
one actor i takes the initiative,
chooses the best possible change $x_{ij} \Rightarrow 1 - x_{ij}$ (or none)
and unilaterally imposes that this change is made.

A2. Unilateral initiative and reciprocal confirmation:
one actor i takes the initiative,
chooses the best possible change $x_{ij} \Rightarrow 1 - x_{ij}$ (or none);
if this is the dissolution of a tie, the change is carried out,
otherwise the new tie is proposed to j,
if this actor agrees then the change is carried out,
otherwise nothing happens.

B1. Pairwise disjunctive (forcing) model:
actors i and j meet and reconsider their tie variable X_{ij};
if at least one wishes a tie, then they set $X_{ij} = 1$, else $X_{ij} = 0$.

B2. Pairwise conjunctive model:
actors i and j meet and reconsider their tie variable X_{ij};
if both wish a tie, then they set $X_{ij} = 1$, else $X_{ij} = 0$.

B3. Pairwise compensatory (additive) model:
actors i and j meet and reconsider their tie variable X_{ij};
on the basis of their summed objective function $f_i(\beta, x) + f_j(\beta, x)$
they decide on the new value of the tie variable.

Other possibilities can be thought of.

In models A, when actor i gets the initiative,
he must choose which tie variable to change;
call the possibly changed tie variable x_{ij}.

The hypothetical new network obtained by changing x_{ij}
is denoted by $x(i \rightsquigarrow j)$.
Formally, let $j = i$ denote that nothing changes
(the current situation is the best): $x(i \rightsquigarrow i) = x$.
Actor i chooses the "best" j by maximizing

$$f_i\Big(\beta, x(i \rightsquigarrow j)\Big) \; + \; U_i(t, x, j) \, .$$

$$\Uparrow$$

random component

If the random component U has a
type 1 extreme value = Gumbel distribution,
the probability that i chooses j is

$$p_{ij}(\beta, x) = \frac{\exp\big(f(i,j)\big)}{\sum_{h=1}^{n} \exp\big(f(i,h)\big)}$$

where

$$f(i,h) = f_i\Big(\beta, x(i \rightsquigarrow h)\Big) \ .$$

This is the multinomial logit form of a *random utility* model.

The Gumbel distribution has variance $\pi^2/6 = 1.645$ and s.d. 1.28.

Note and remember for later interpretation:

The scale of the model parameters is defined
by the fixed s.d. of the random component.

If two model specifications
have different but proportional parameters,
this is equivalent to having the same parameters
but different s.d.s of the random component
$\sim$ different amounts of unexplained variability.

Yes/no decisions occur in Models B,
and for the agreement by the second actor in model A2.
These are based on the comparison between the actor
of the network $x^{+}(i,j)$ with tie $i-j$,
and the network $x^{-}(i,j)$ without this tie.
The comparison is made similar as before.

This means that the probability
that actor j wishes the tie $i-j$ is given by

$$\frac{\exp\Big(f_i(\beta, x^{+}(i,j))\Big)}{\exp\Big(f_i(\beta, x^{-}(i,j))\Big) + \exp\Big(f_i(\beta, x^{+}(i,j))\Big)}$$

Model specification :

The objective function f_i reflects
network effects (endogenous) and covariate effects (exogenous).

Covariates can be actor-dependent: v_i
or dyad-dependent: w_{ij} .

Convenient definition of f_i is a weighted sum

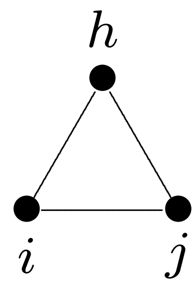
$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \, ,$$

where weights β_k are statistical parameters
indicating strength of effect $s_{ik}(x)$.

Choose possible network effects for actor i, e.g.:
(others to whom actor i is tied are called here i's 'friends')

1. *out-degree effect*, number of friends
$s_{i1}(x) = x_{i+} = \sum_j x_{ij}$
2. *popularity/activity effect*, sum of degrees of i's friends
$s_{i2}(x) = \sum_j x_{ij}\, x_{+j} = \sum_j x_{ij} \sum_h x_{hj}$
3. *transitive triads effect*,

number of transitive patterns in i's ties
$(i - j,\ \ j - h,\ \ i - h)$
$s_{i3}(x) = \sum_{j,h}\ x_{ij}\, x_{jh}\, x_{ih}$



transitive triad

4. *indirect relations effect*,

   number of actors j to whom i is indirectly related
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   but not directly ($x_{ij} = 0$),
   = number of geodesic distances equal to 2,
   $s_{i4}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih}\, x_{hj}) > 0\}$

Many other network effects are possible.

Two kinds of effect associated with actor covariate v_i :

4. *covariate-related degrees*,
   $s_{i4}(x) = v_i \, x_{i+}$;
   positive effect contributes
   to correlation between degrees and V.

5. *covariate-related similarity*,
   sum of measure of covariate similarity
   between i and his friends, e.g.
   $s_{i5}(x) = \sum_j x_{ij} \left(1 - |\, v_i - v_j \,|\right)$
   if V has values between 0 and 1;
   positive effect contributes to network autocorrelation of V.

Objective function effect for dyadic covariate w_{ij} :

6. *covariate-related preference*,
   values of w_{ij} summed over all others to whom i is tied,
   $s_{i6}(x) = \sum_j x_{ij}\, w_{ij}$;
   positive effect contributes to correlation
   between X_{ij} and W_{ij} .

This model definition, with components

A / B. individual – dyadic initiative, with rate function λ_i

1 / 2 / 3. disjunctive – conjunctive – compensatory decisions
based on objective function $f_i(x)$, endowment function $g_i(x)$

yields (given that λ_i and $f_i(x),\ g_i(x)$ are specified)
a model for the network dynamics that can be simulated.

Parameters can be estimated using Method of Moments estimators implemented by stochastic approximation as in Snijders (2001); this is implemented in SIENA

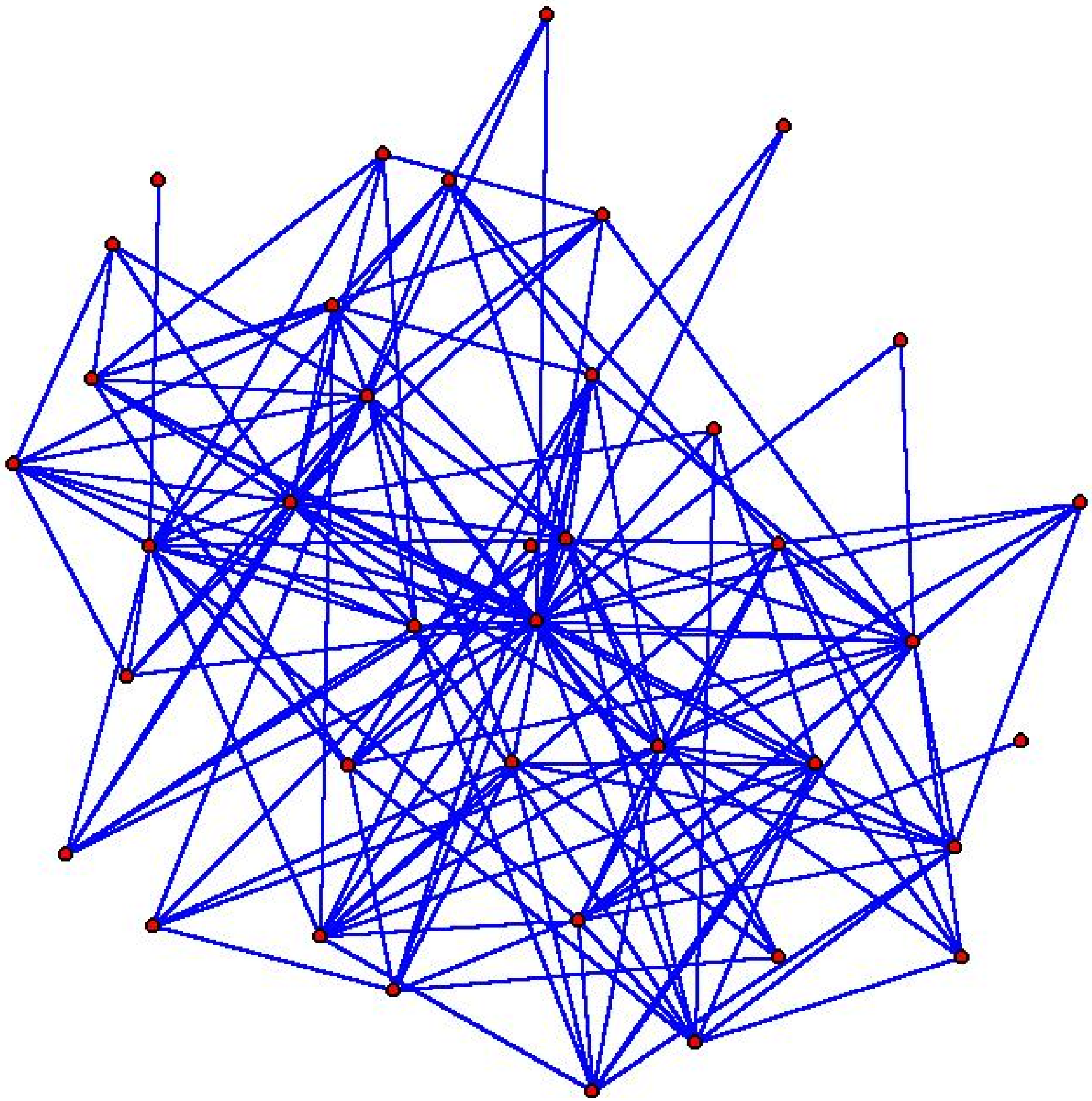
`http://stat.gamma.rug.nl/stocnet`

Example: Assistance Relation in Kapferer's Taylor Shop

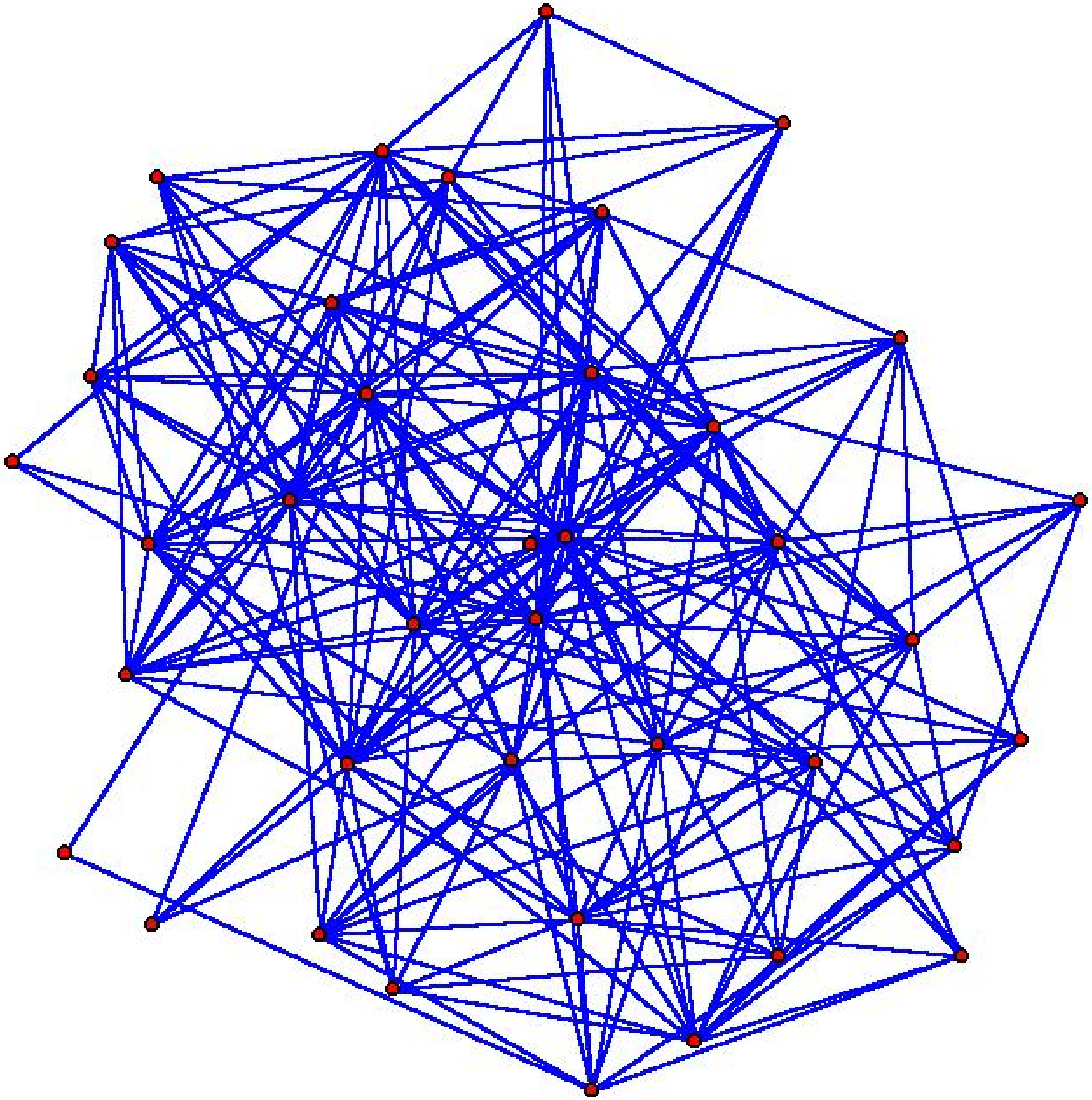
Kapferer (1972):
Interactions in tailor shop in Zambia, period of 10 months, within which there was an abortive strike.

Work- and assistance-related relation.

Dichotomous covariate *status*:
contrast { head tailor (nr 19), cutter (16),
line 1 tailor $(1\!-\!3, 5\!-\!7, 9, 11\!-\!14, 21, 24)$, button machiner $(25\!-\!26)$}
to
{ line 3 tailor $(8, 15, 20, 22-23, 27-28)$, ironer $(29, 33, 39)$,
cotton boy $(30-32, 34-38)$, line 2 tailor $(4, 10, 17-18)$ } .



Assistance network
time 1.
Average degree 8.3



Assistance network
time 2.
Average degree 11.7

Preliminary (trial) analyses yielded the following suggestions:

Rate function:

higher status workers change ties more often:

$$\lambda_i = \lambda_0 \, e^{v_i}$$

Objective function:

status homophily

transitivity, as expressed by preference for transitive triads

No evidence for other components
in objective function, or for endowment function.

(This holds for all model specifications.)

Models with individual initiative

	Model 1 (forcing)		Model 2 (agreement)	
Effect	*par.*	(*s.e.*)	*par.*	(*s.e.*)
Rate	7.60	1.10	12.09	1.51
Status ⇒ rate	1.32	0.39	1.27	0.36
Degree	−1.08	(0.13)	−0.55	0.11
Transitive triads	0.22	(0.03)	0.17	0.03
Status similarity	0.42	(0.11)	0.33	0.09

Models with two-sided initiative

	Model 3 (disjunctive)		Model 4 (conjunctive)		Model 5 (compensatory)	
Effect	*par.*	*(s.e.)*	*par.*	*(s.e.)*	*par.*	*(s.e.)*
Rate	0.88	0.06	0.79	0.06	0.79	0.05
Status ⇒ rate	0.70	0.21	0.71	0.19	0.69	0.20
Degree	−0.85	0.17	−2.03	(0.24)	−1.01	0.16
Transitive triads	0.28	0.06	0.43	(0.08)	0.21	0.07
Status similarity	0.48	0.15	0.73	(0.23)	0.37	0.11

Conclusions:

Results similar in all five models;
higher-status workers change their ties more often;
status-related homophily, transitivity.

Rate functions higher for models with individual initiative
(follows from different definition).

Out-degree effect higher when more agreement is required,
because this tends in itself to depress number of ties.

Effects are somewhat different
but transitive triads and status similarity effects
are roughly proportional.

Different model specification:

include equal job titles as additional dyadic covariate

Models with individual initiative

	Model 1 (forcing)		Model 2 (agreement)	
Effect	*par.*	*(s.e.)*	*par.*	*(s.e.)*
Rate	8.04	1.04	13.02	1.67
Status ⇒ rate	1.16	0.38	1.09	0.41
Degree	−1.08	(0.11)	−0.54	0.10
Transitive triads	0.21	(0.03)	0.16	0.03
Status similarity	0.24	(0.13)	0.19	0.09
Same job title	0.46	(0.16)	0.36	0.13

Models with two-sided initiative

	Model 3 (disjunctive)		Model 4 (conjunctive)		Model 5 (compensatory)	
Effect	*par.*	*(s.e.)*	*par.*	*(s.e.)*	*par.*	*(s.e.)*
Rate	0.92	0.06	0.82	0.06	0.81	0.06
Status ⇒ rate	0.58	0.21	0.60	0.21	0.59	0.23
Degree	−0.85	(0.16)	−1.98	0.21	−1.00	0.17
Transitive triads	0.26	(0.06)	0.38	0.07	0.19	0.08
Status similarity	0.28	(0.16)	0.42	0.31	0.21	0.11
Same job title	0.65	(0.22)	0.90	0.25	0.45	0.17

Conclusions from extended model specifications:

Having the same job title is clearly significant,
when controlling for status similarity
(dichotomized status);

status similarity is not quite, or just, significant
when controlling for having same job title,
depending on precise model specification.

Parameter for transitive triads is still significant,
but smaller than in earlier model:
job title represents a bigger portion of the observed transitivity.

Summary / Discussion

This method extends the methodology
for analysing network dynamics of Snijders (*Soc. Meth.* 2001)
to nondirected relations.

(For directed relations,
a two-sided decision can also be appropriate!)

The methods are included in the program SIENA
included in the StOCNET package

`http://stat.gamma.rug.nl/stocnet`

The fact that two-sided decisions must be modeled
leads to a variety of different models.
Choosing between them might be difficult
if there is a lack of convincing theory.

In the example, conclusions were similar across these models,
but not quite the same.
Similarity of conclusions obviously
saves us some practical complications.