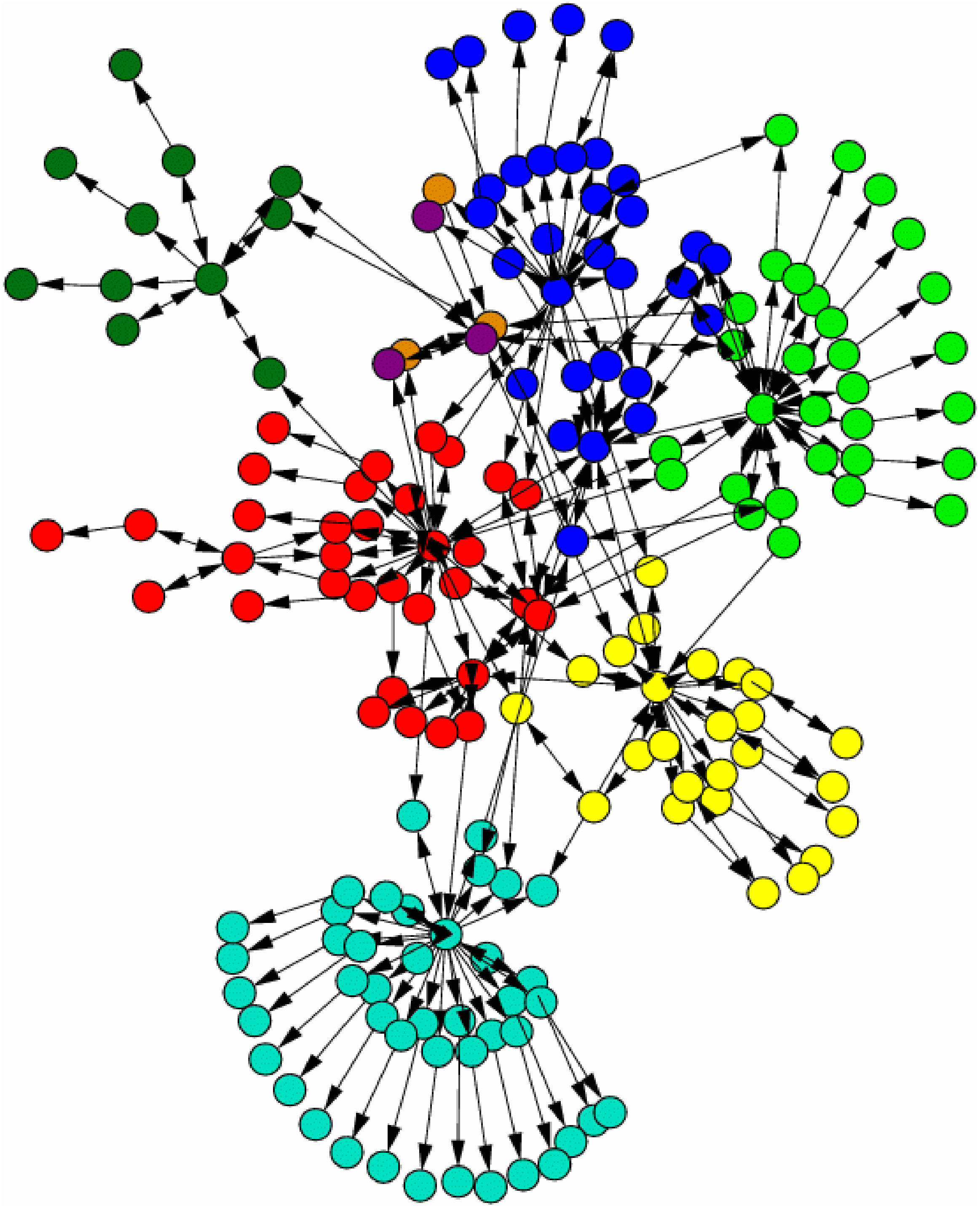
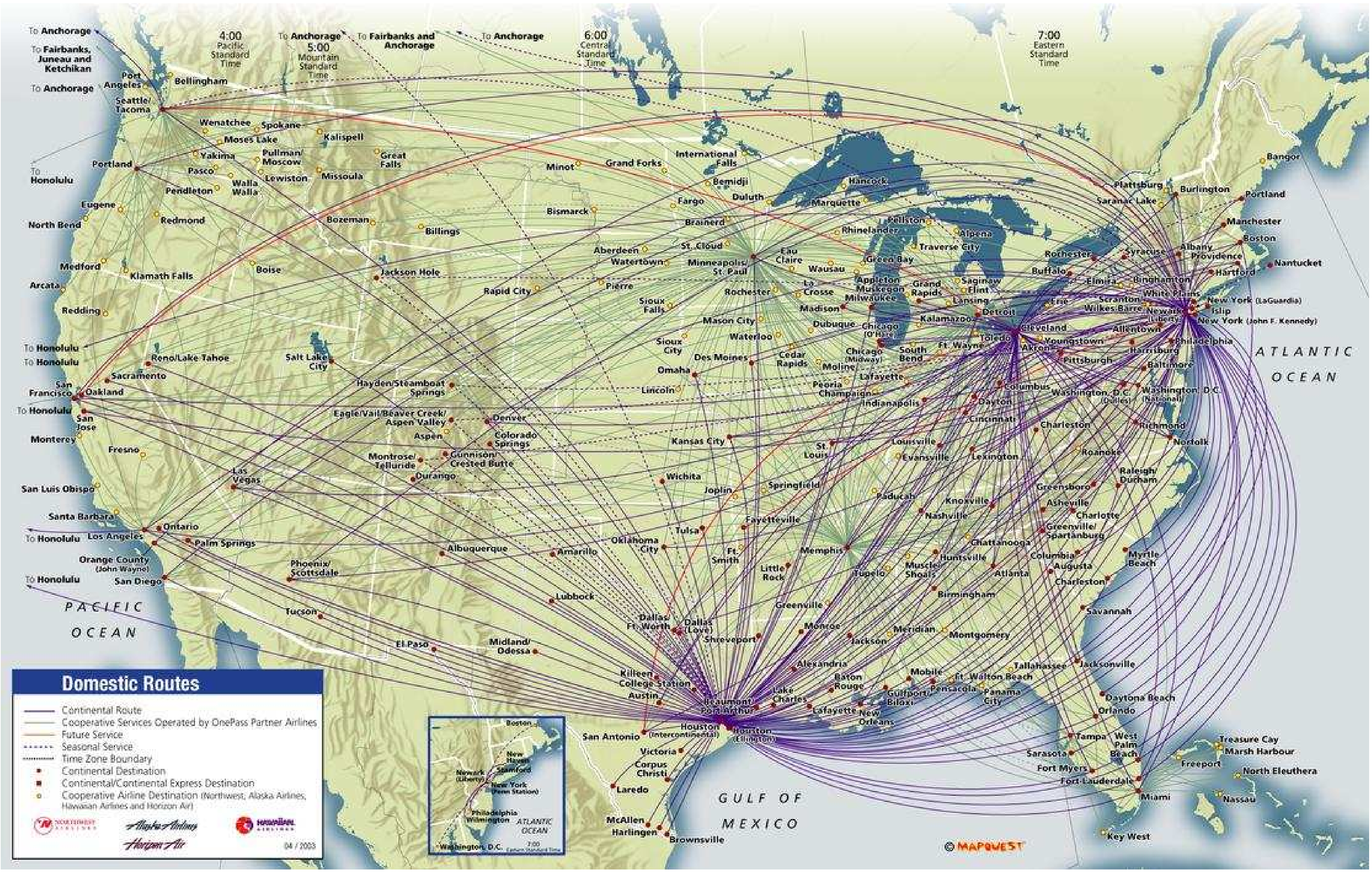
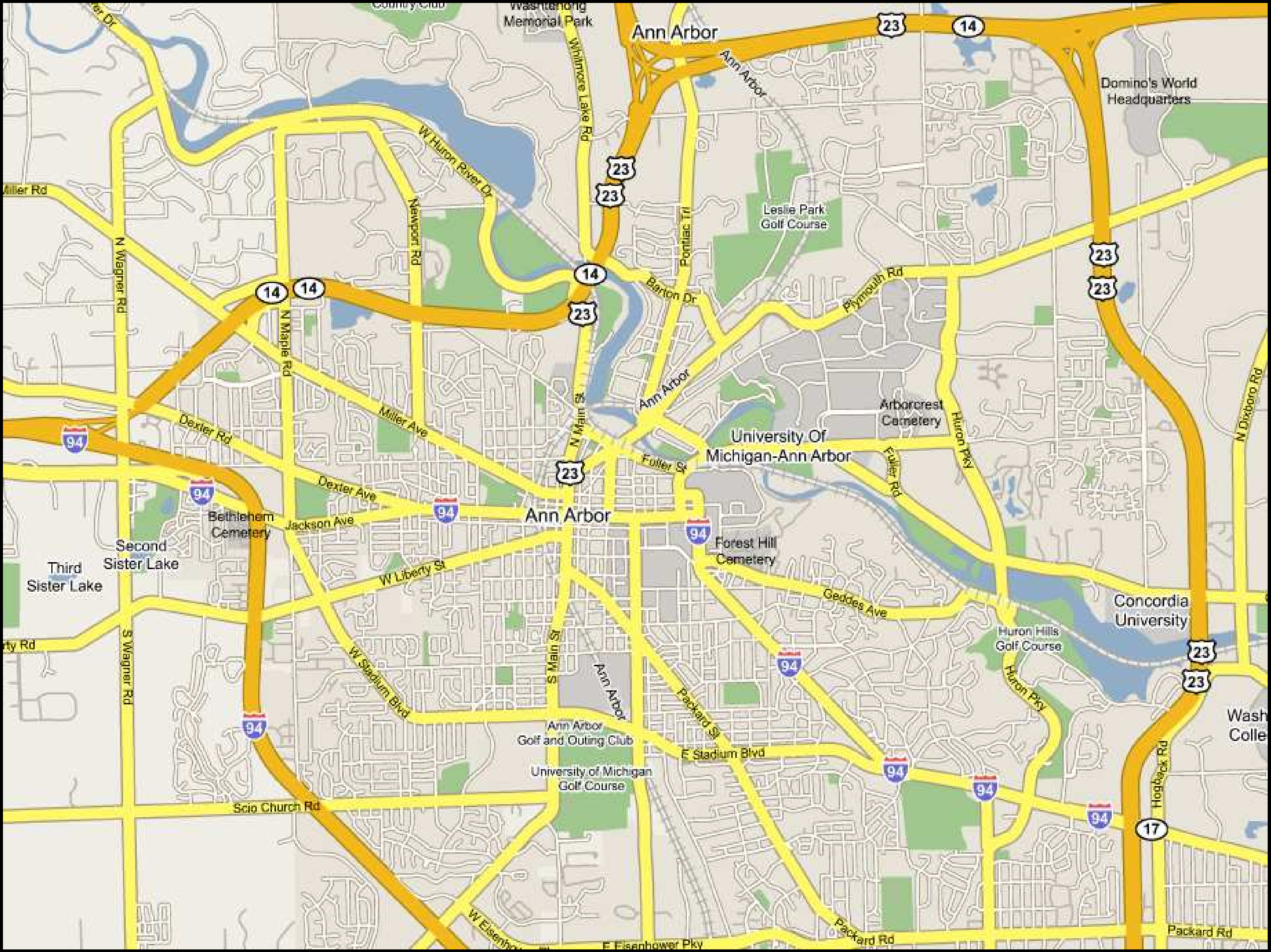
Networks in Space

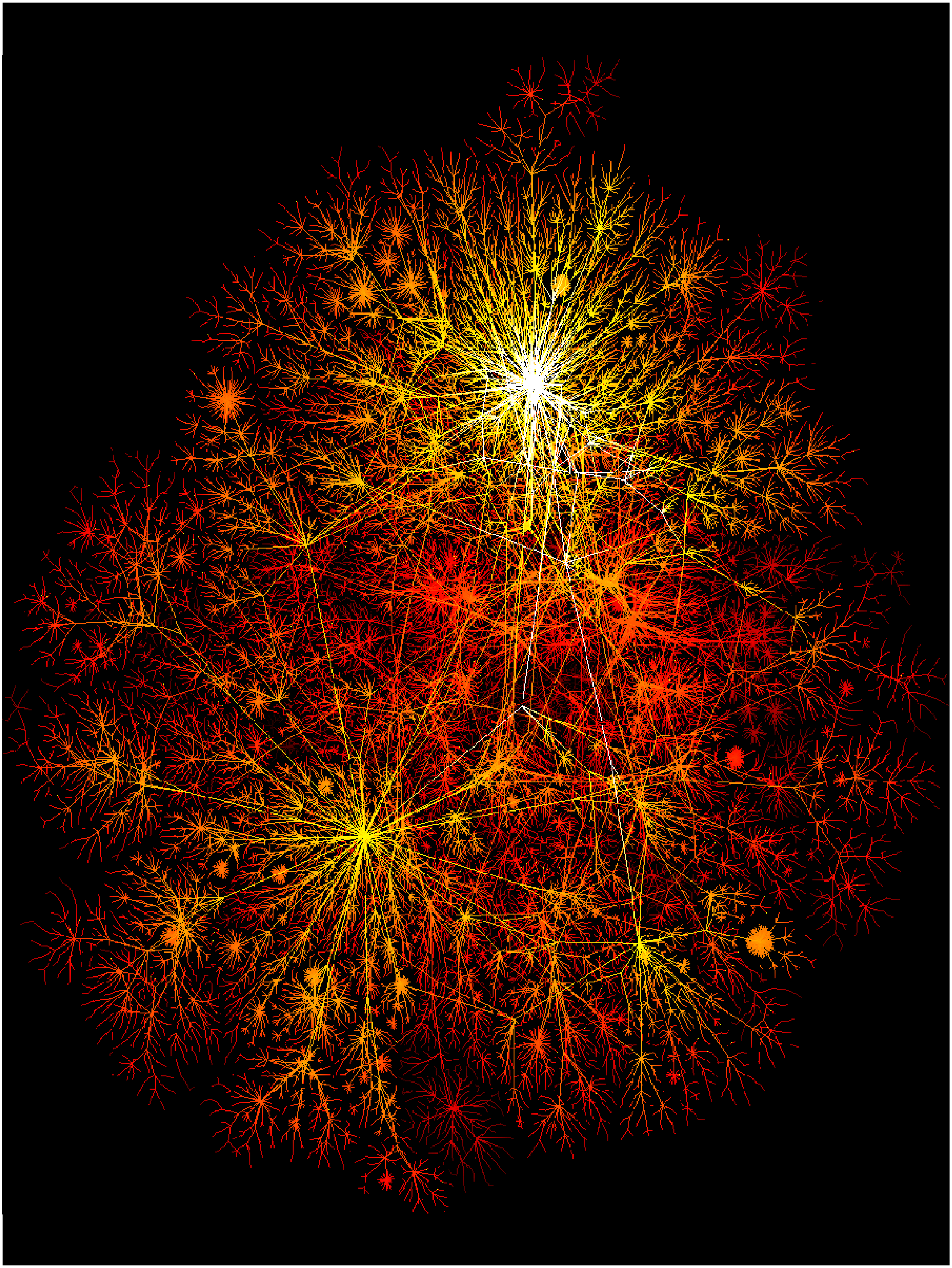
Mark Newman
Michael Gastner

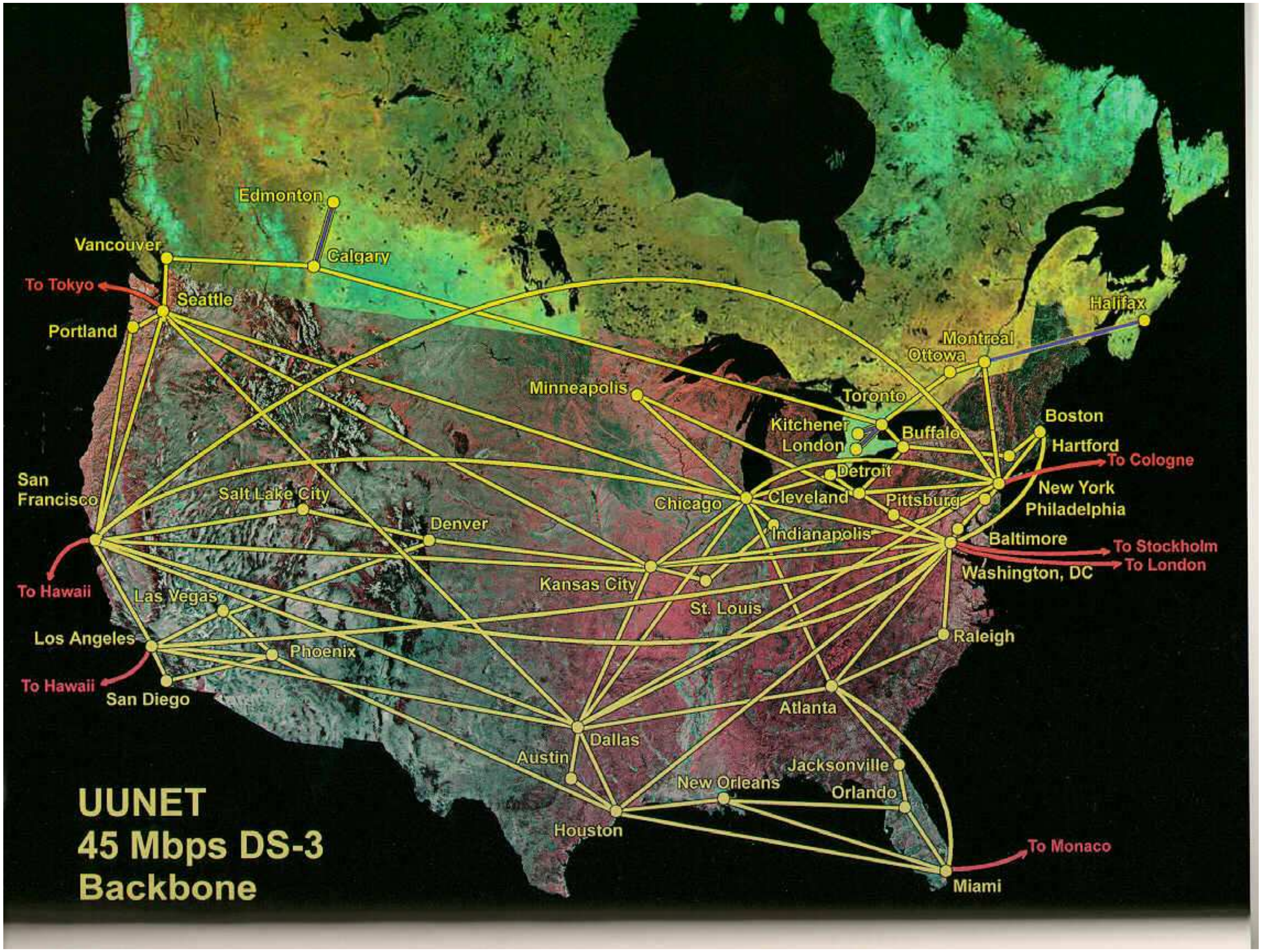
University of Michigan




Domestic Routes
Continental Route
Cooperative Services Operated by OnePass Partner Airlines
Future Service
Seasonal Service
Time Zone Boundary
Continental Destination
Continental/Continental Express Destination
Cooperative Airline Destination (Northwest, Alaska Airlines, Hawaiian Airlines and Horizon Air)
Northwest Airlines
Alaska Airlines
Horizon Air
Hawaiian Airlines
04 / 2003
4:00 Pacific Standard Time
5:00 Mountain Standard Time
6:00 Central Standard Time
7:00 Eastern Standard Time
To Anchorage
To Fairbanks, Juneau and Ketchikan
To Anchorage
To Fairbanks and Anchorage
To Anchorage
To Honolulu
To Honolulu
To Honolulu
To Honolulu
To Honolulu
To Honolulu
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
Port Angeles
Bellingham
Seattle/Tacoma
Wenatchee
Spokane
Moses Lake
Kalispell
Yakima
Pullman/Moscow
Great Falls
Portland
Pasco
Lewiston
Missoula
Walla Walla
Pendleton
Eugene
Redmond
North Bend
Bozeman
Billings
Medford
Klamath Falls
Boise
Jackson Hole
Arcata
Redding
Reno/Lake Tahoe
Salt Lake City
Sacramento
San Francisco
Oakland
San Jose
Monterey
Fresno
San Luis Obispo
Santa Barbara
Los Angeles
Ontario
Palm Springs
Orange County (John Wayne)
San Diego
Las Vegas
Phoenix/Scottsdale
Tucson
El Paso
Hayden/Steamboat Springs
Eagle/Vail/Beaver Creek/Aspen Valley
Aspen
Denver
Colorado Springs
Montrose/Telluride
Gunnison/Crested Butte
Durango
Albuquerque
Amarillo
Lubbock
Midland/Odessa
Rapid City
Minot
Bismarck
Grand Forks
Aberdeen
Watertown
Pierre
Fargo
International Falls
Bemidji
Duluth
Brainerd
St. Cloud
Minneapolis/St. Paul
Sioux Falls
Sioux City
Omaha
Lincoln
Des Moines
Mason City
Waterloo
Rochester
Eau Claire
Wausau
La Crosse
Madison
Dubuque
Cedar Rapids
Moline
Peoria
Champaign
Kansas City
Wichita
Joplin
Springfield
Oklahoma City
Tulsa
Fayetteville
Ft. Smith
Little Rock
Dallas/Ft. Worth
Dallas (Love)
Shreveport
Killeen
College Station
Austin
San Antonio
Houston (Intercontinental)
Houston (Ellington)
Beaumont/Port Arthur
Victoria
Corpus Christi
Laredo
McAllen
Harlingen
Brownsville
Hancock
Marquette
Rhinelander
Pellston
Alpena
Traverse City
Green Bay
Appleton
Muskegon
Grand Rapids
Saginaw
Flint
Lansing
Milwaukee
Kalamazoo
Detroit
Chicago (O'Hare)
Chicago (Midway)
South Bend
Ft. Wayne
Toledo
Lafayette
Indianapolis
St. Louis
Louisville
Evansville
Paducah
Memphis
Tupelo
Greenville
Monroe
Jackson
Meridian
Montgomery
Alexandria
Baton Rouge
Lake Charles
Lafayette
New Orleans
Gulfport/Biloxi
Mobile
Pensacola
Ft. Walton Beach
Panama City
Tallahassee
Nashville
Knoxville
Muscle Shoals
Huntsville
Chattanooga
Birmingham
Atlanta
Cleveland
Akron
Youngstown
Pittsburgh
Erie
Columbus
Dayton
Cincinnati
Lexington
Charleston
Roanoke
Greensboro
Asheville
Charlotte
Greenville/Spartanburg
Columbia
Augusta
Charleston
Myrtle Beach
Raleigh/Durham
Savannah
Jacksonville
Daytona Beach
Orlando
Tampa
Sarasota
Fort Myers
West Palm Beach
Fort Lauderdale
Miami
Key West
Treasure Cay
Marsh Harbour
Freeport
North Eleuthera
Nassau
Rochester
Buffalo
Syracuse
Elmira
Binghamton
Scranton
Wilkes-Barre
Allentown
Harrisburg
Philadelphia
Baltimore
Washington, D.C. (Dulles)
Washington, D.C. (National)
Richmond
Norfolk
Plattsburg
Saranac Lake
Burlington
Albany
Providence
Hartford
White Plains
Newark (Liberty)
New York (LaGuardia)
Islip
New York (John F. Kennedy)
Bangor
Portland
Manchester
Boston
Nantucket
Boston
New Haven
Stamford
Newark (Liberty)
New York (Penn Station)
Philadelphia
Wilmington
ATLANTIC OCEAN
Washington, D.C.
7:00 Eastern Standard Time
© MAPQUEST


Country Club
Washtenong Memorial Park
Ann Arbor
23
14
Domino's World Headquarters
Ann Arbor
Whitmore Lake Rd
W Huron River Dr
Miller Rd
N Wagner Rd
Newport Rd
Leslie Park Golf Course
Pontiac Trl
Barton Dr
Plymouth Rd
N Maple Rd
Ann Arbor
Arborcrest Cemetery
Huron Pky
N Dixboro Rd
Miller Ave
N Main St
Dexter Rd
University Of Michigan-Ann Arbor
Fuller St
Fuller Rd
94
Dexter Ave
Ann Arbor
Bethlehem Cemetery
Jackson Ave
Forest Hill Cemetery
Second Sister Lake
Third Sister Lake
W Liberty St
Geddes Ave
Concordia University
rty Rd
S Wagner Rd
Huron Hills Golf Course
S Main St
Ann Arbor
W Stadium Blvd
Packard St
Huron Pky
Wash
Colle
Ann Arbor Golf and Outing Club
E Stadium Blvd
University of Michigan Golf Course
Hogback Rd
Scio Church Rd
17
W Eisenho
E Eisenhower Pky
Packard Rd
Packard Rd



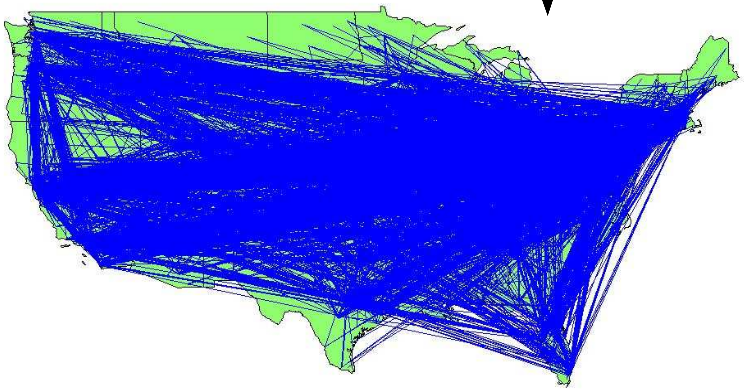


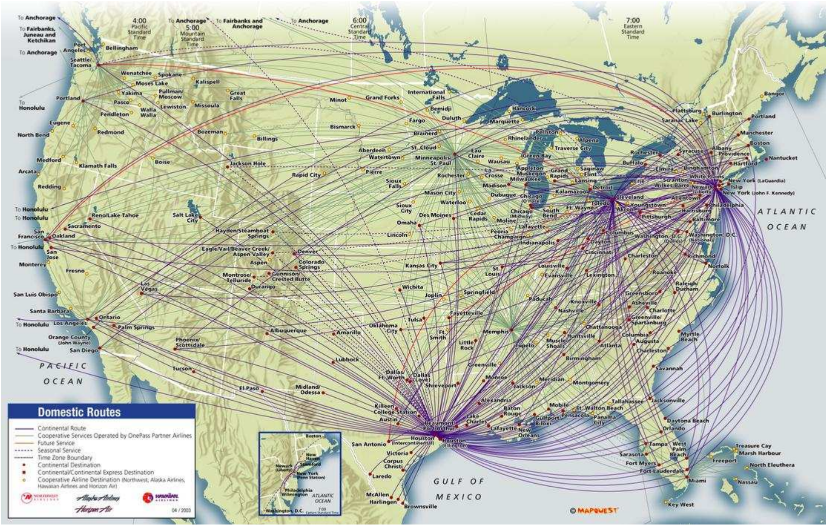
UUNET
45 Mbps DS-3
Backbone

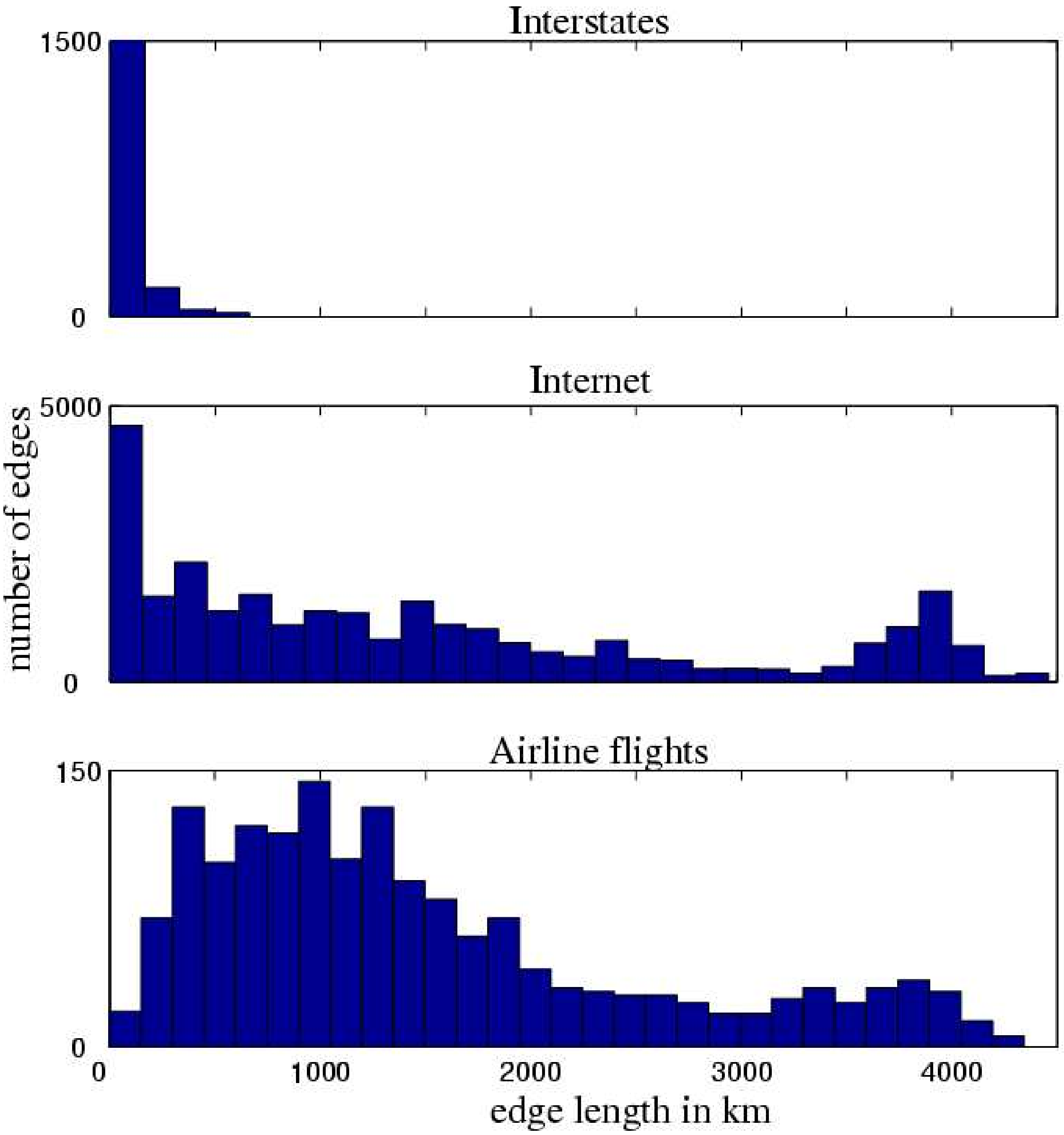
- US Interstate highways

- Passenger airline

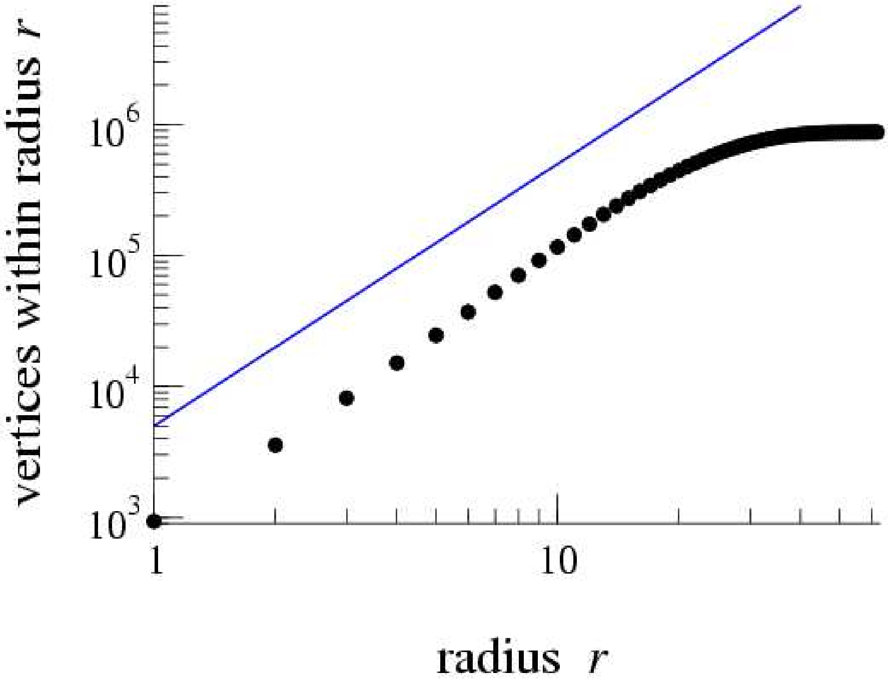
- US portion of the Internet

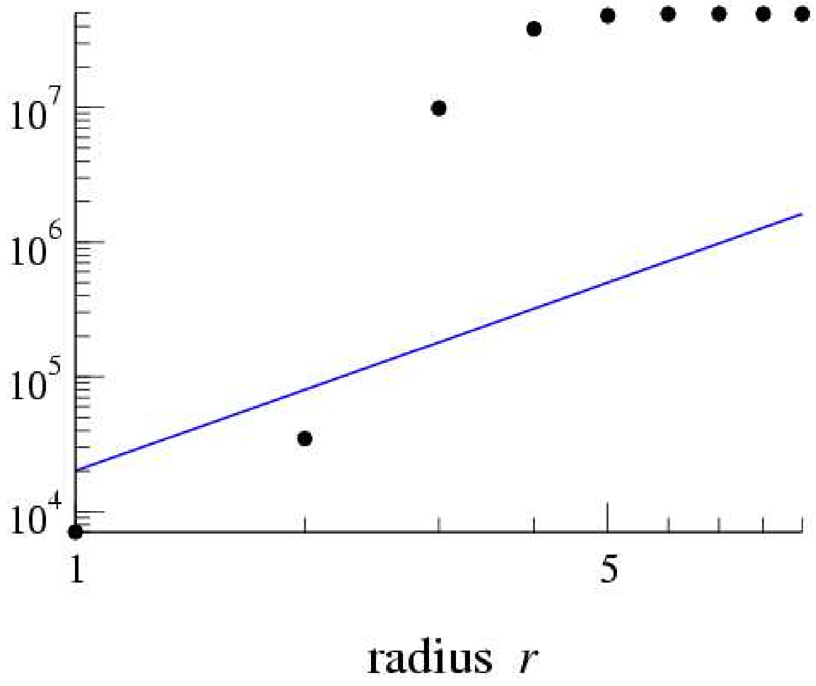


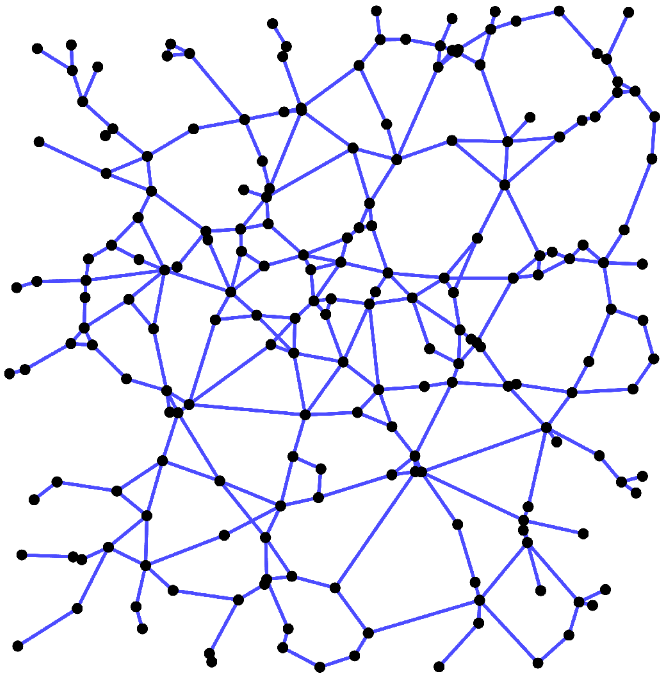


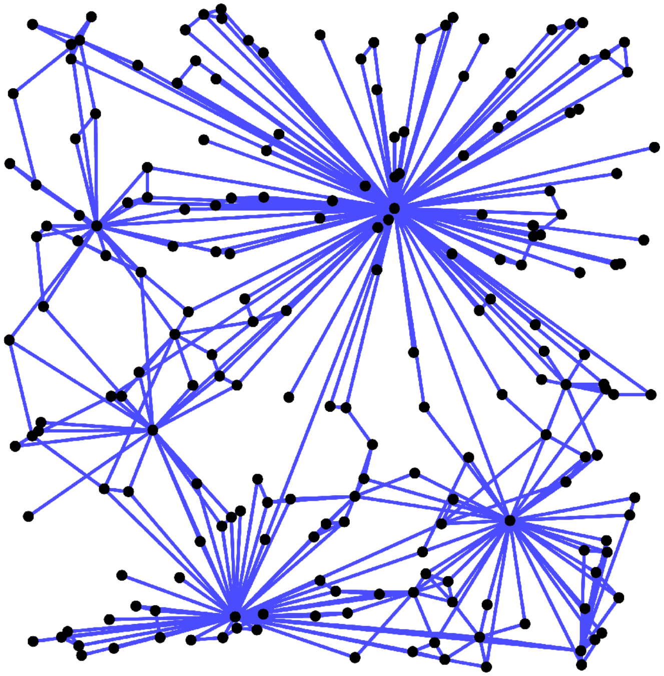

Interstates
1500
0
Internet
5000
number of edges
0
Airline flights
150
0
0
1000
2000
3000
4000
edge length in km

Effective dimension: measures the volume of a neighborhood as a function of radius









	Interstates	model
vertices	935	200
average degree (avg. number of edges connected to a vertex)	2.9	2.9
maximum degree (max. number of edges connected to a vertex)	4 (0.4% of the network)	7 (3.5% of the network)
diameter (max # of edges between two vertices	61 $\propto \sqrt{\text{number of vertices}}$	21
planar?	No, but only 9 pairs of edges cross	No, but only 4 pairs of edges cross

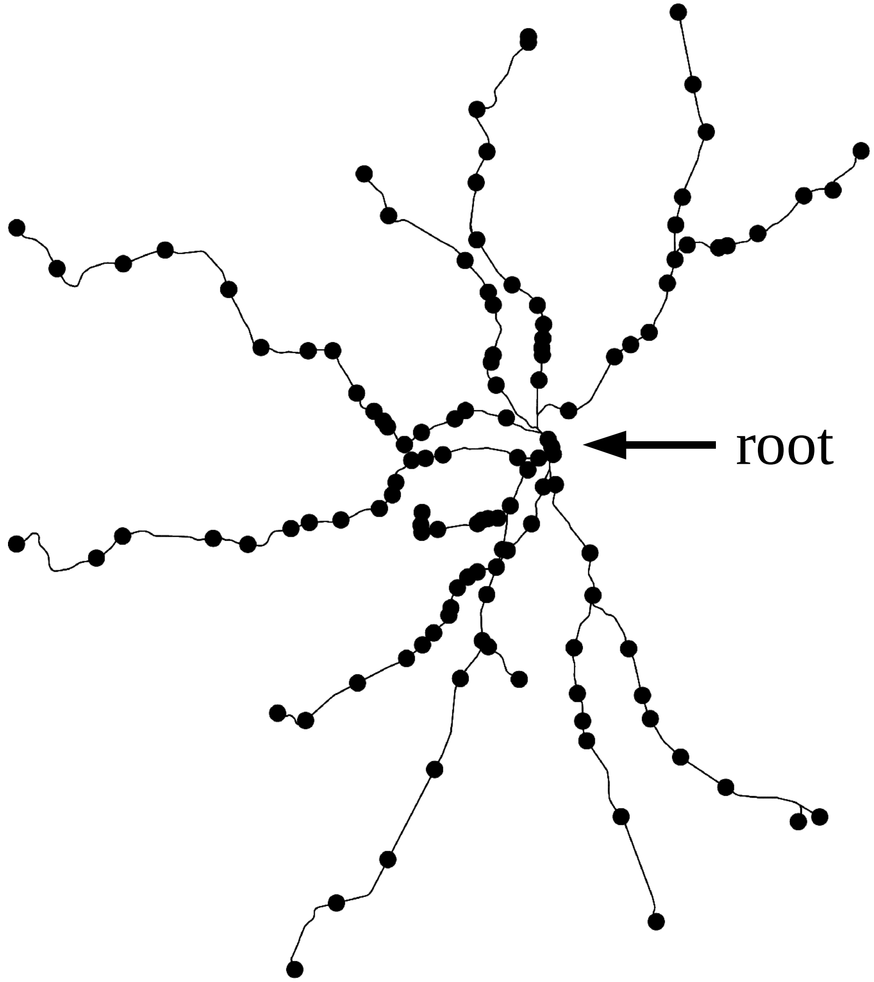
	airline	model
vertices	187	200
average degree (avg. number of edges connected to a vertex)	8.8	8.8
maximum degree (max. number of edges connected to a vertex)	141 (75% of the network)	143 (72% of the network)
diameter (max # of edges between two vertices	3	4
planar?	No.	No.

Distribution and collection networks

- There is a “root vertex” acting as a source or sink of the commodity distributed, e.g., oil, trains, sewage

- Vertices are households, businesses, train stations.

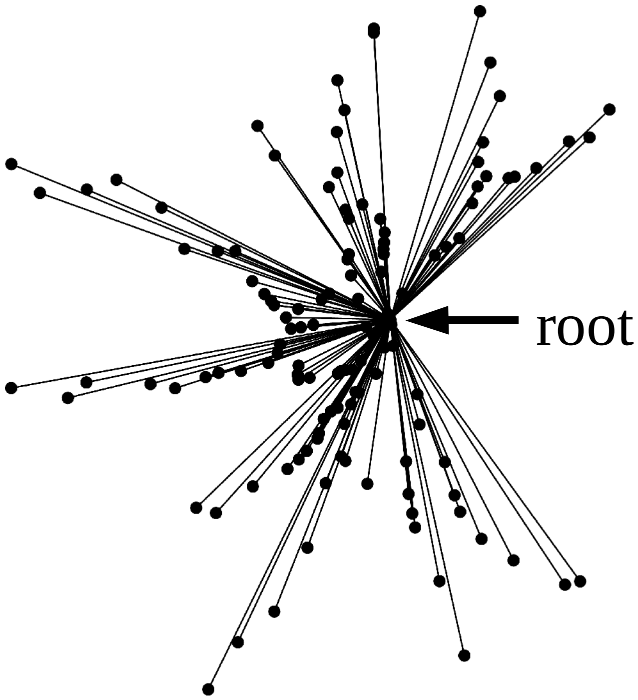
- Edges: pipes, tracks, roads, cables.



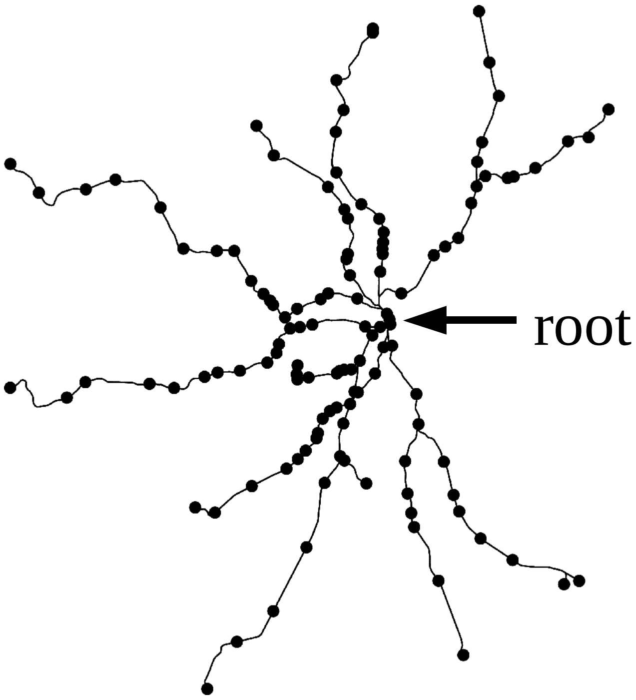
Boston commuter train network

Cost of building the network

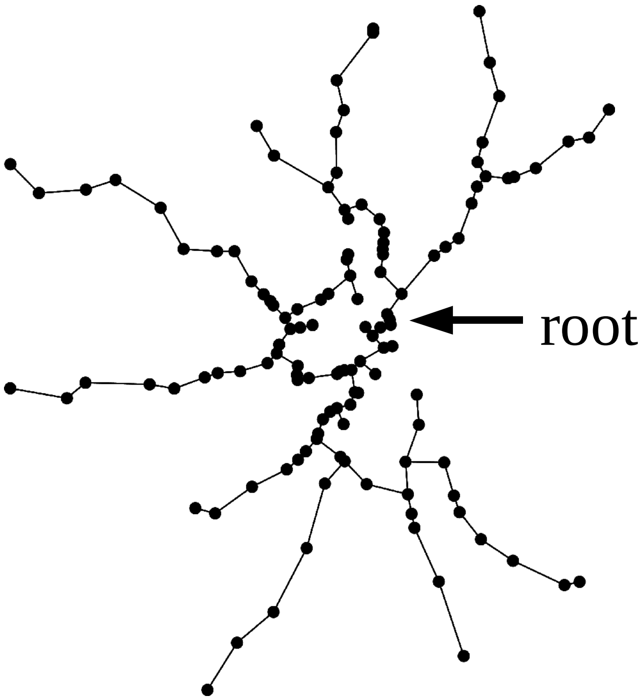
Cost is assumed proportional to the sum of the lengths of the edges



star graph



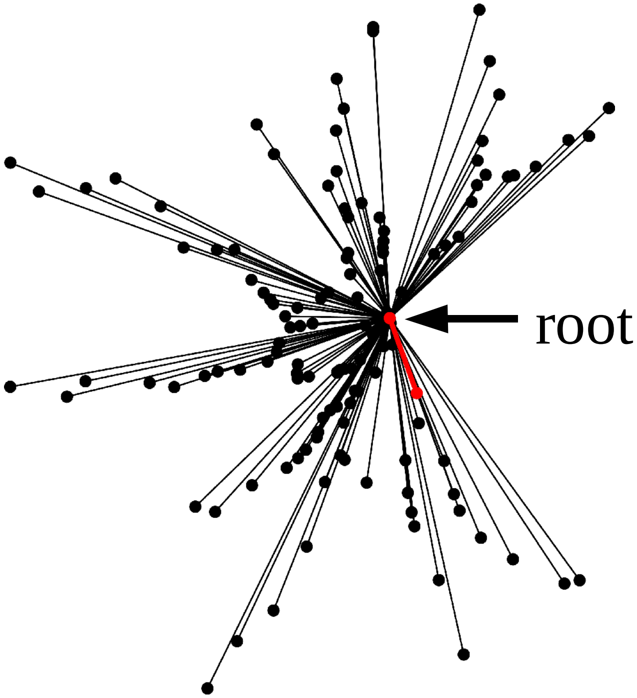
real network



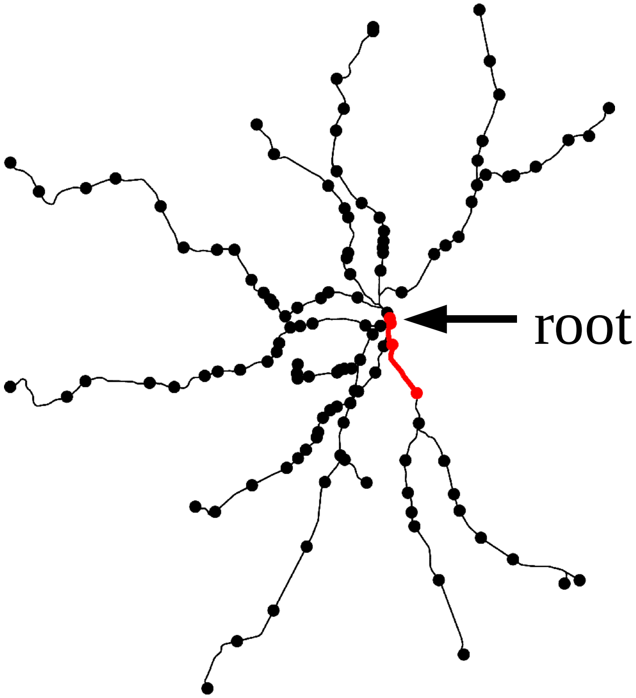
minimum spanning tree
(MST)

Efficiency

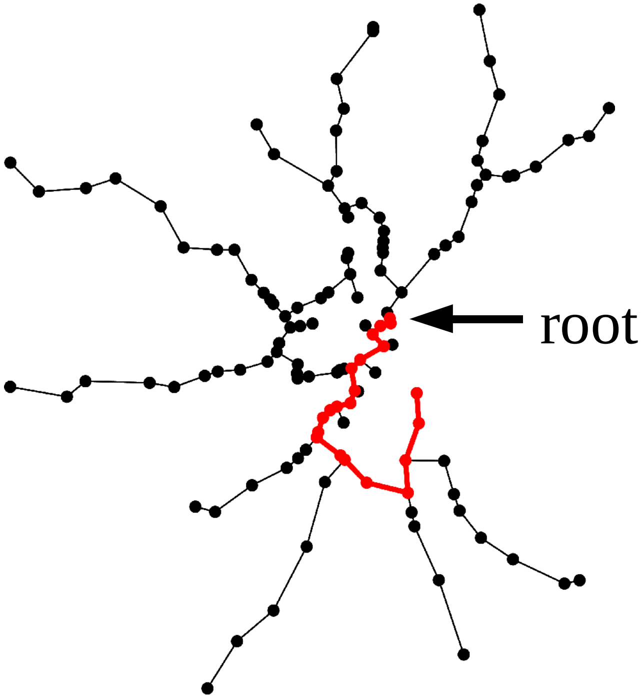
Paths to the root should be straight, so that journeys are efficient



star graph



real network



minimum spanning tree
(MST)

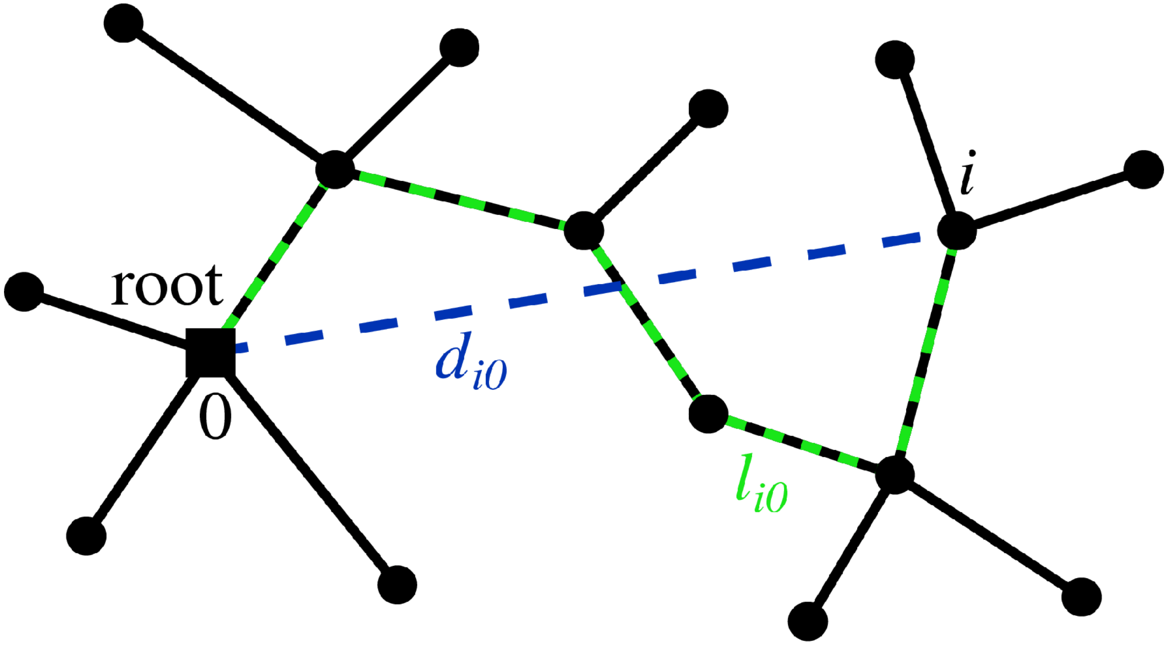
Route factor

The route factor for vertex *i* compares actual and ideal path length:

$$\boxed{q_i = \frac{l_{i0}}{d_{i0}}}$$

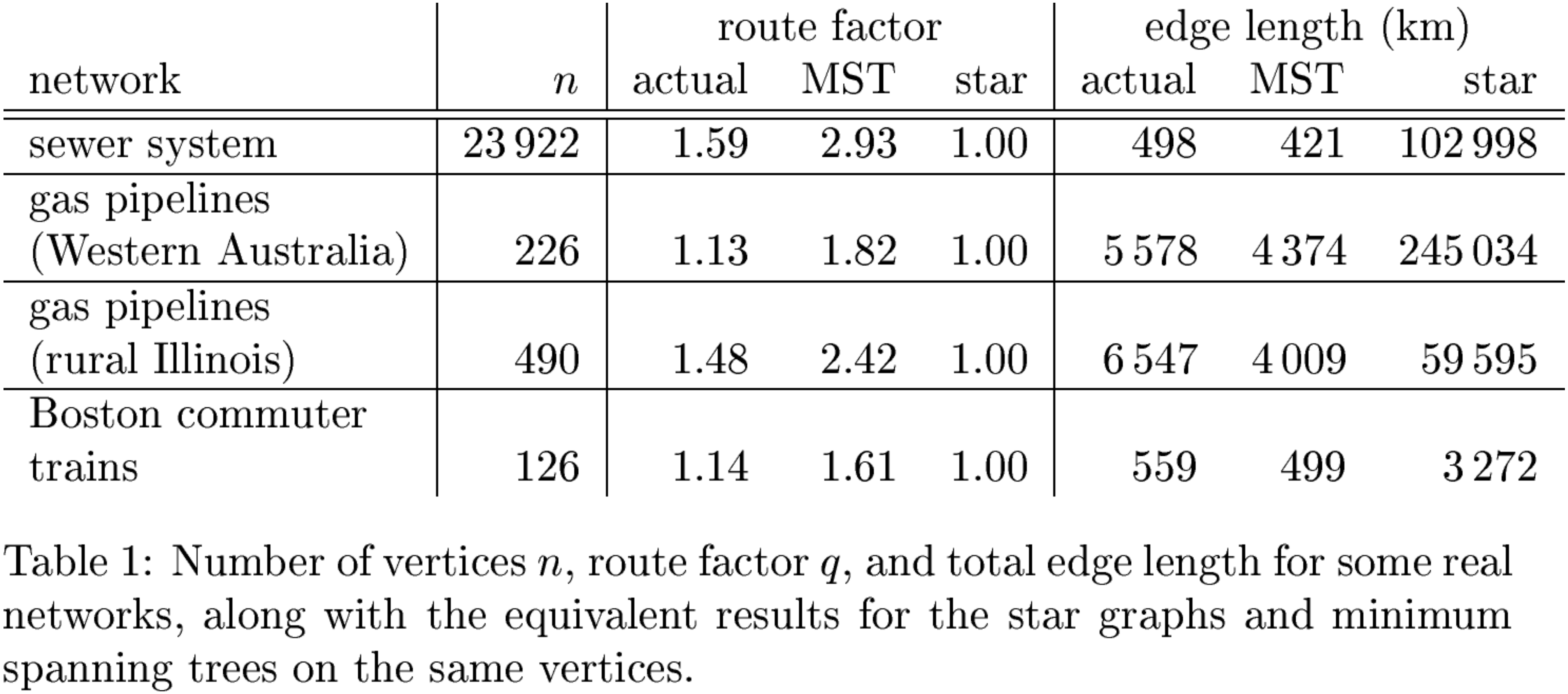
where l_{i0} is the distance along the edges of the network from vertex *i* to the root and d_{i0} is the direct Euclidean distance.

Example:



$q_i = 1.53$

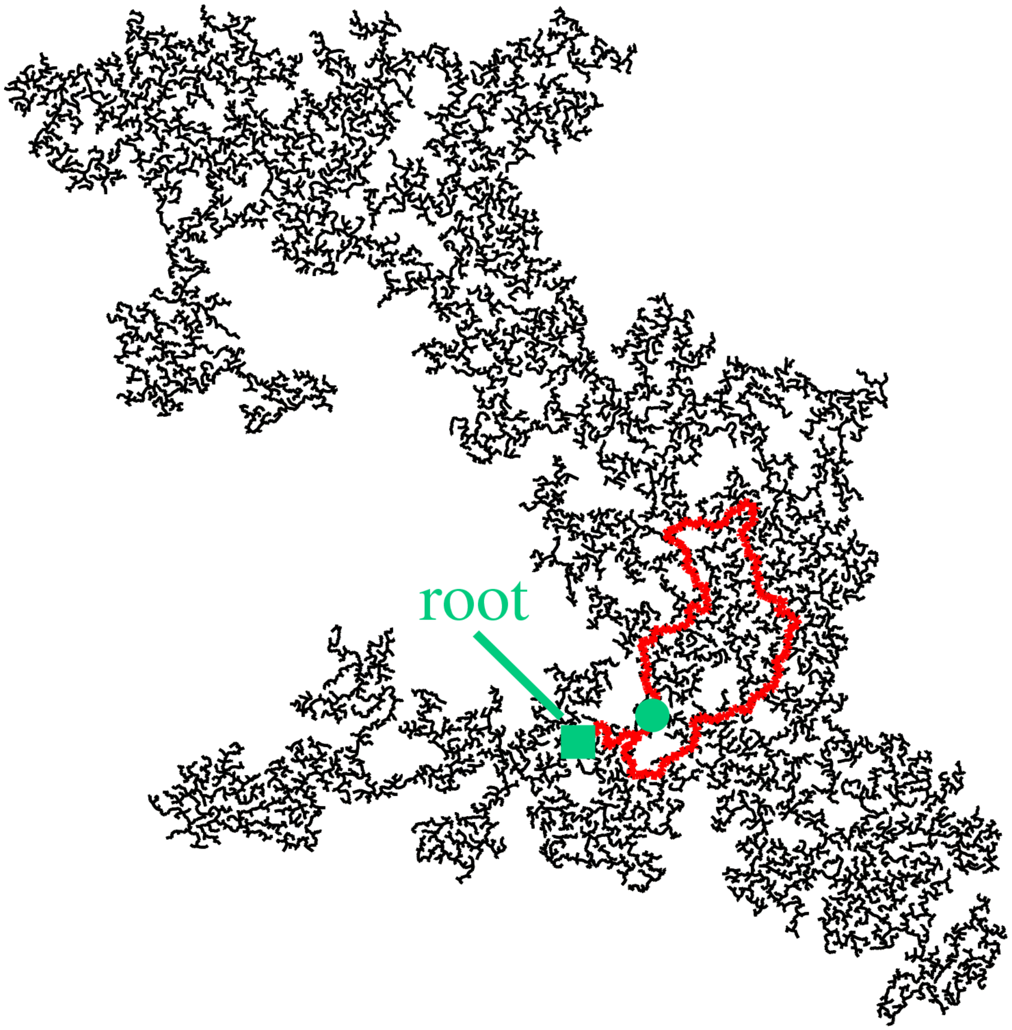
Real networks are close to ideal



network	n	route factor			edge length (km)		
		actual	MST	star	actual	MST	star
sewer system	23 922	1.59	2.93	1.00	498	421	102 998
gas pipelines (Western Australia)	226	1.13	1.82	1.00	5 578	4 374	245 034
gas pipelines (rural Illinois)	490	1.48	2.42	1.00	6 547	4 009	59 595
Boston commuter trains	126	1.14	1.61	1.00	559	499	3 272

Table 1: Number of vertices n, route factor q, and total edge length for some real networks, along with the equivalent results for the star graphs and minimum spanning trees on the same vertices.

Network growth model



Improving the route factor

Define a weight for each possible edge between vertices i (unconnected) and j (connected): $w_{ij} = d_{ij} + \beta l_{j0}$ with

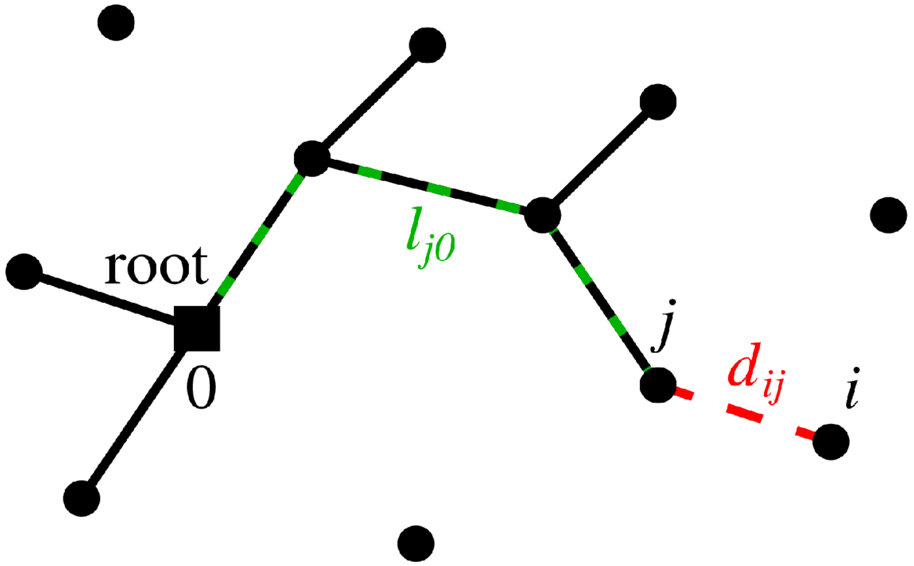
length of new edge

distance to root

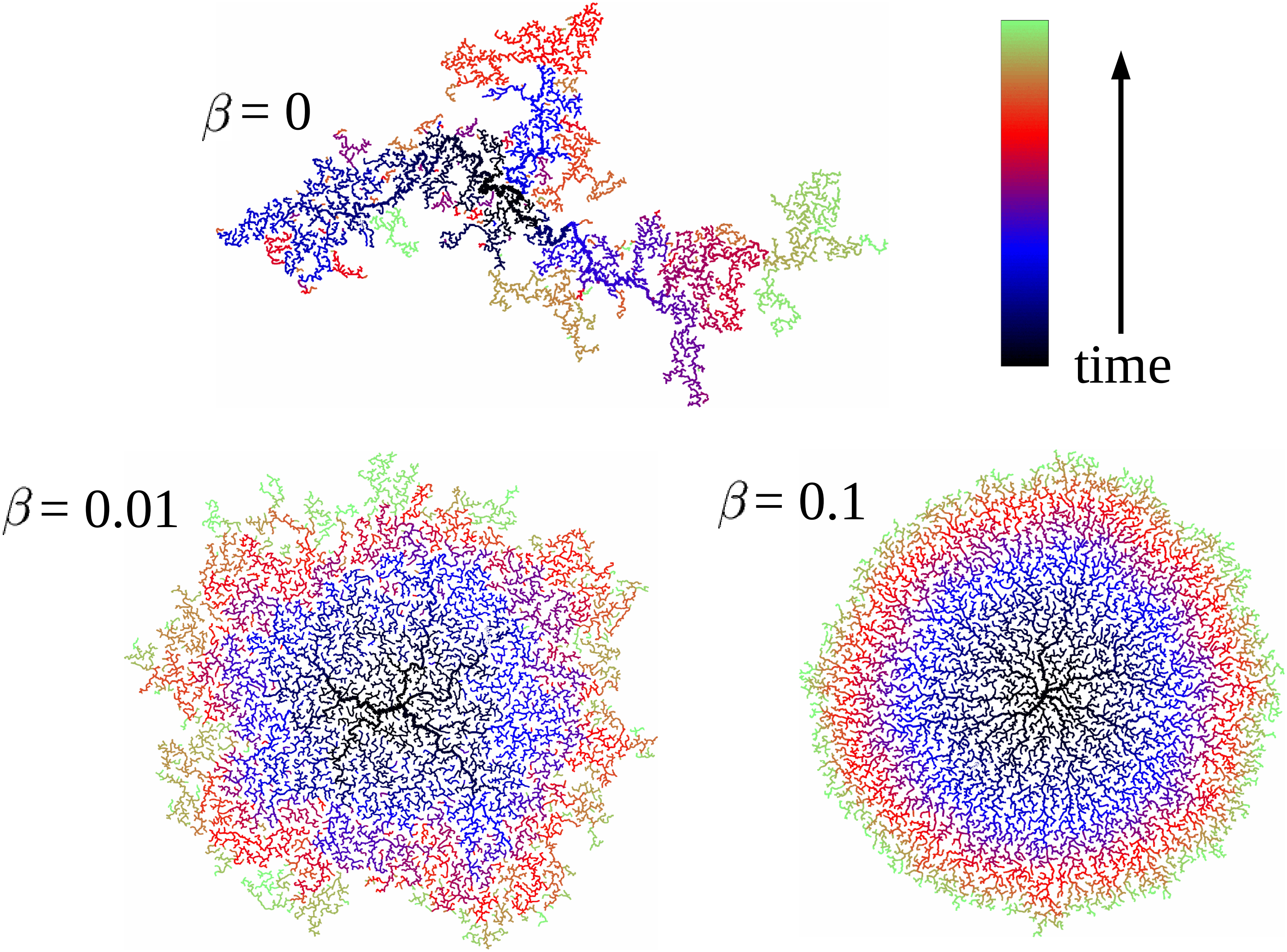
d_{ij} : length of edge between i and j,

l_{j0} : distance to the root along the shortest path through network,

β : non-negative parameter.



Add the edge with the global minimum weight.


$\beta = 0$
time
$\beta = 0.01$
$\beta = 0.1$

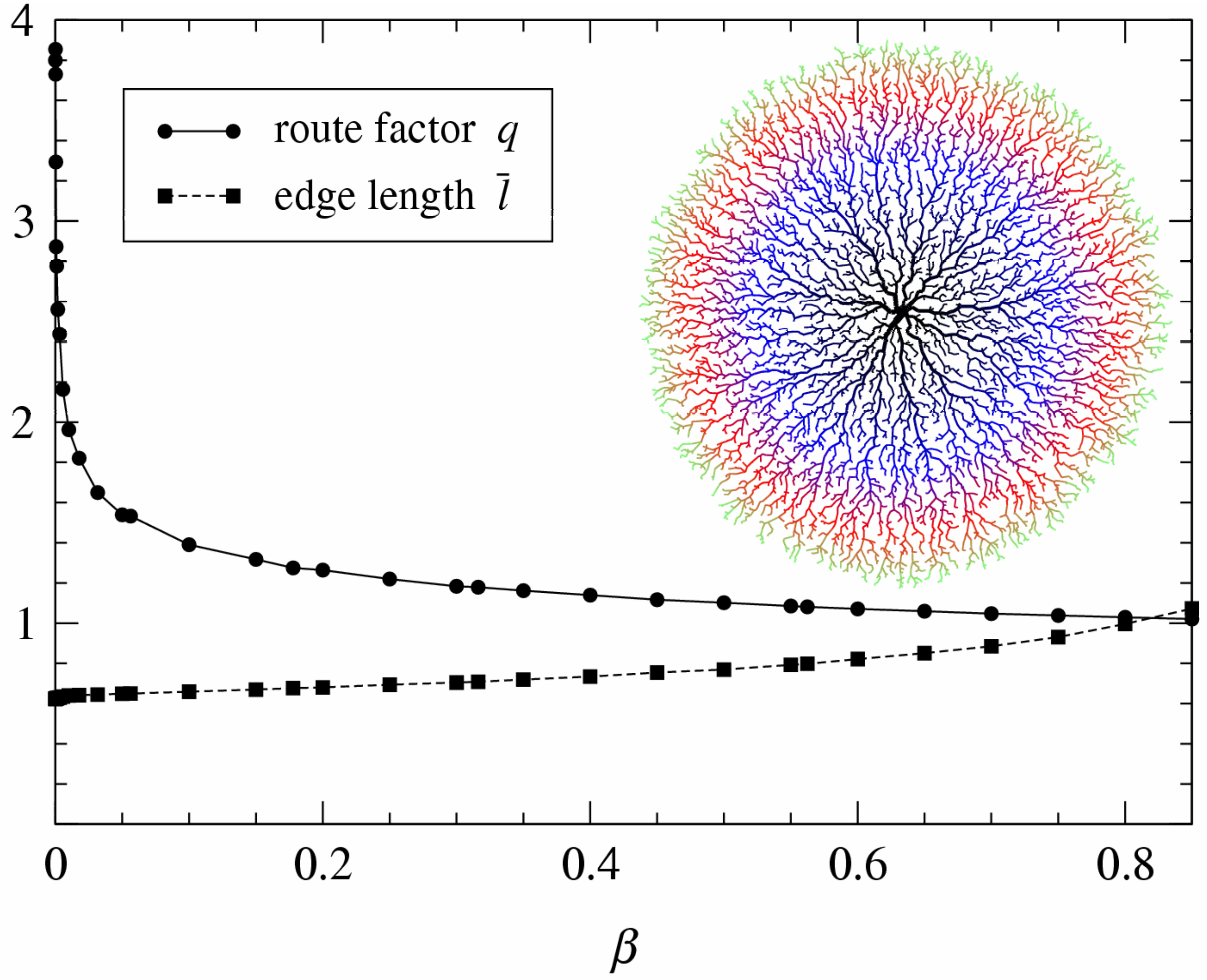
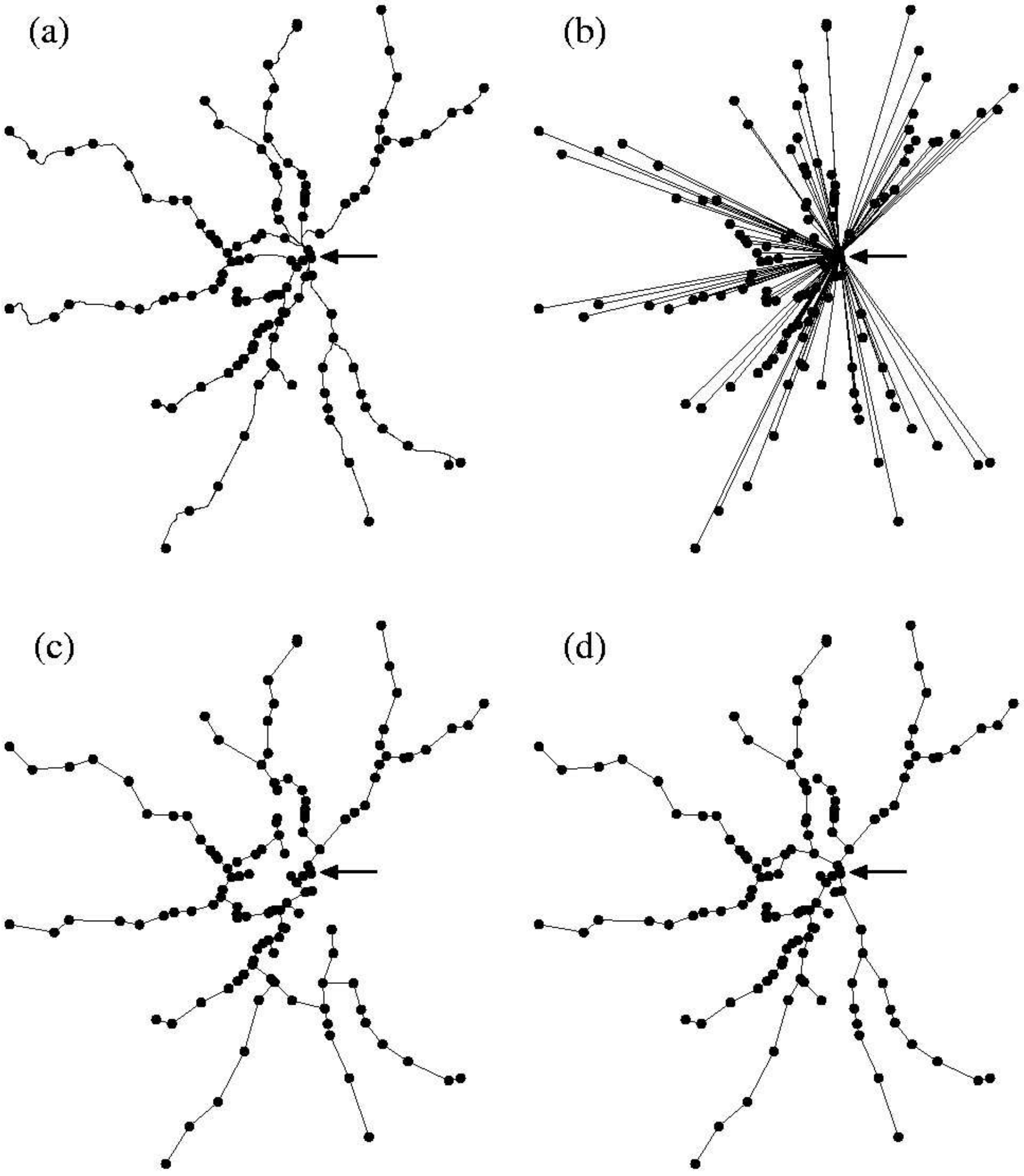


Figure 1: Route factor q and average edge length $\bar{l}$ as a function of β for our second model ($n = 10\,000$). Inset: an example model network with $\beta = 0.4$.

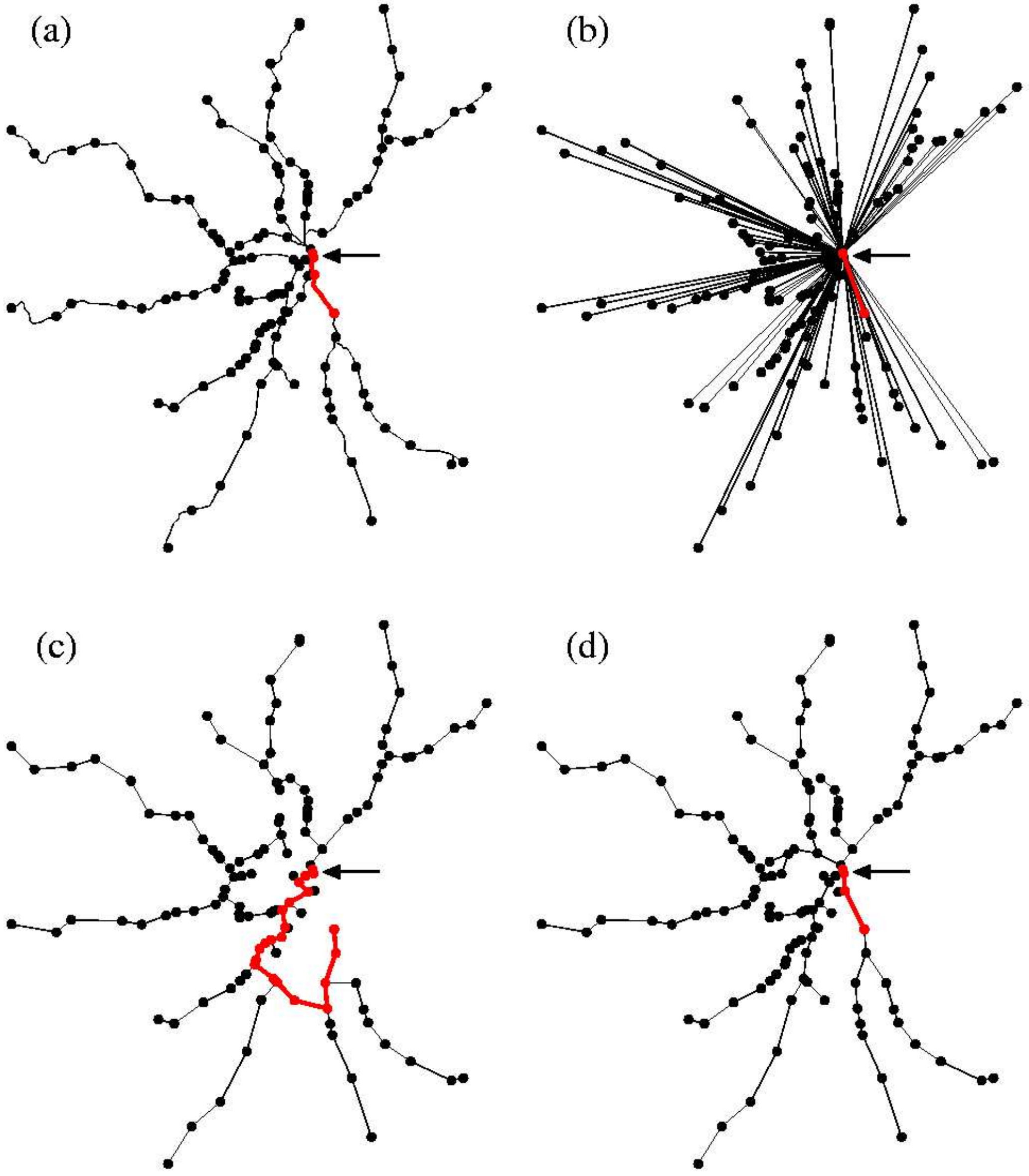


(a) MBTA
commuter
trains

(b) star graph

(c) Minimum
spanning
tree

(d) model
($\beta = 0.4$)



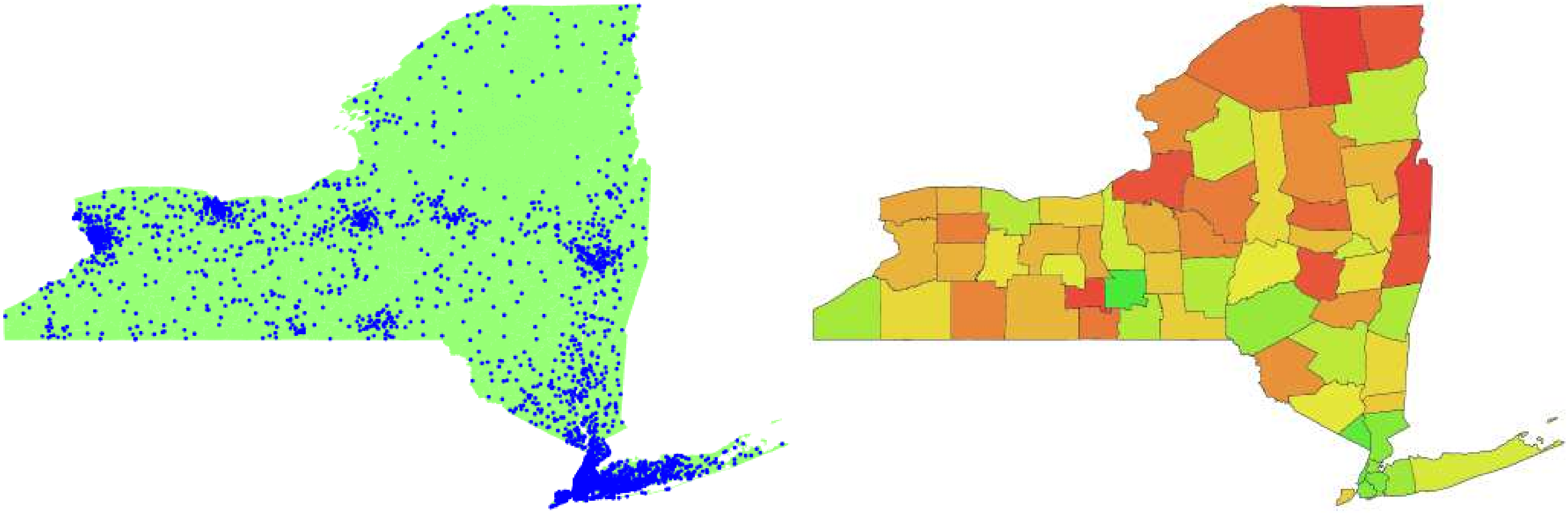
(a) MBTA
commuter
trains

(b) star graph

(c) Minimum
spanning
tree

(d) model
($\beta = 0.4$)

Cartograms



Lung cancer cases among the male population of the state of New York

1993 to 1997

The diffusion cartogram

We need a process that moves population away from high-density areas into low-density ones until everything is uniform.

$$\mathbf{J} = \mathbf{v}(\mathbf{r},t)\,\rho(\mathbf{r},t) \qquad \text{and} \qquad \mathbf{J} = -\nabla\rho,$$

The diffusing population is conserved locally:

$$\nabla \cdot \mathbf{J} + \frac{\partial\rho}{\partial t} = 0.$$

Hence

$$\nabla^2\rho - \frac{\partial\rho}{\partial t} = 0 \qquad \text{and} \qquad \mathbf{v}(\mathbf{r},t) = -\frac{\nabla\rho}{\rho}.$$

Express population density as a discrete cosine transform:

$$\rho(\mathbf{r},t) = \frac{4}{L_x L_y} \sum_{\mathbf{k}} \tilde{\rho}(\mathbf{k}) \cos(k_x x) \cos(k_y y) \exp(-k^2 t),$$

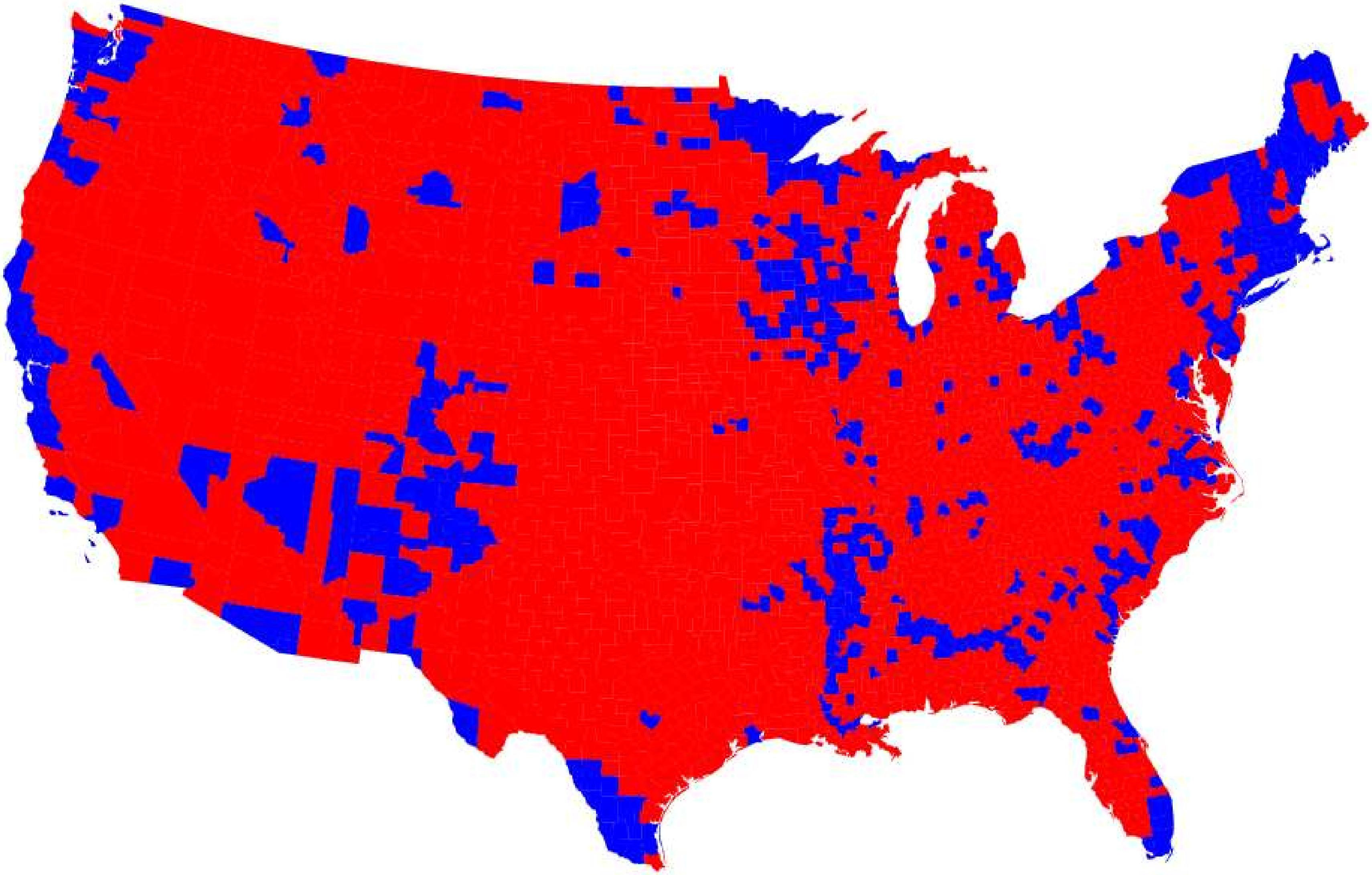
Then the components of the velocity are given by

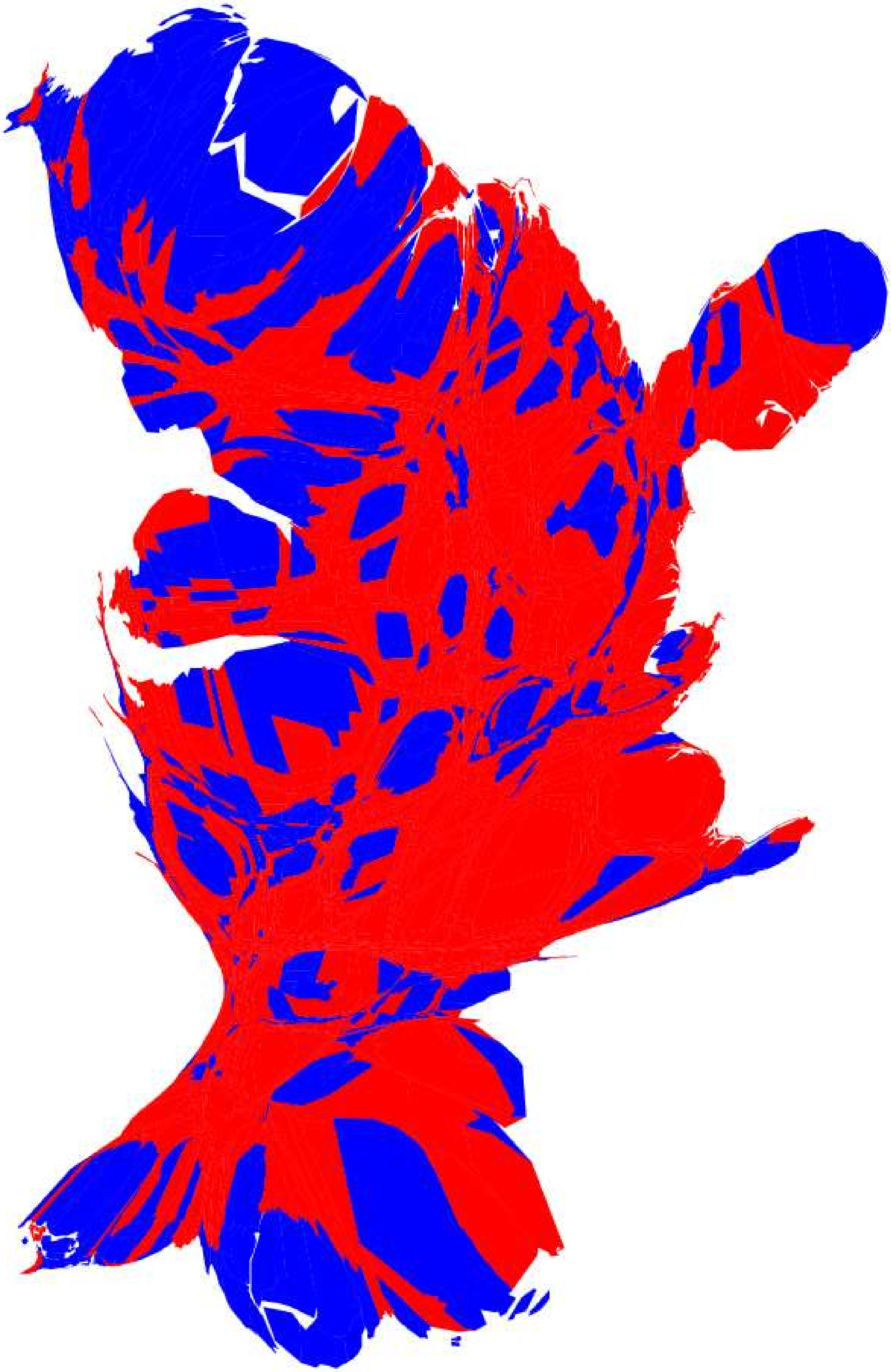
$$\begin{aligned} v_x(\mathbf{r},t) &= \frac{\sum_{\mathbf{k}} k_x \tilde{\rho}(\mathbf{k}) \sin(k_x x) \cos(k_y y) \exp(-k^2 t)}{\sum_{\mathbf{k}} \tilde{\rho}(\mathbf{k}) \cos(k_x x) \cos(k_y y) \exp(-k^2 t)}, \\ v_y(\mathbf{r},t) &= \frac{\sum_{\mathbf{k}} k_y \tilde{\rho}(\mathbf{k}) \cos(k_x x) \sin(k_y y) \exp(-k^2 t)}{\sum_{\mathbf{k}} \tilde{\rho}(\mathbf{k}) \cos(k_x x) \cos(k_y y) \exp(-k^2 t)}. \end{aligned}$$

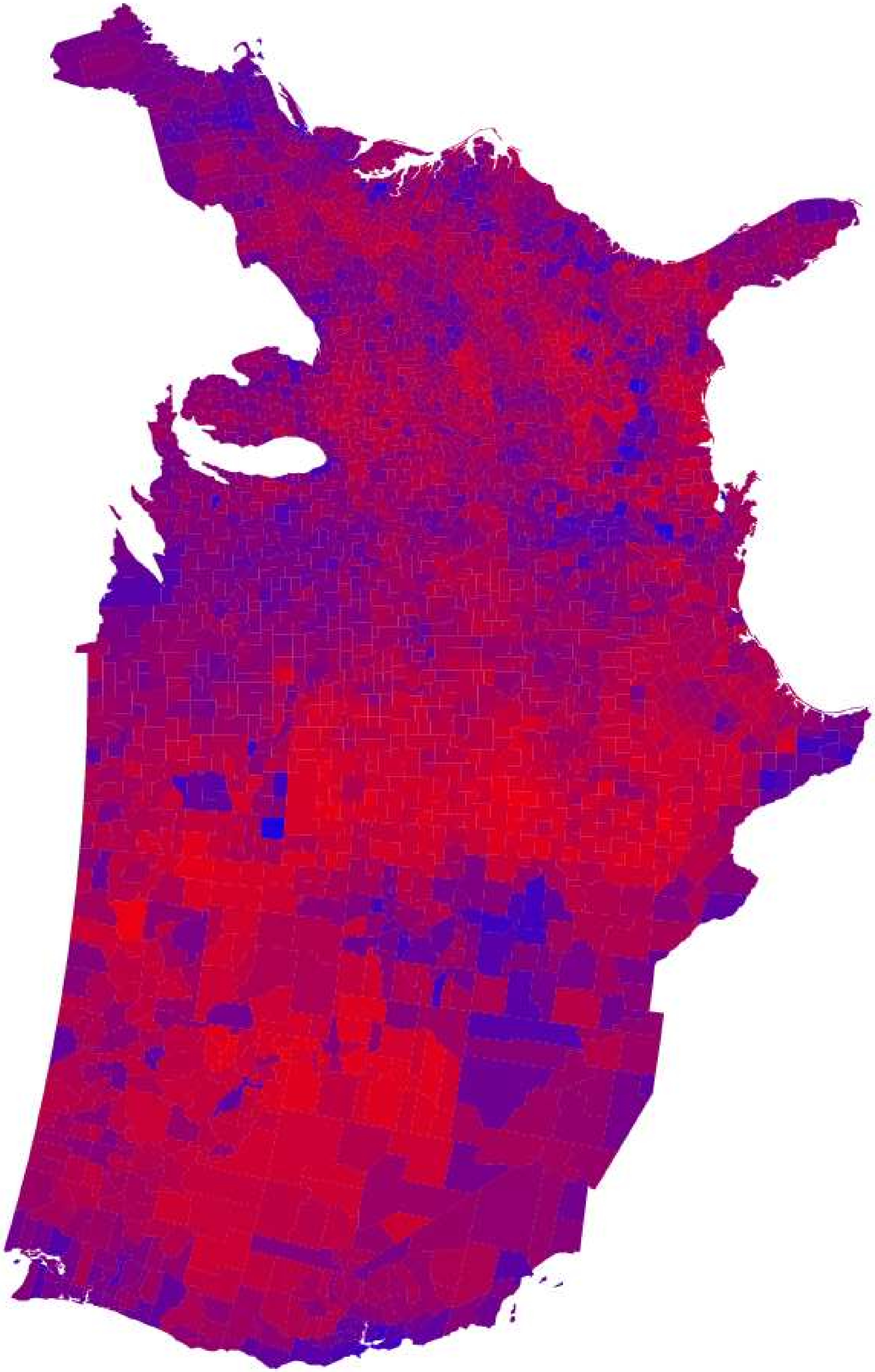
And the cartogram is defined by

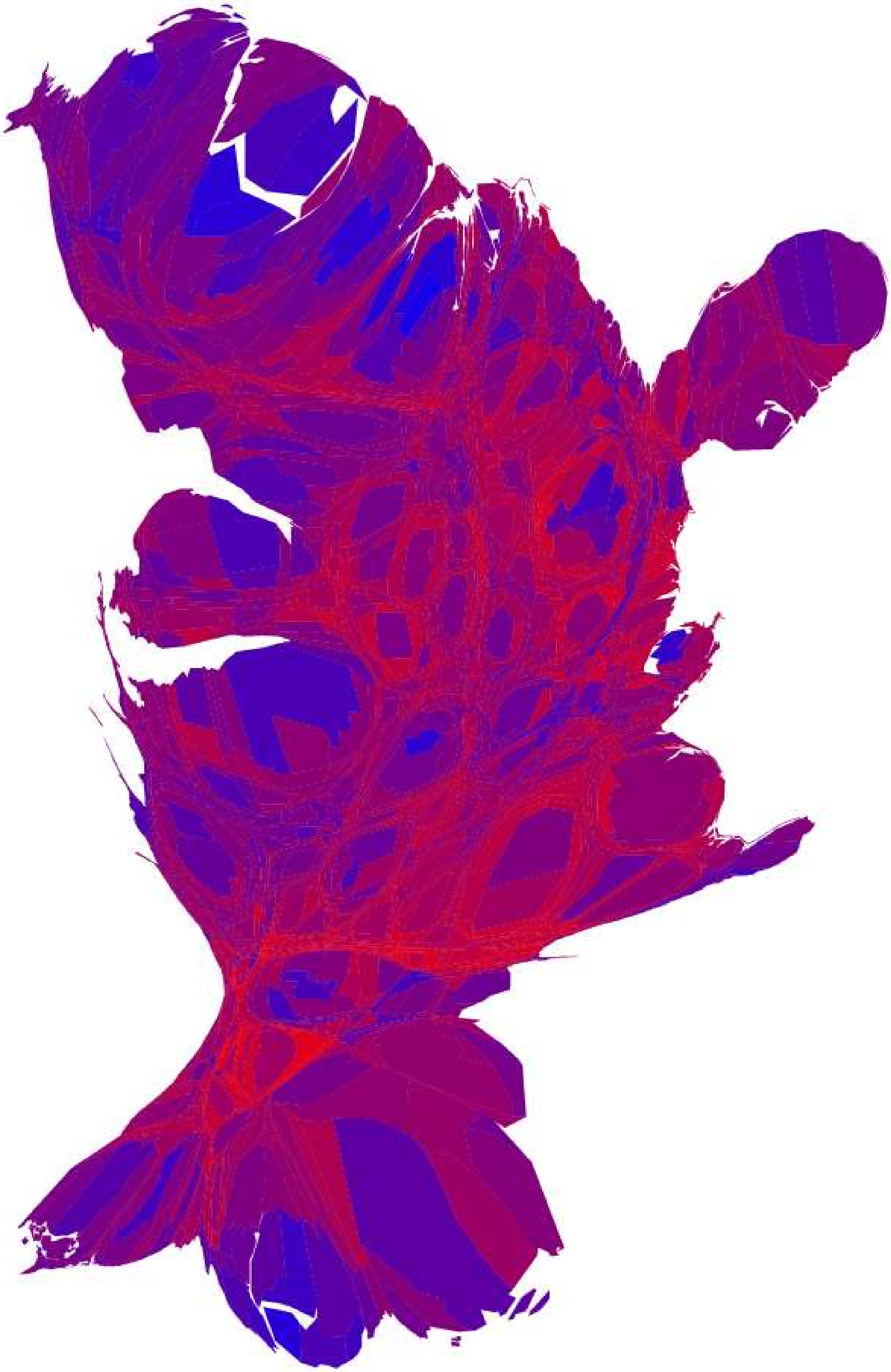
$$\mathbf{r}(t) = \mathbf{r}(0) + \int_0^t \mathbf{v}(\mathbf{r},t')\,\mathrm{d}t',$$

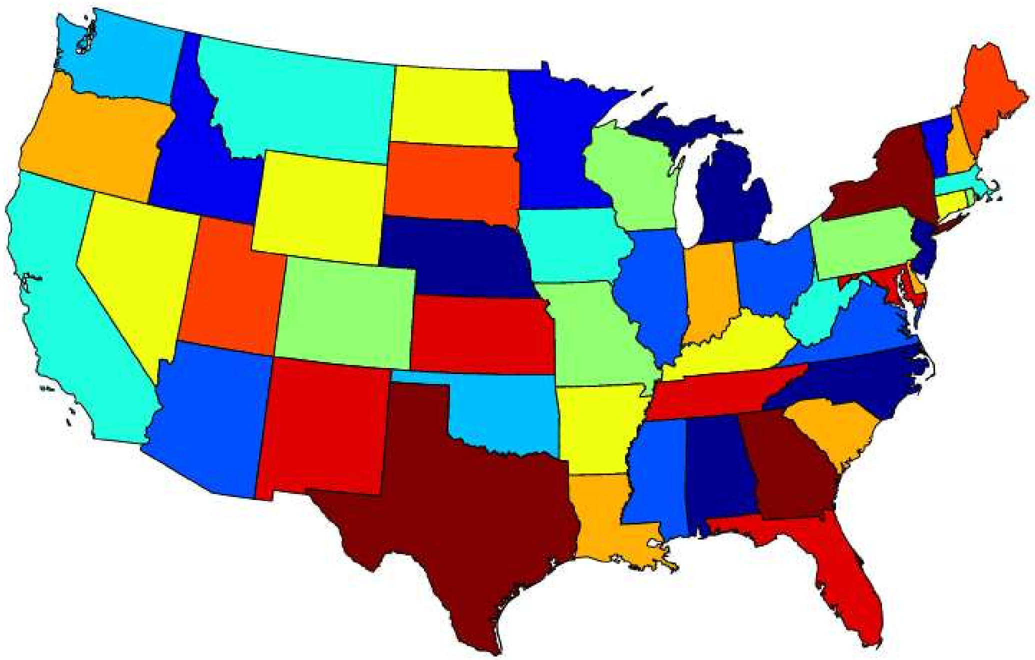
which can be integrated using a standard predictor–corrector algorithm.

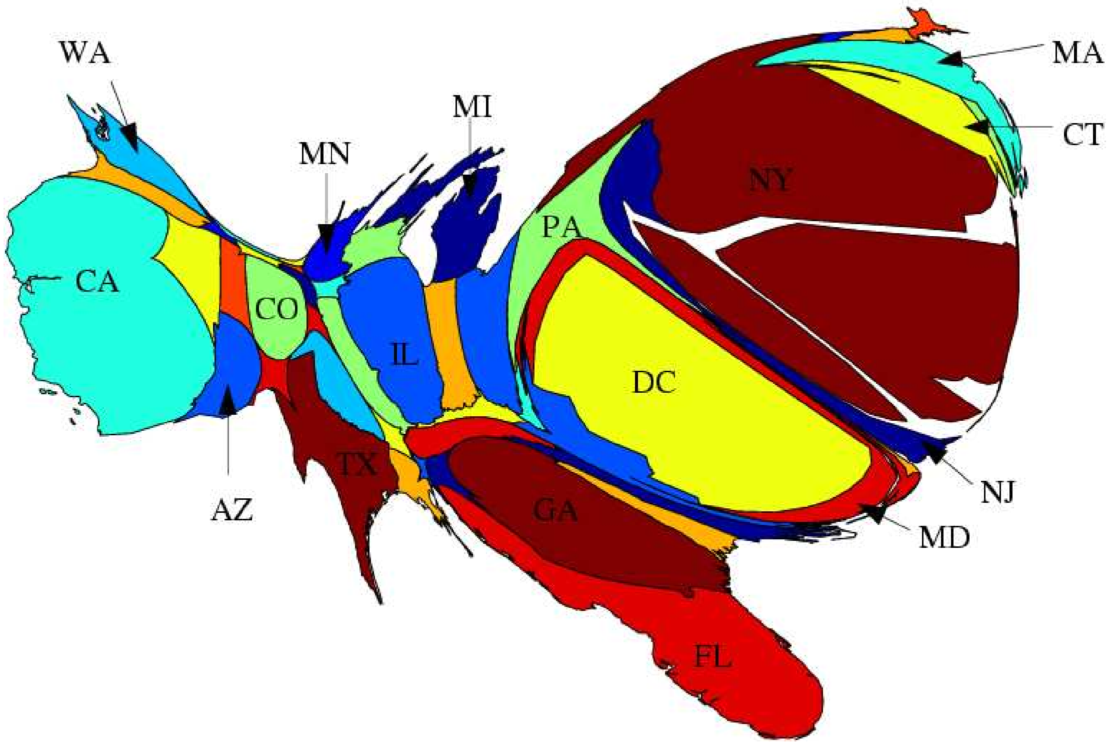






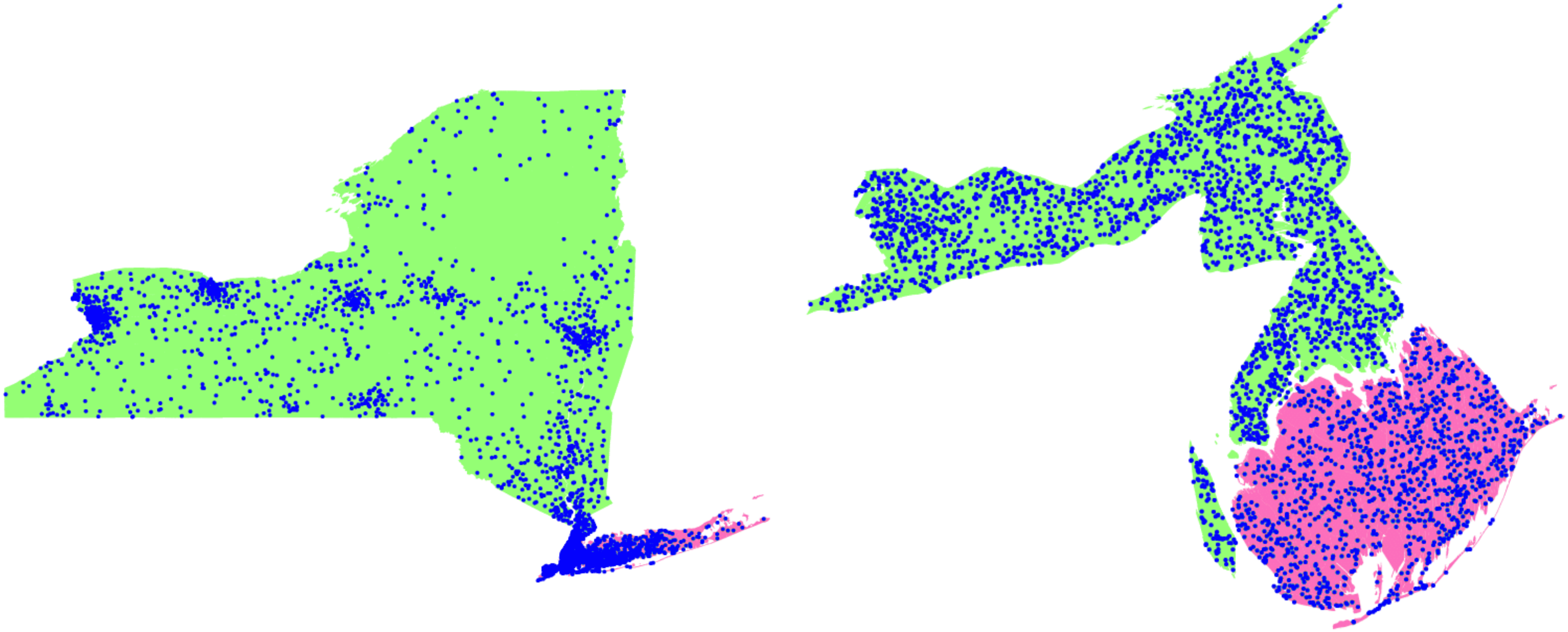


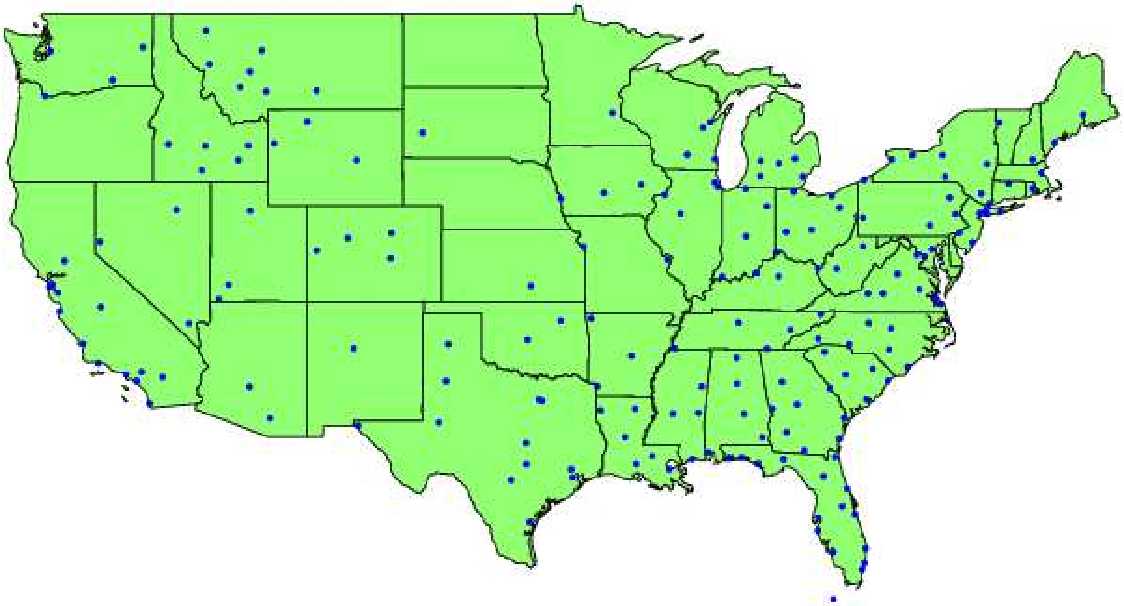


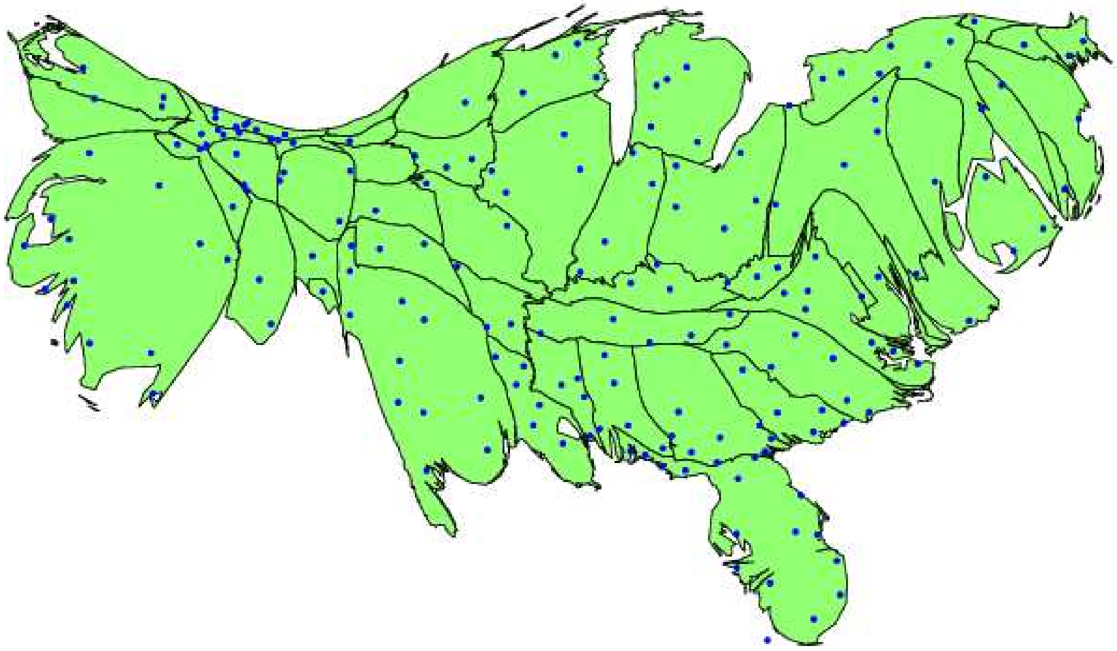


- 70,000 AP newswire stories, 1994-1998
- States scaled in proportion to number of stories from that state

New York lung cancer cases:







Thanks to . . .

- Elizabeth Leicht
- Cosma Shalizi
- UM Numeric and Spatial Data Services
- National Science Foundation
- McDonnell Foundation