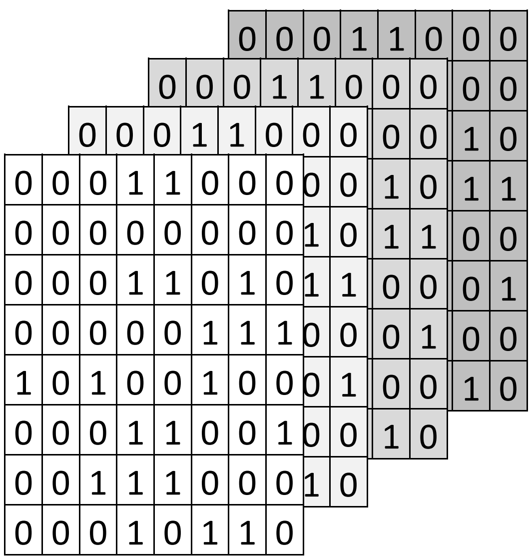
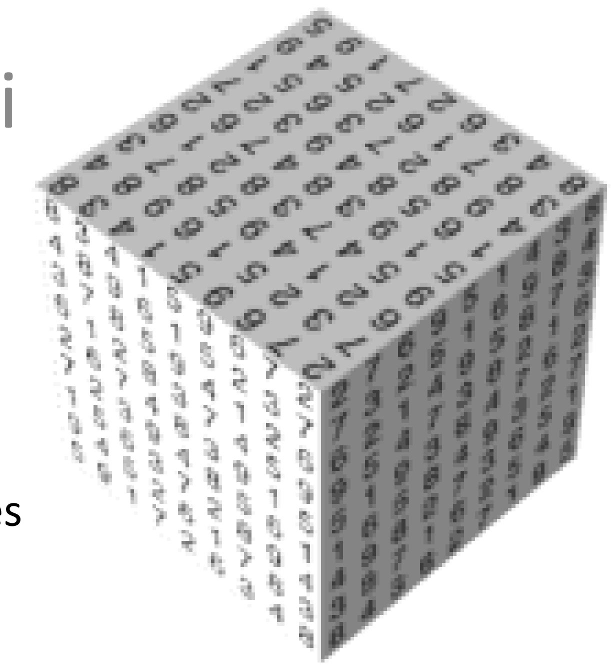

# Multi-Dimensional Matrices

Steve Borgatti

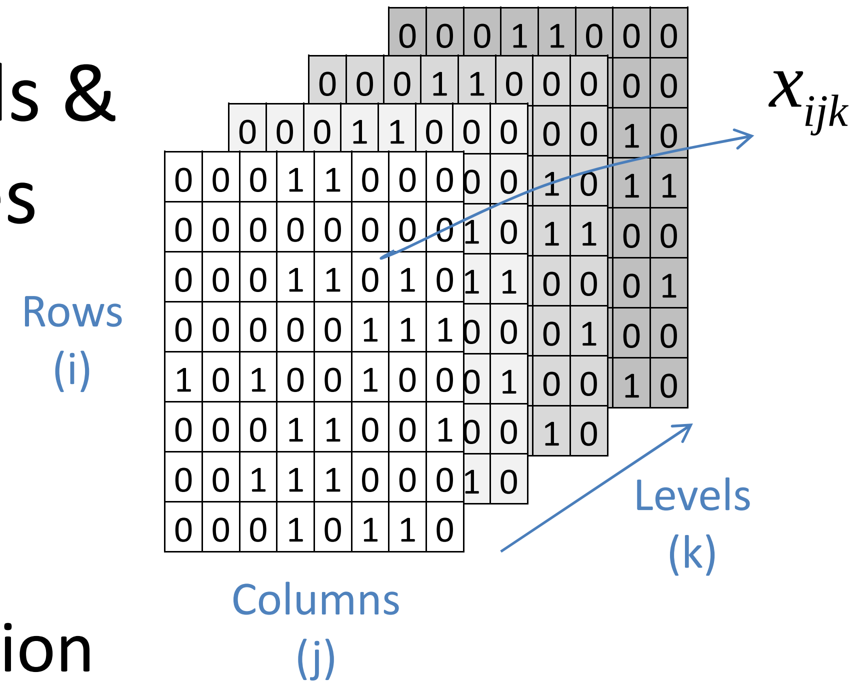


One
multidimensional
matrix, or layers of
lower dimensional matrices

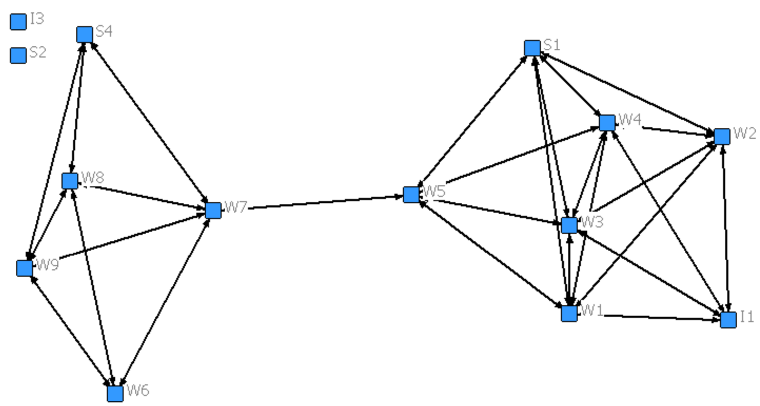


# 3-dimensional matrices

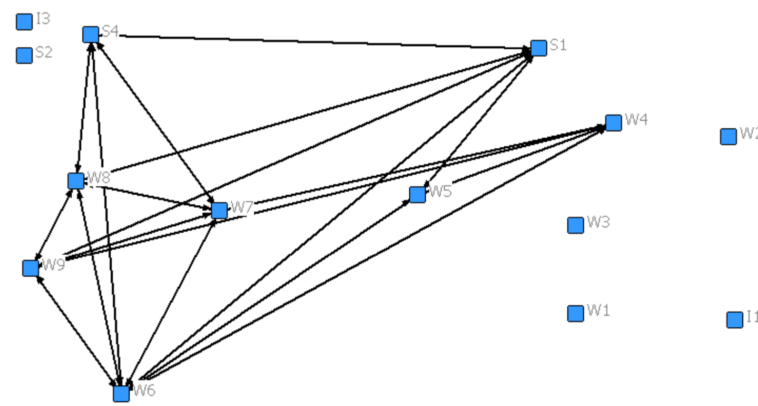
- 3-way matrix X with cells $x_{ijk}$. By convention, first subscript is rows, second subscript is columns, third subscript is levels
- 3-way, 3-mode: rows, cols & levels all different entities
  - Person by prod over time
- 3-way, 2-mode
  - Person by person by time
  - Person by person by relation



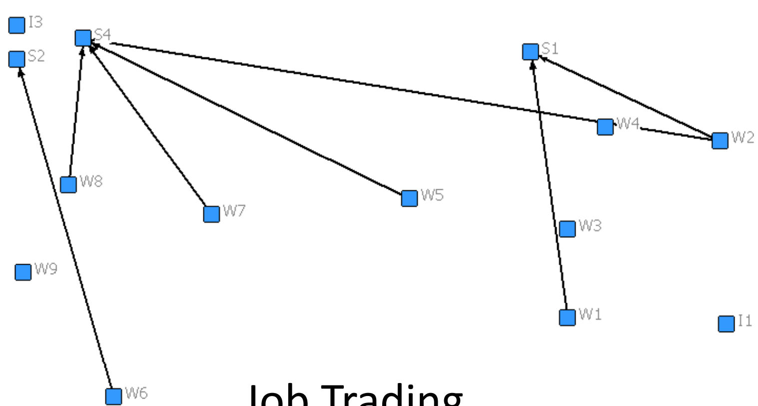
# Bank Wiring Room data




Games




Conflict




Job Trading

- Multiple social relations measured for the same group of actors
- 3-way, 2-mode matrices
  - Multiple 1-mode matrices

# File formats

- VNA
- DL

# VNA file with multiple relations

- For each ordered pair of nodes (rows) the columns indicate whether the pair have a given relation

```
*Node data
"ID"
"I1"
"I3"
"W1"
"W2"
"W3"
"W4"
"W5"
"W6"
"W7"
"W8"
"W9"
"S1"
"S2"
"S4"
*Tie data
FROM TO
"RDGAM","RDCON","RDPOS","RDNEG","RDHLP","RDJOB"
"I1","W1",1,0,0,0,0,0
"I1","W2",1,0,0,1,0,0
"I1","W3",1,0,1,0,0,0
"I1","W4",1,0,0,0,0,0
"W1","I1",1,0,0,0,0,0
"W1","W2",1,0,0,0,0,0
"W1","W3",1,0,1,0,1,0
"W1","W4",1,0,1,0,0,0
"W1","W5",1,0,0,0,0,0
"W1","S1",1,0,1,0,1,2
```

# DL nodelist1 format

- Each relation separated by ! mark

```
DL N=14 NM=6 FORMAT = NODELIST1
LABELS EMBEDDED
MATRIX LABELS EMBEDDED
DATA:
"RDGAM"
"I1" "W1" "W2" "W3" "W4"
"I3"
"W1" "I1" "W2" "W3" "W4" "W5" "S1"
"W2" "I1" "W1" "W3" "W4" "S1"
"W3" "I1" "W1" "W2" "W4" "W5" "S1"
"W4" "I1" "W1" "W2" "W3" "W5" "S1"
"W5" "W1" "W3" "W4" "W7" "S1"
"W6" "W7" "W8" "W9"
"W7" "W5" "W6" "W8" "W9" "S4"
"W8" "W6" "W7" "W9" "S4"
"W9" "W6" "W7" "W8" "S4"
"S1" "W1" "W2" "W3" "W4" "W5"
"S2"
"S4" "W7" "W8" "W9"
!
"RDCON"
"I1"
"I3"
"W1"
"W2"
"W3"
"W4" "W5" "W6" "W7" "W9"
"W5" "W4" "W6" "S1"
```

# DL edgelist1 format

- Just like nodelist1 format, the relations are separated by “!” marks

```
DL N=14 NM=6 FORMAT = EDGELIST1
LABELS EMBEDDED
MATRIX LABELS EMBEDDED
DATA:
RDGAM
 I1 W1 1
 I1 W2 1
 I1 W3 1
 I1 W4 1
 W1 I1 1
 W1 W2 1
 W1 W3 1
…
!
RDCON
 W4 W5 1
 W4 W6 1
 W4 W7 1
…
!
RDNEG
...
```

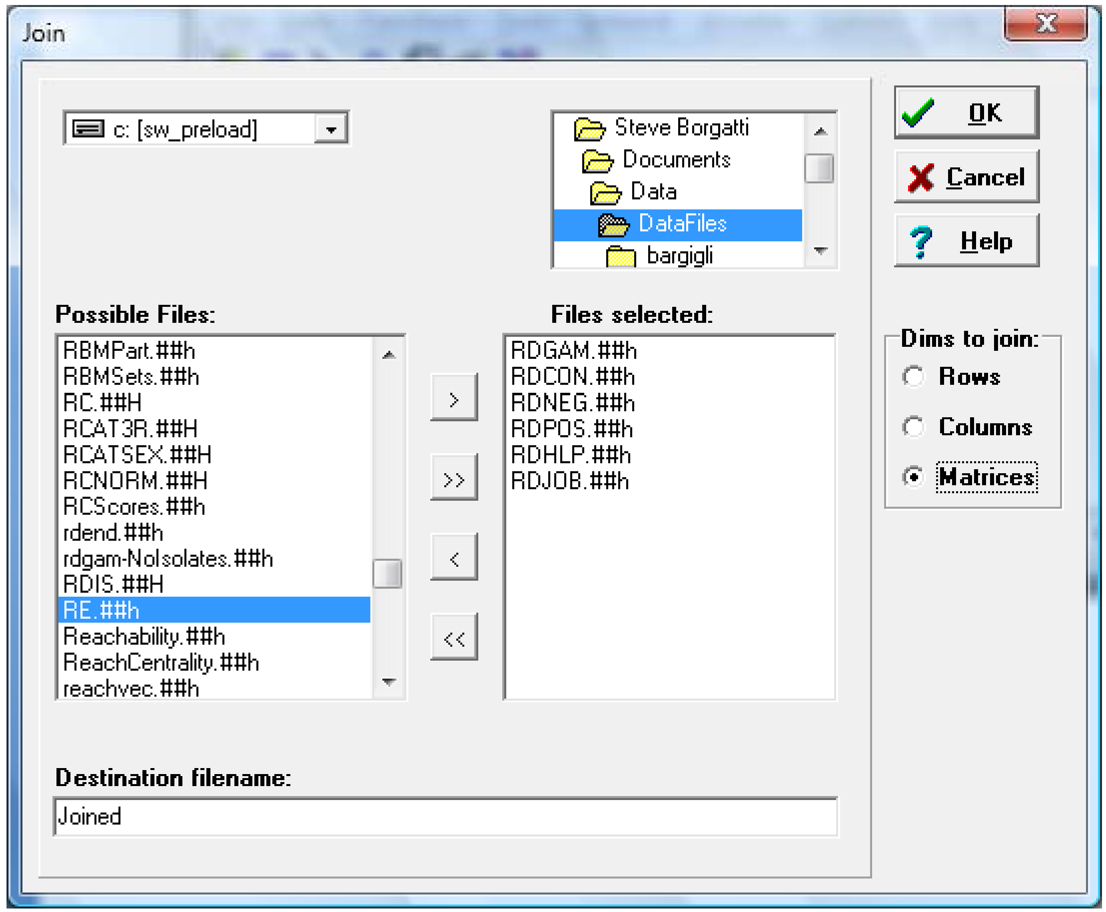
# DL fullmatrix format

- No “!” marks needed since the number of numbers to be read is fixed and calculable in advance

```
DL N=14 NM=6 FORMAT = FULLMATRIX
LABELS EMBEDDED
MATRIX LABELS EMBEDDED
DATA:
"RDGAM"
    "I1" "I3" "W1" "W2" "W3" "W4" "W5" "W6" "W7" "W8" "W9" "S1" "S2" "S4"
"I1"  0  0  1  1  1  1  0  0  0  0  0  0  0  0
"I3"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"W1"  1  0  0  1  1  1  1  0  0  0  0  1  0  0
"W2"  1  0  1  0  1  1  0  0  0  0  0  1  0  0
"W3"  1  0  1  1  0  1  1  0  0  0  0  1  0  0
"W4"  1  0  1  1  1  0  1  0  0  0  0  1  0  0
"W5"  0  0  1  0  1  1  0  0  1  0  0  1  0  0
"W6"  0  0  0  0  0  0  0  0  1  1  1  0  0  0
"W7"  0  0  0  0  0  0  1  1  0  1  1  0  0  1
"W8"  0  0  0  0  0  0  0  1  1  0  1  0  0  1
"W9"  0  0  0  0  0  0  0  1  1  1  0  0  0  1
"S1"  0  0  1  1  1  1  1  0  0  0  0  0  0  0
"S2"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"S4"  0  0  0  0  0  0  0  1  1  1  0  0  0  0
"RDCON"
    "I1" "I3" "W1" "W2" "W3" "W4" "W5" "W6" "W7" "W8" "W9" "S1" "S2" "S4"
"I1"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"I3"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"W1"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"W2"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"W3"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"W4"  0  0  0  0  0  0  1  1  1  0  1  0  0  0
"W5"  0  0  0  0  0  1  0  1  0  0  0  1  0  0
"W6"  0  0  0  0  0  1  1  0  1  1  1  1  0  1
"W7"  0  0  0  0  0  1  0  1  0  1  1  0  0  1
"W8"  0  0  0  0  0  0  0  1  1  0  1  1  0  1
"W9"  0  0  0  0  0  1  0  1  1  1  0  1  0  0
"S1"  0  0  0  0  0  0  1  1  0  1  1  0  0  1
"S2"  0  0  0  0  0  0  0  0  0  0  0  0  0  0
"S4"  0  0  0  0  0  0  0  1  1  1  0  1  0  0
…
```

# Packing and Unpacking

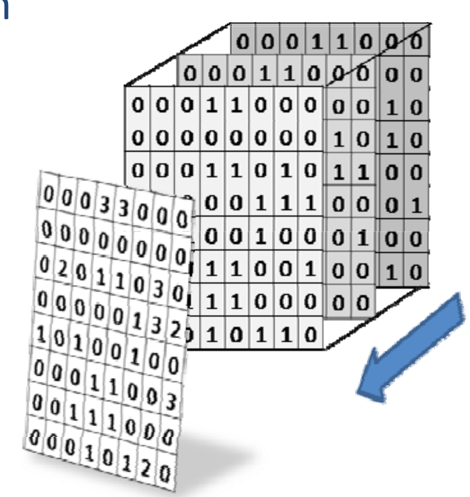
- UCINET data|unpack command can create separate files for each relation in a 3-dimensional dataset
- UCINET data|join can combine separate 2D files (of the same size) into one 3D dataset
  - Make sure to pick “matrices” as the dimension



# Aggregations

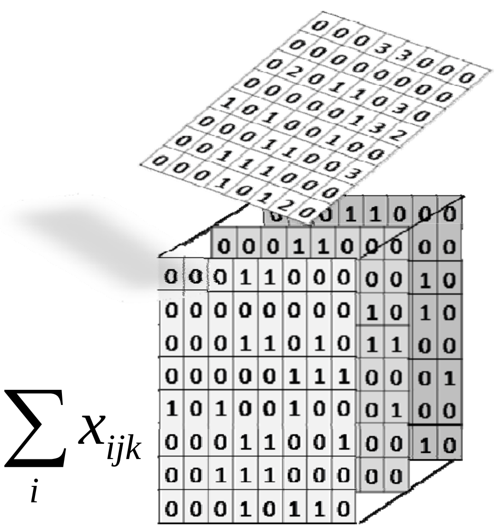
Summing across one dimension

$$y_{ij} = \sum_k x_{ijk}$$



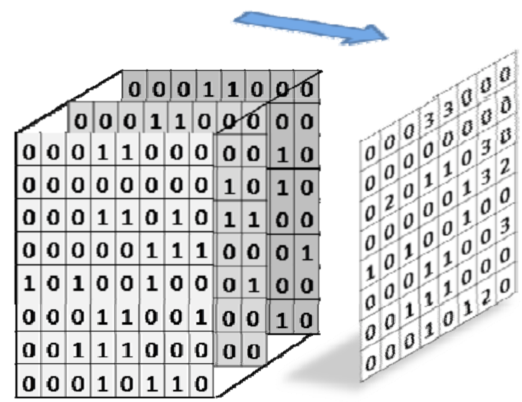
- Summing across levels, creating 2D matrix from 3D matrix
  - E.g., summing over relations to get total number ties binding i to j
  - E.g., summing over time to get total number of collaborations between i and j over all time

$$y_{jk} = \sum_i x_{ijk}$$



- Summing across rows, creating 2D horizontal "slice"
  - E.g., summing over all egos to get total number ties that alter j is receiving at time k (indegree over time)

$$y_{ik} = \sum_j x_{ijk}$$

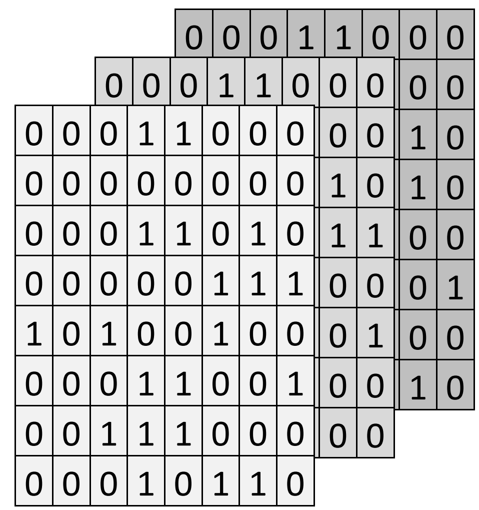


- Summing across columns, creating 2D vertical "slice"
  - E.g., summing over all alters to get total number ties that i is sending at time k (outdegree over time)

# Aggregations – cont.

Summing across two dimensions at same time

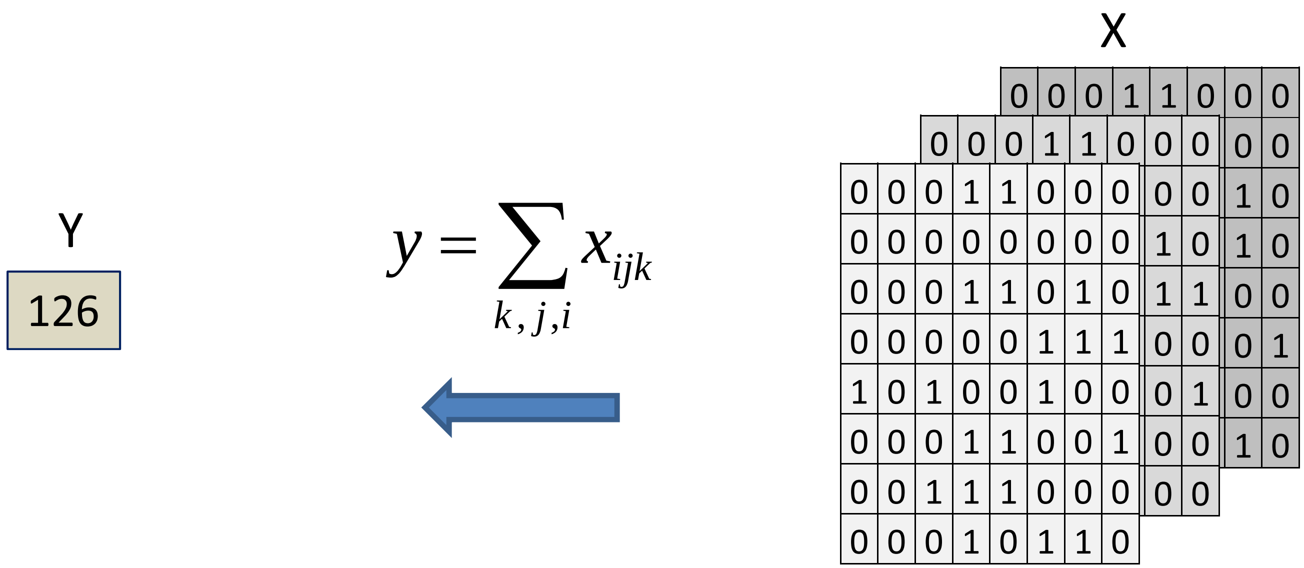
- Row sums
  - Sum across columns and levels
    - Squish left and right (accordion), then back and front
    - Results in a single column vector
  - E.g., total number of outgoing ties with all others across time periods
- Column sums
  - Sum across rows and levels
    - Squish up and down, then back and front
    - Results in a single row vector
  - E.g. total number of incoming ties with all others across time periods
- Level sums
  - Sum across rows columns, to get number for each level
    - Squish each layer left and right, and top to bottom
    - Results in a "level" vector
  - E.g., total number of ties in each relation, or at each time period



# Aggregations – cont.

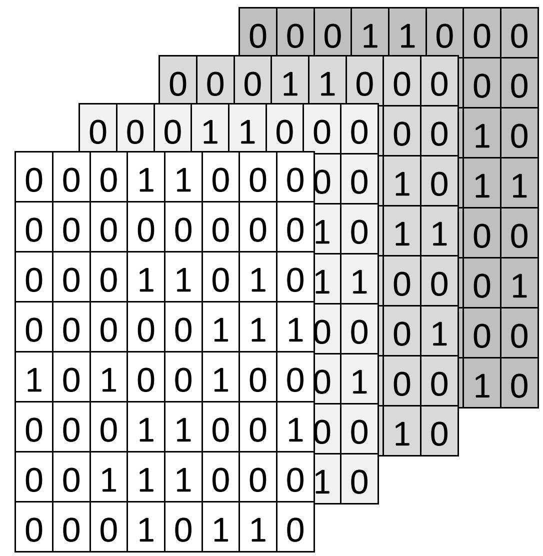
- Summing across three dimensions, creating single number from 3D matrix
  - E.g., summing over all cells to get total number ties of all kinds.
  - Can also average, take median, maximum, minimum, std dev, etc.
    - E.g., overall density

$$y = \sum_{k,j,i} x_{ijk}$$



# Aggregations across dimensions (in matrix algebra program)

- Aggregations across one dimension – delivers 2-dim matrix
  - RC = total(wiring row col)
  - RM = total(wiring row lev)
  - CM = total(wiring col lev)
- Aggregations across two dimensions – delivers a vector
  - R = total(wiring row)
  - C = total(wiring col)
  - M = total(wiring lev)
- Aggregations across three dim – delivers a single number
  - T = total(wiring)



The dimensions <u>NOT</u> mentioned in a Total command are the ones being aggregated across. So, total(wiring row col) aggregates across level.

# Application

(using Wiring dataset)

- Measuring extent of multiplexity
  - For a given pair of actors, how many different kinds of ties bind them together?
  - Sum across all the relations to obtain a single aggregate matrix M in which $m_{ij}$ = number of different relations that connect i to j
- In matrix algebra:
  - > M = total(wiring rows cols)
- From menus:
  - Transform|matrix ops|within datasets|aggregations

# Bundles of relations

### Marriage ties

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

### Business ties

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

### Multiplex

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 3 | 0 | 0 | 0 | 2 | 0 |
| 0 | 0 | 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 3 | 0 | 0 | 0 | 2 | 0 |
| 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 2 | 0 | 3 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 0 | 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 3 | 0 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| 0 | 0 | 1 | 3 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |

Padgett Dataset

Legend

- 0 = no tie on either relation
- 1 = business tie only
- 2 = marriage tie only
- 3 = both business and marriage tie

# Correlating slices of 3-dimensional matrices

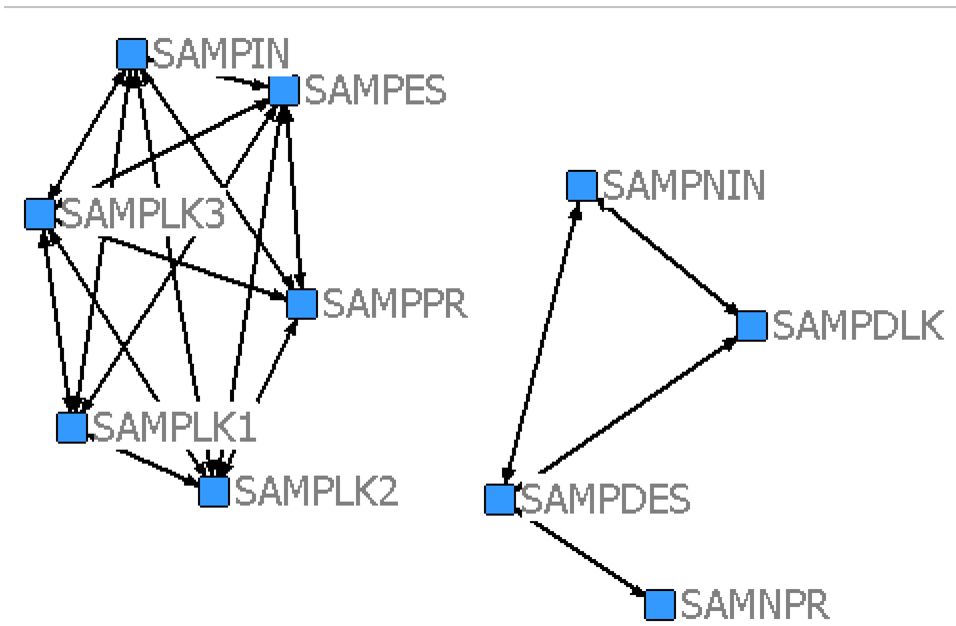
- UCINET commands
  - For slices contained in a single 3D object
    - Tools|similarity > select matrices as dimension to correlation
  - For slices stored as separate matrices:
    - Tools|testing hyps|dyadic|qap correlation
- Wiring example

| | RDGAM | RDCON | RDPOS | RDNEG | RDHLP | RDJOB |
|---|---|---|---|---|---|---|
| RDGAM | 1.000 | 0.243 | 0.544 | -0.050 | 0.374 | 0.136 |
| RDCON | 0.243 | 1.000 | 0.099 | 0.002 | 0.159 | 0.087 |
| RDPOS | 0.544 | 0.099 | 1.000 | -0.210 | 0.259 | 0.129 |
| RDNEG | -0.050 | 0.002 | -0.210 | 1.000 | -0.160 | -0.076 |
| RDHLP | 0.374 | 0.159 | 0.259 | -0.160 | 1.000 | 0.085 |
| RDJOB | 0.136 | 0.087 | 0.129 | -0.076 | 0.085 | 1.000 |

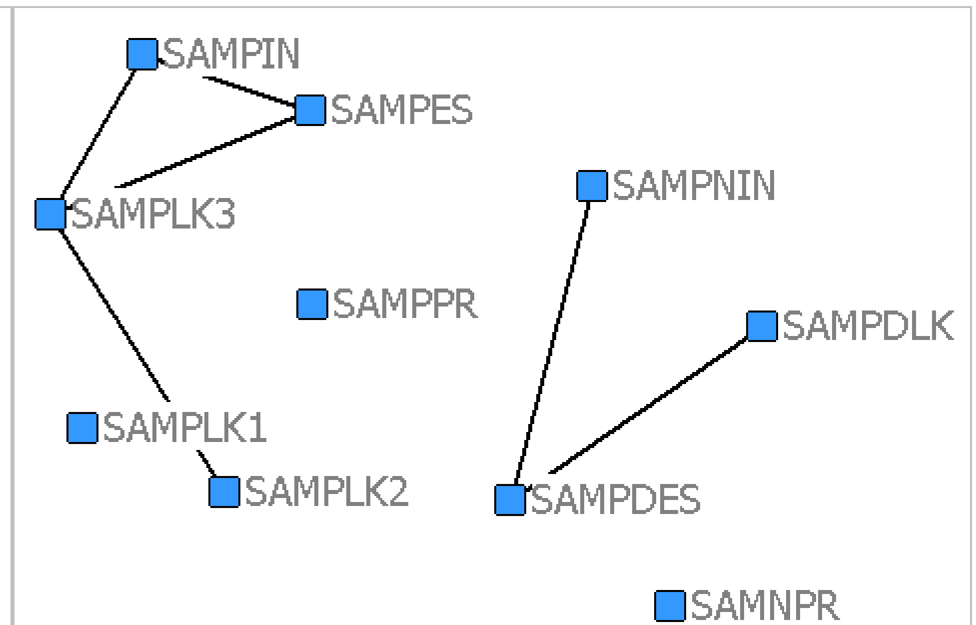
# Visualizing correlations among relations

- Open the correlation matrix in netdraw as if it were a relation
  - Adjust strength of tie criterion to show only strong ties

Sampson Monastery Dataset




r > 0.4

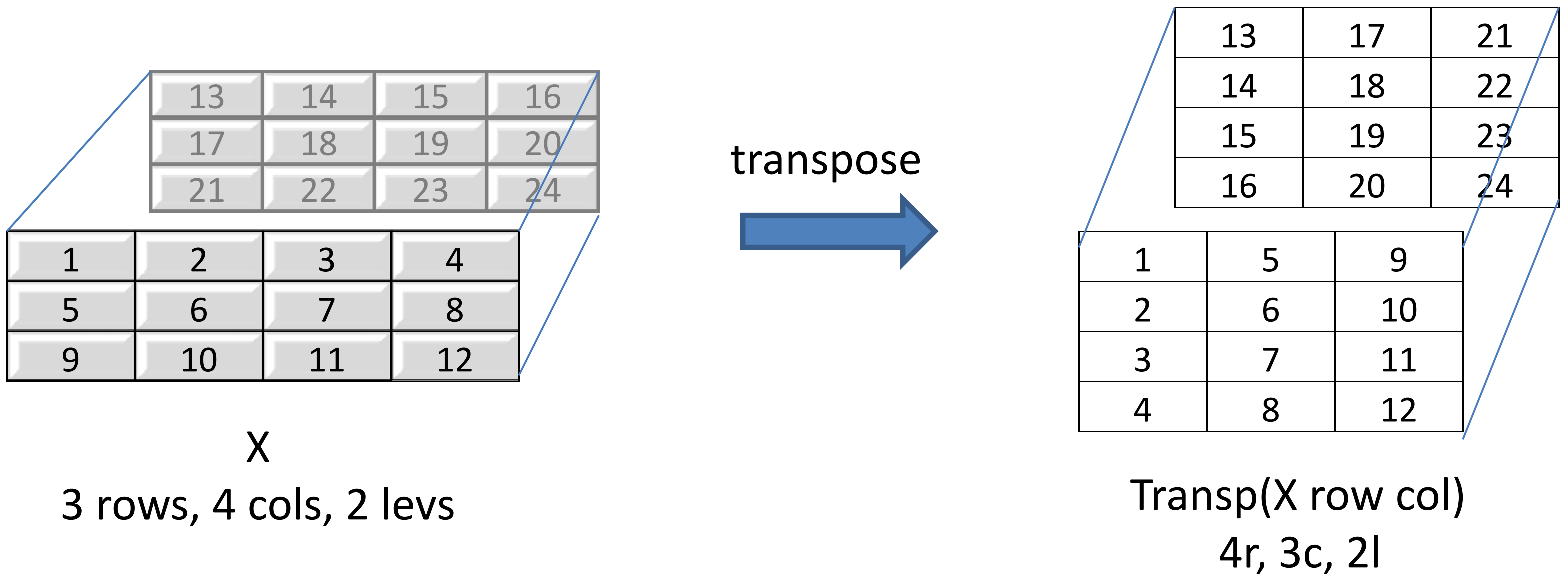



r > 0.65

# Transposing multiple dimensions

| Description | Math | UCINET Matrix Algebra syntax |
|---|---|---|
| Interchange rows with columns for all levels<br>- invert direction of ties | $y_{ijk} = x_{jik}$ | CRL = transp(wiring row col) |
| Interchange levels with columns for all levels | $y_{ijk} = x_{ikj}$ | RLC = transp(wiring lev col) |
| Interchange levels with rows for all levels | $y_{ijk} = x_{kji}$ | LCR = transp(wiring lev row) |
| Interchange rows with cols then levs with (new) cols | $y_{ijk} = x_{jki}$ | CRL = transp(wiring row col)<br>CLR= transp(CRL col lev) |
| Interchange rows with cols then levs with (new) rows | $y_{ijk} = x_{kij}$ | CRL = transp(wiring row col)<br>LRC= transp(CRL row lev) |

# Transposing



# Transposing

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 |

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |

Original matrix X
3 rows, 4 cols, 2 levs

| | M1 | M2 |
|---|---|---|
| R1 | 1 | 13 |
| R2 | 5 | 17 |
| R3 | 9 | 21 |

| | M1 | M2 |
|---|---|---|
| R1 | 2 | 14 |
| R2 | 6 | 18 |
| R3 | 10 | 22 |

| | M1 | M2 |
|---|---|---|
| R1 | 3 | 15 |
| R2 | 7 | 19 |
| R3 | 11 | 23 |

| | M1 | M2 |
|---|---|---|
| R1 | 4 | 16 |
| R2 | 8 | 20 |
| R3 | 12 | 24 |

Transp(X col lev)
3r, 2c, 4l

| | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| M1 | 1 | 2 | 3 | 4 |
| M2 | 13 | 14 | 15 | 16 |

| | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| M1 | 5 | 6 | 7 | 8 |
| M2 | 17 | 18 | 19 | 20 |

| | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| M1 | 9 | 10 | 11 | 12 |
| M2 | 21 | 22 | 23 | 24 |

Transp(X row lev)
2r, 4c, 3l

# Transposing - cont

| | M1 | M2 |
|---|---|---|
| C1 | 1 | 13 |
| C2 | 2 | 14 |
| C3 | 3 | 15 |
| C4 | 4 | 16 |

| | M1 | M2 |
|---|---|---|
| C1 | 5 | 17 |
| C2 | 6 | 18 |
| C3 | 7 | 19 |
| C4 | 8 | 20 |

| | M1 | M2 |
|---|---|---|
| C1 | 9 | 21 |
| C2 | 10 | 22 |
| C3 | 11 | 23 |
| C4 | 12 | 24 |

Transp(transp(X row col) col lev)

| | R1 | R2 | R3 |
|---|---|---|---|
| M1 | 1 | 5 | 9 |
| M2 | 13 | 17 | 21 |

| | R1 | R2 | R3 |
|---|---|---|---|
| M1 | 2 | 6 | 10 |
| M2 | 14 | 18 | 22 |

| | R1 | R2 | R3 |
|---|---|---|---|
| M1 | 3 | 7 | 11 |
| M2 | 15 | 19 | 23 |

| | R1 | R2 | R3 |
|---|---|---|---|
| M1 | 4 | 8 | 12 |
| M2 | 16 | 20 | 24 |

Transp(transp(X row col) row lev)

# Multiplication table for all possible combinations of transpositions

| | i | rc | rl | cl | rc*rl | rc*cl |
|---|---|---|---|---|---|---|
| i | i | rc | rl | cl | rc*rl | rc*cl |
| rc | rc | i | rc*rl | rc*cl | rl | cl |
| rl | rl | rc*cl | i | rc*rl | cl | rc |
| cl | cl | rc*rl | rc*cl | i | rc | rl |
| rc*rl | rc*rl | cl | rc | rl | i | i |
| rc*cl | rc*cl | rl | cl | rc | i | i |

# Transpose Example

- Run Tools|Univariate stats on the Matrices in Wiring dataset

```
Matrix: RDGAM
  1      Mean    0.308
  2   Std Dev    0.462
  3       Sum   56.000
  4  Variance    0.213
  5       SSQ   56.000
  6     MCSSQ   38.769
  7  Euc Norm    7.483
  8   Minimum    0.000
  9   Maximum    1.000
 10  N of Obs  182.000

Matrix: RDCON
  1      Mean    0.209
  2   Std Dev    0.406
... < snip >
```

- Use Tools|matrix algebra to transpose rows with levels

| | | RDGAM | RDCON | RDPOS | RDNEG | RDHLP | RDJOB |
|---|---|---|---|---|---|---|---|
| 1 | Mean | 0.308 | 0.209 | 0.143 | 0.209 | 0.132 | 0.269 |
| 2 | Std Dev | 0.462 | 0.406 | 0.350 | 0.406 | 0.338 | 1.827 |
| 3 | Sum | 56.000 | 38.000 | 26.000 | 38.000 | 24.000 | 49.000 |
| 4 | Variance | 0.213 | 0.165 | 0.122 | 0.165 | 0.114 | 3.340 |
| 5 | SSQ | 56.000 | 38.000 | 26.000 | 38.000 | 24.000 | 621.000 |
| 6 | MCSSQ | 38.769 | 30.066 | 22.286 | 30.066 | 20.835 | 607.808 |
| 7 | Euc Norm | 7.483 | 6.164 | 5.099 | 6.164 | 4.899 | 24.920 |
| 8 | Minimum | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | Maximum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 20.000 |
| 10 | N of Obs | 182.000 | 182.000 | 182.000 | 182.000 | 182.000 | 182.000 |

# Relational Composition

- Compositions of relations, such as a friend of a friend (F°F) or enemy of a friend (F°E) can be computed by boolean matrix multiplication

$$FE = F \times E$$

| | Mary | Bill | John | Larry |
|---|---|---|---|---|
| Mary | 0 | 1 | 0 | 1 |
| Bill | 1 | 0 | 1 | 0 |
| John | 0 | 0 | 0 | 1 |
| Larry | 0 | 0 | 0 | 0 |

F

| | Mary | Bill | John | Larry |
|---|---|---|---|---|
| Mary | 0 | 0 | 1 | 1 |
| Bill | 1 | 0 | 1 | 0 |
| John | 0 | 0 | 0 | 1 |
| Larry | 0 | 1 | 0 | 0 |

E

| | Mary | Bill | John | Larry |
|---|---|---|---|---|
| Mary | 1 | 1 | 1 | 0 |
| Bill | 0 | 0 | 0 | 1 |
| John | 0 | 1 | 0 | 0 |
| Larry | 0 | 0 | 0 | 0 |

FE

# Relational composition example (wiring dataset)

- Matrices
  - P = positive affect (rdpos)
  - C = conflict with (rdcon)
  - Construct matrix PC
- Interpretation
  - iPCj (i.e, cell $(pc)_{ij} > 0$ means that person i has positive feeling toward someone who has a conflict with j
- Uses
  - Predict that i will have negative affect toward j
  - Outdegree on PC relation predicts lower morale

# Another relational composition example

- Matrices
  - F = friends with
  - W = works with
  - FW = works with a friend of
    - $fw_{ij} > 0$ means that j works with a friend of i's (i has a friend who works with j)
    - $Fw_{ij} > 0$ means j is an indirect source of information for i
- Interpretation
  - High degree centrality on FW relation should mean the person has lots of access to organizational information

# An Improvement …

- Matrices
  - F = friends with
  - W = works with
  - FW = works with a friend of
    - $fw_{ij} > 0$ means that j works with a friend of i's (i has a friend who works with j)
    - $Fw_{ij} > 0$ means j is an indirect source of information for I
  - X = F+FW = is a friend of <u>or</u> works with a friend of
    - $x_{ij} > 0$ means j is either a friend of i or works with a friend of i
- Application
  - High degree centrality on X relation should mean the person has lots of access to organizational information

# Running analyses with multiple relations

- Two types
  - Automatically repeating analysis on each relation in turn
    - Works on many but not all ucinet procedures
  - Taking all relations into account simultaneously
    - Only for role analysis / blockmodeling techniques

# Automatically Repeating Analyses

- Example: eigenvector centrality on Wiring
  - Output is 3-dimensional matrix in which each matrix slice gives centralities for one relation

```
Matrix #1: RDGAM

        Eigenv nEigen
        ------ ------
  1 I1   0.307 43.368
  2 I3   0.000  0.000
  3 W1   0.417 58.960
  4 W2   0.365 51.669
  5 W3   0.417 58.960
  6 W4   0.417 58.960
  7 W5   0.323 45.718
  8 W6   0.029  4.070
  9 W7   0.085 12.011
 10 W8   0.033  4.719
 11 W9   0.033  4.719
 12 S1   0.368 52.043
 13 S2   0.000  0.000
 14 S4   0.029  4.070

-------------------------------------------
```

```
Matrix #2: RDCON

        Eigenv nEigen
        ------ ------
  1 I1   0.000  0.000
  2 I3   0.000  0.000
  3 W1   0.000  0.000
  4 W2   0.000  0.000
  5 W3   0.000  0.000
  6 W4   0.290 40.946
  7 W5   0.224 31.719
  8 W6   0.466 65.942
  9 W7   0.369 52.170
 10 W8   0.380 53.742
 11 W9   0.375 53.090
 12 S1   0.356 50.308
 13 S2   0.000 -0.000
 14 S4   0.317 44.828

        < SNIP >
```

# Displaying results for each relation side by side

- Having just run eigenvector centrality (which by default creates dataset "eigenvectorcentrality" as output) In matrix algebra:
  - > dsp transp(eigenvectorcentrality col lev)

| | | RDGAM | RDCON | RDPOS | RDNEG | RDHLP | RDJOB |
|---|---|---|---|---|---|---|---|
| 1 | I1 | 0.307 | 0.000 | 0.147 | 0.173 | 0.000 | 0.000 |
| 2 | I3 | 0.000 | 0.000 | -0.000 | 0.474 | 0.000 | 0.000 |
| 3 | W1 | 0.417 | 0.000 | 0.438 | 0.000 | -0.291 | -0.009 |
| 4 | W2 | 0.365 | 0.000 | -0.000 | 0.261 | -0.214 | -0.185 |
| 5 | W3 | 0.417 | 0.000 | 0.472 | 0.000 | -0.328 | 0.000 |
| 6 | W4 | 0.417 | 0.290 | 0.438 | 0.114 | -0.374 | 0.000 |
| 7 | W5 | 0.323 | 0.224 | 0.000 | 0.485 | -0.078 | -0.224 |
| 8 | W6 | 0.029 | 0.466 | -0.000 | 0.310 | -0.417 | 0.000 |
| 9 | W7 | 0.085 | 0.369 | 0.254 | 0.360 | -0.298 | -0.064 |
| 10 | W8 | 0.033 | 0.380 | 0.159 | 0.287 | -0.323 | -0.641 |
| 11 | W9 | 0.033 | 0.375 | 0.159 | 0.287 | -0.322 | 0.000 |
| 12 | S1 | 0.368 | 0.356 | 0.498 | 0.114 | -0.192 | -0.102 |
| 13 | S2 | 0.000 | -0.000 | 0.000 | 0.114 | -0.100 | 0.000 |
| 14 | S4 | 0.029 | 0.317 | 0.099 | 0.112 | -0.315 | -0.700 |