Hunting for numbers: optimal strategies in mental searches

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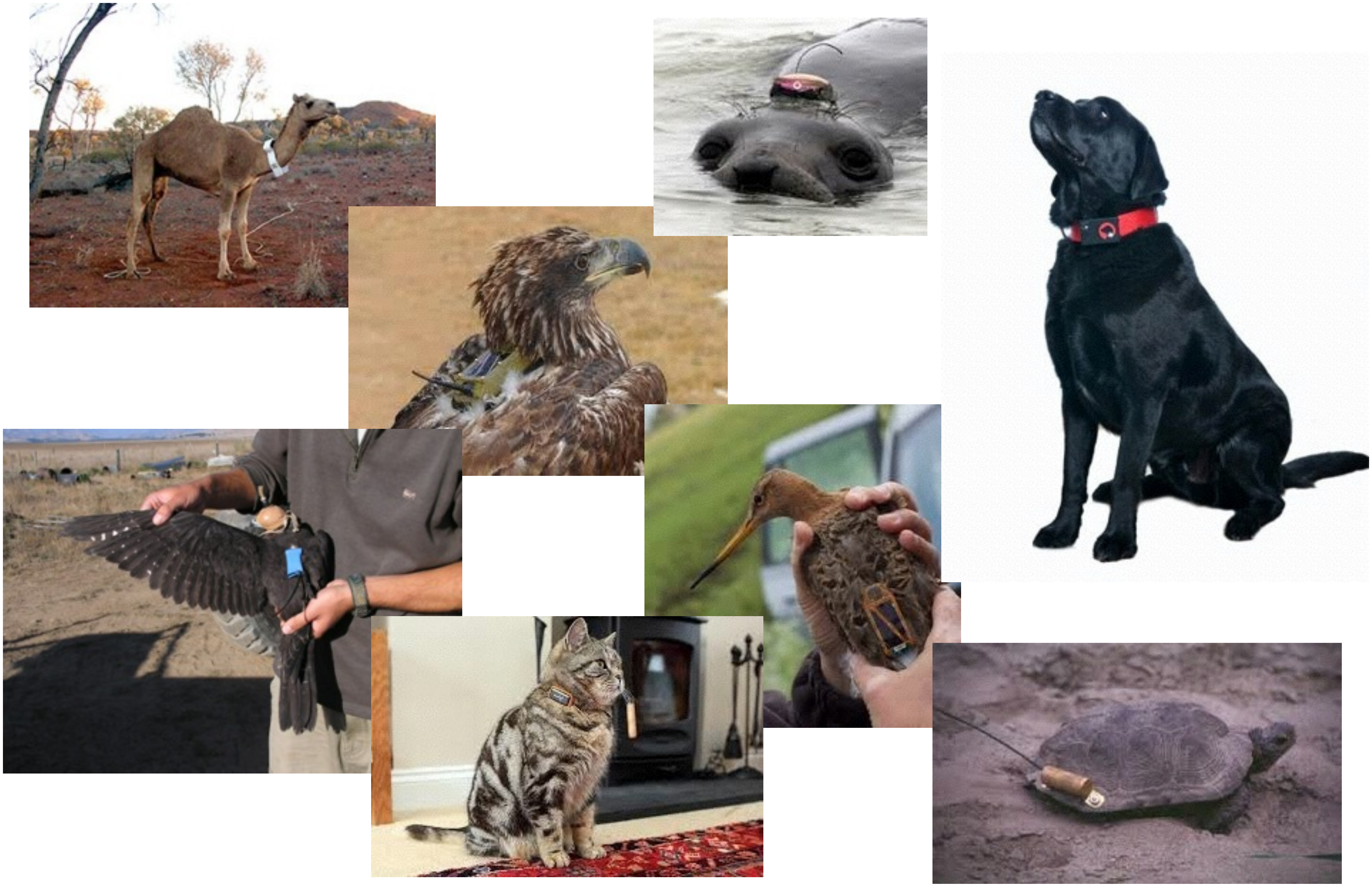
Tracking animals

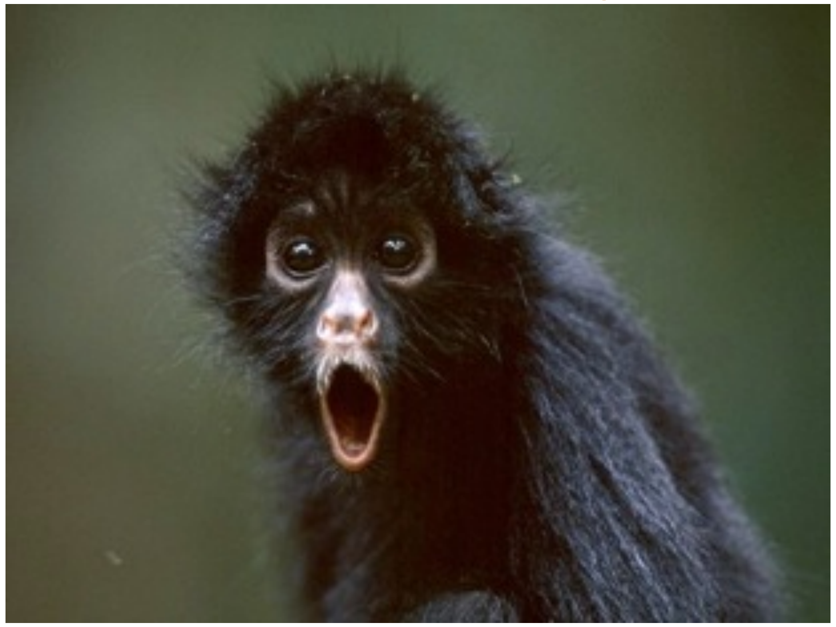


Movement patterns in nature

albatross

Viswanathan GM, et al., Nature 381, 413–415 (1996)

spider monkey



Fernandez G, et al., Beahv Ecol Sociobiol 55, 223–230 (2004)

marine predators

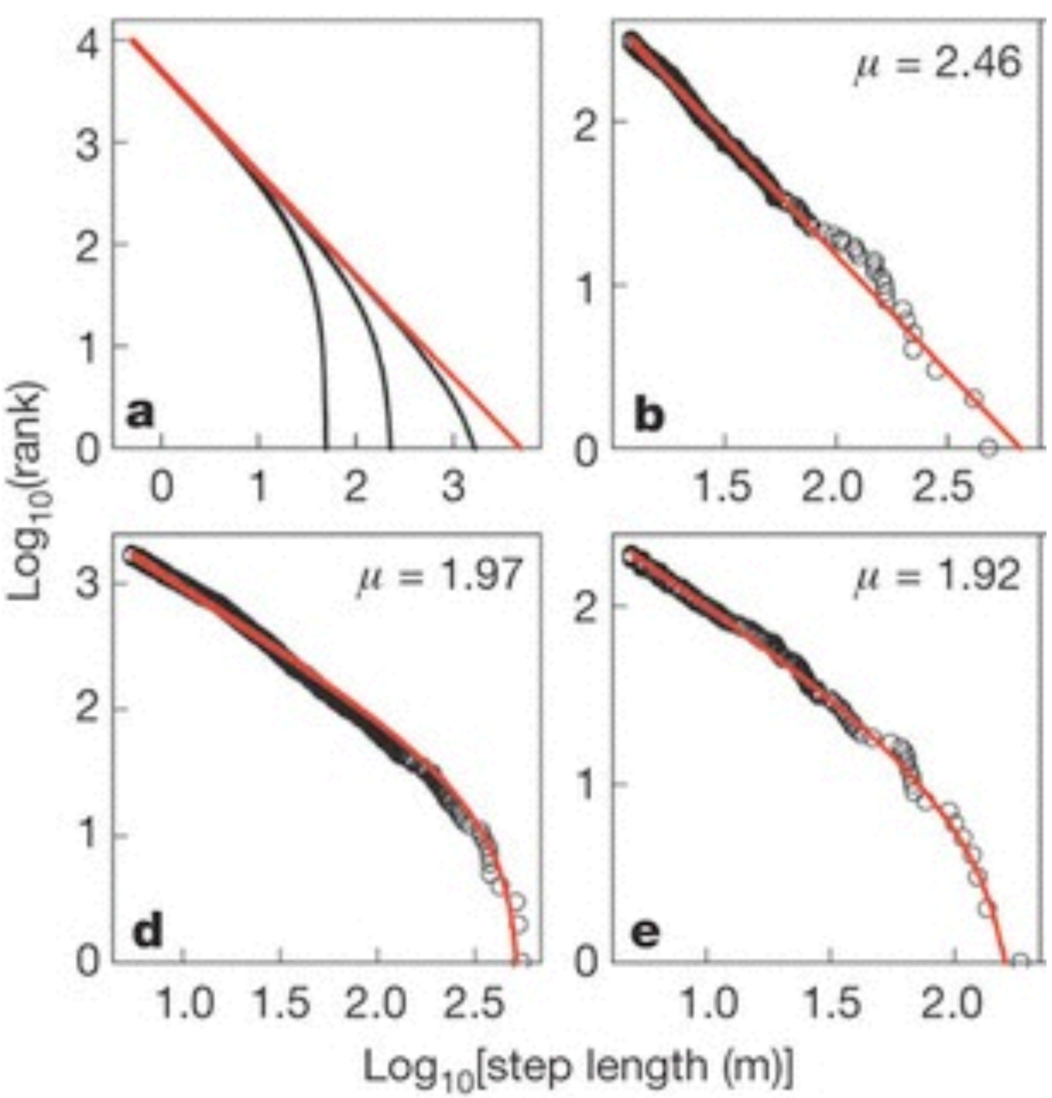
Sims DW, et al., Nature 451, 1098–1102 (2008)

honey bee

Reynolds AM,et al., J Exp Bio 210, 3763-3770 (2007)

Movement patterns in nature

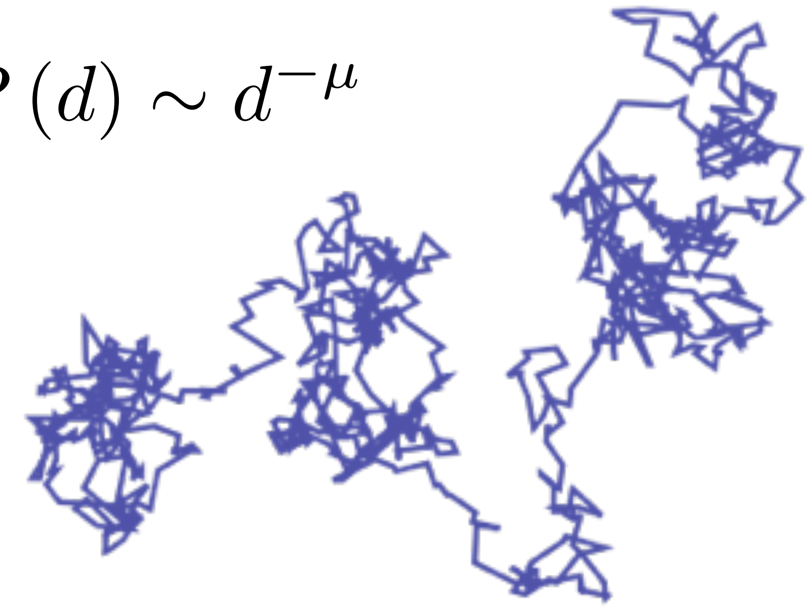
Humphries NE, et al., Nature 465, 1066–1069 (2010)



Lévy flight

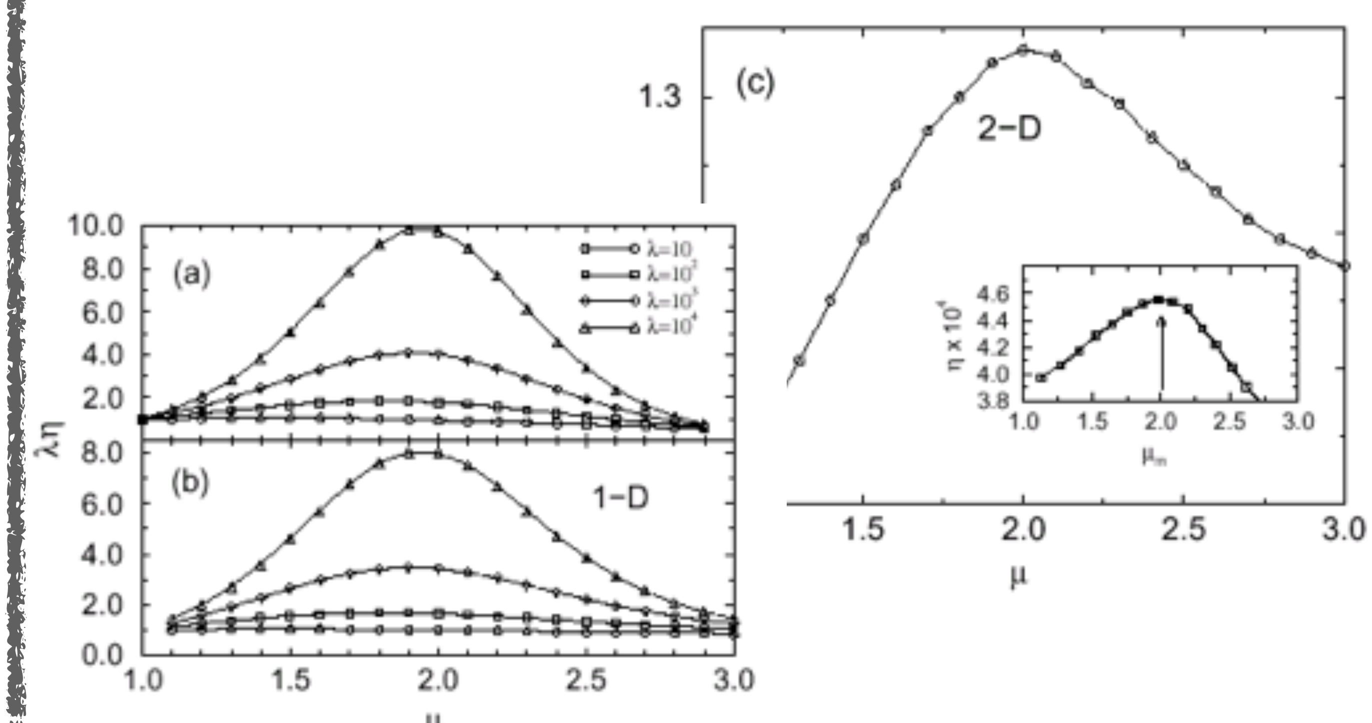
Shlesinger MF, Zaslavsky GM, Klafter J, Nature 363, 1–37

$$P\left(d\right) \sim d^{-\mu}$$

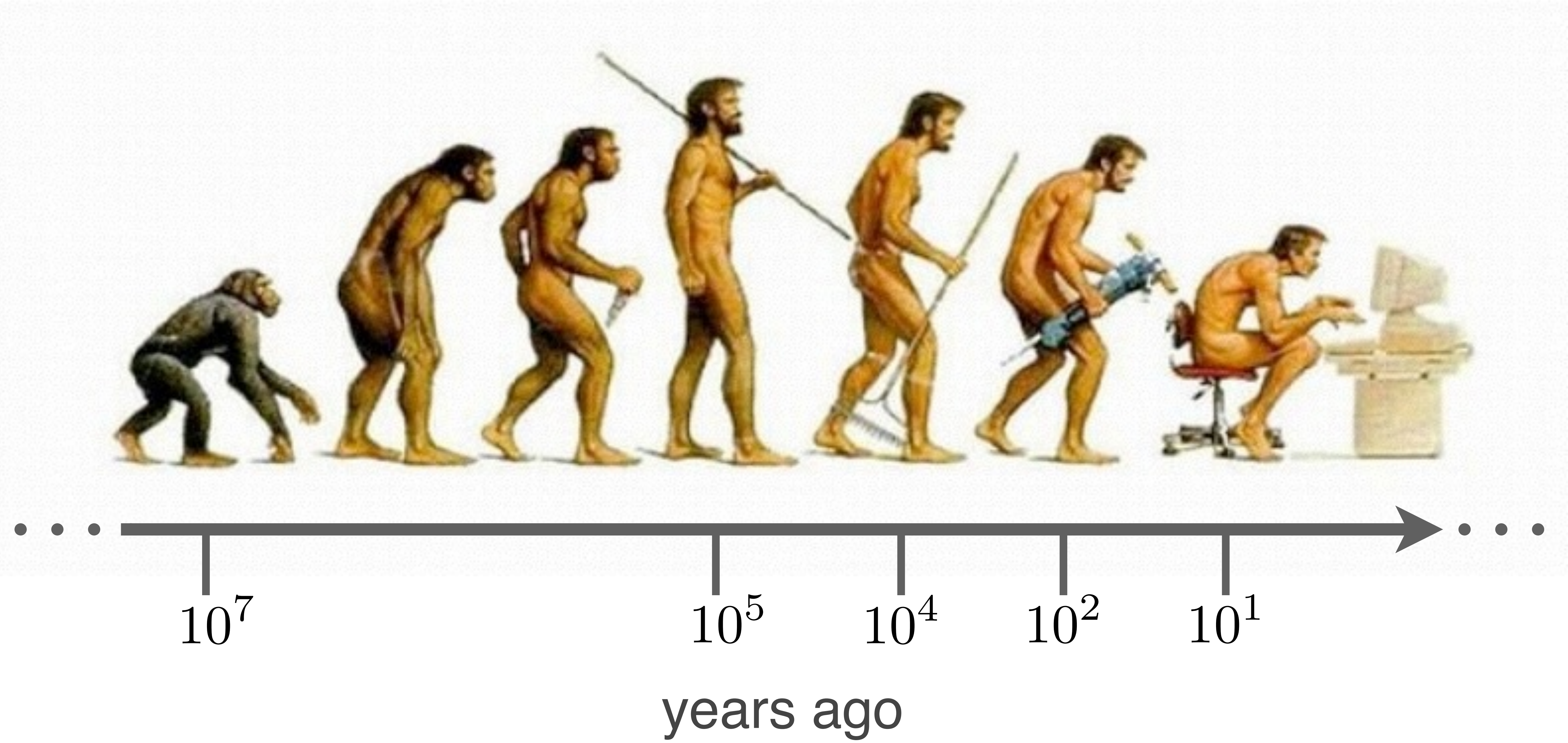


Lévy flights are optimal search strategies

Viswanathan GM, et al., Nature 401, 911-914 (1999)



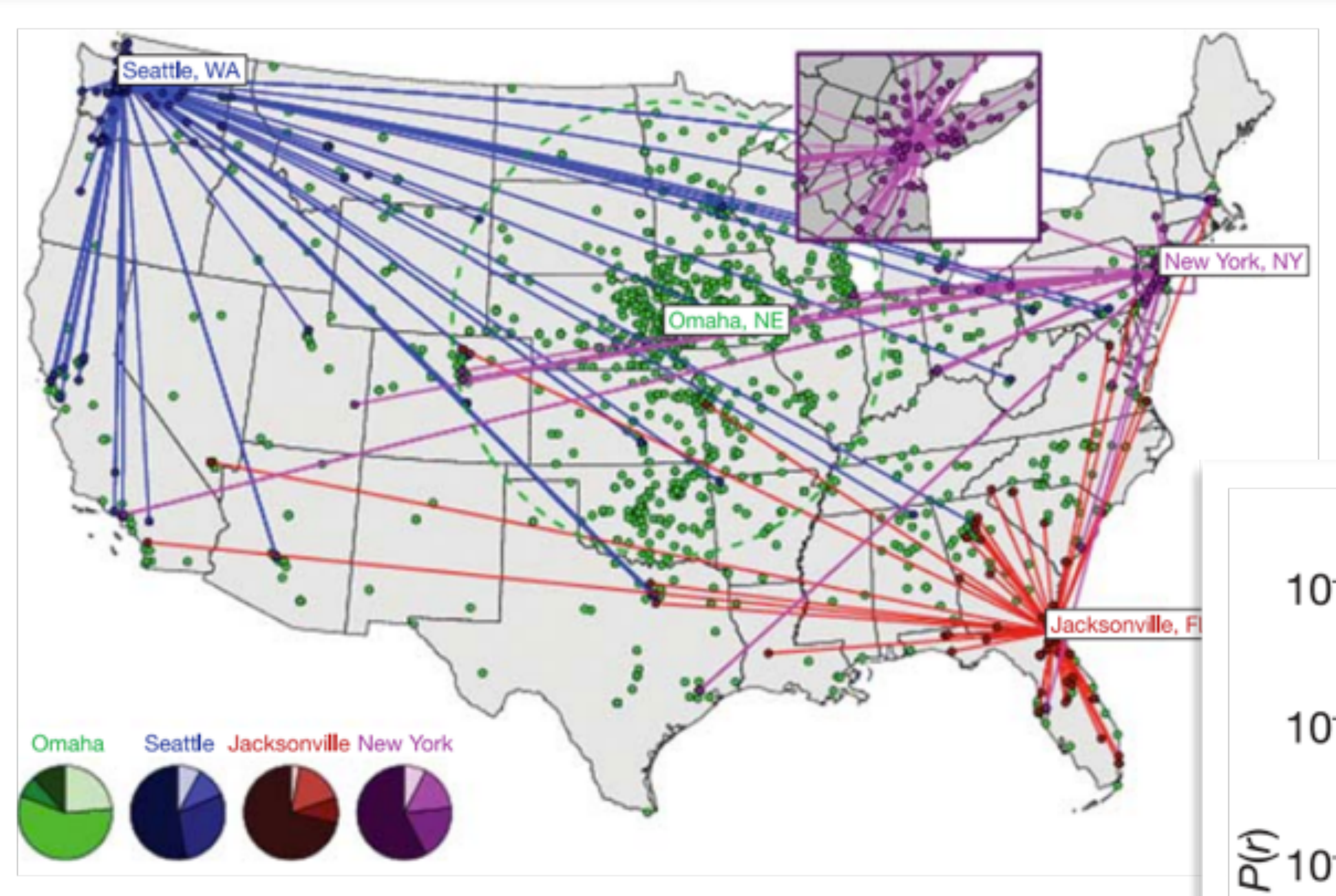
What about humans?

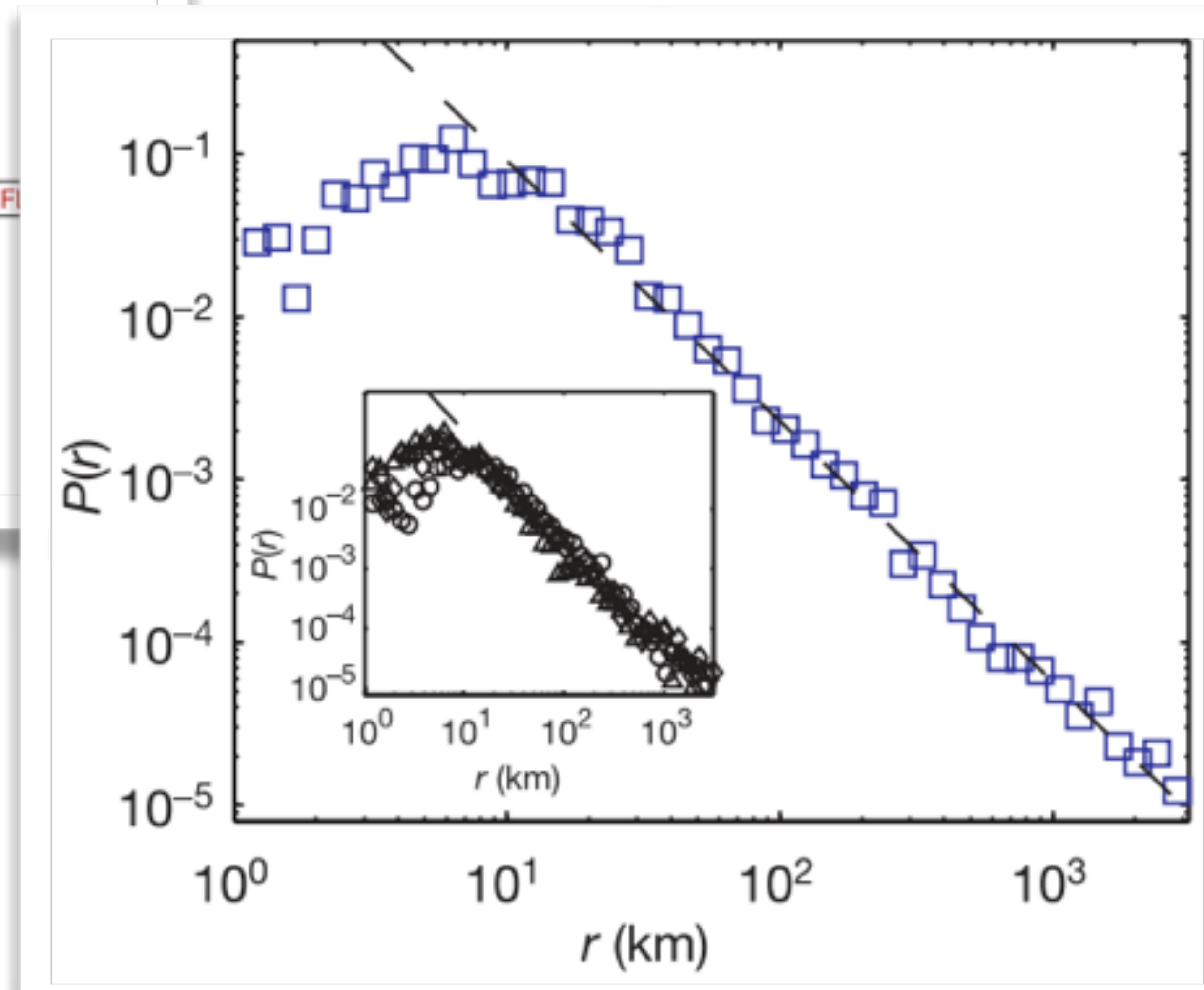


Human mobility patterns

currency tracking

Brockmann D, et al., Nature 439, 462-465 (2006)

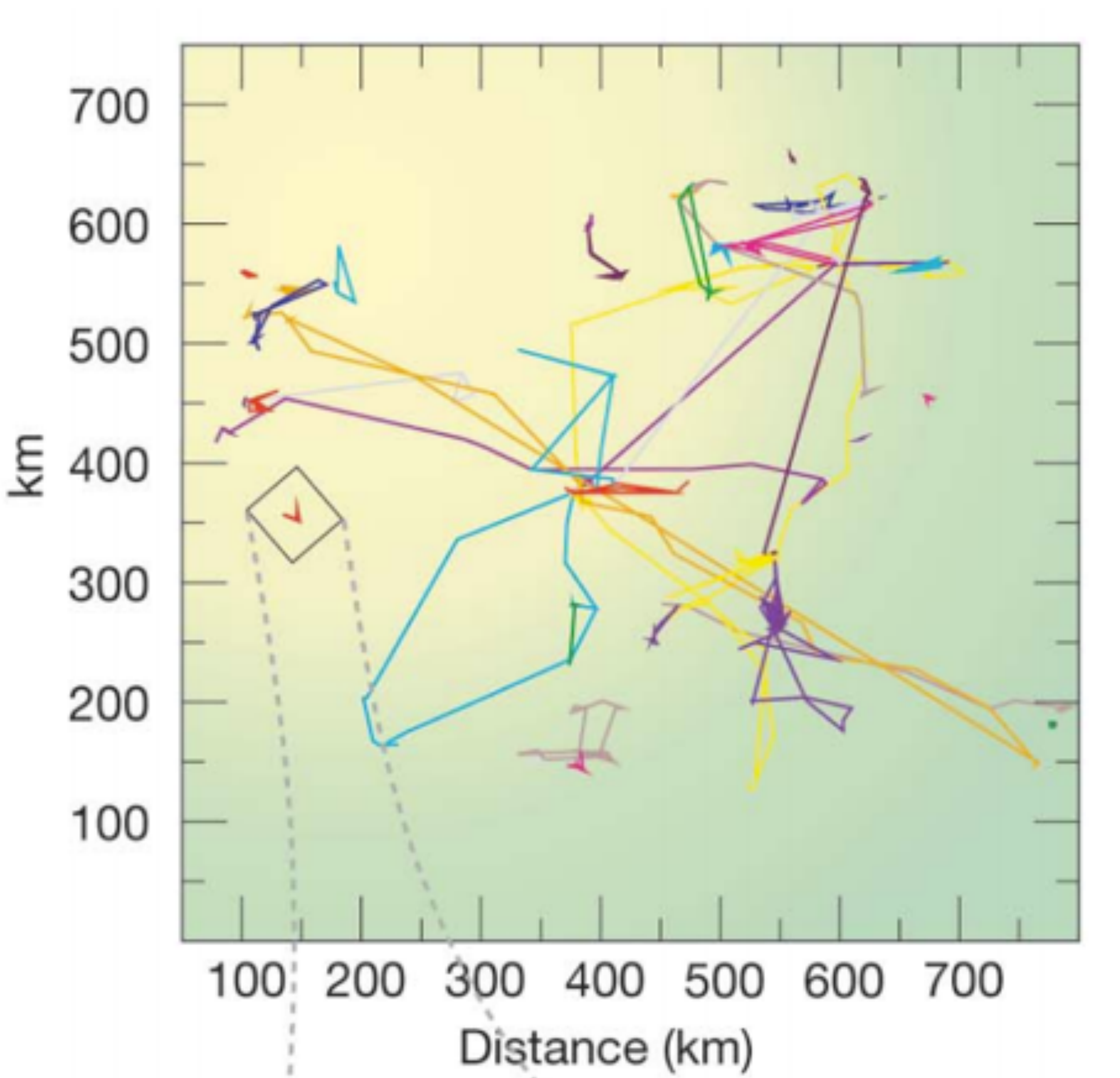


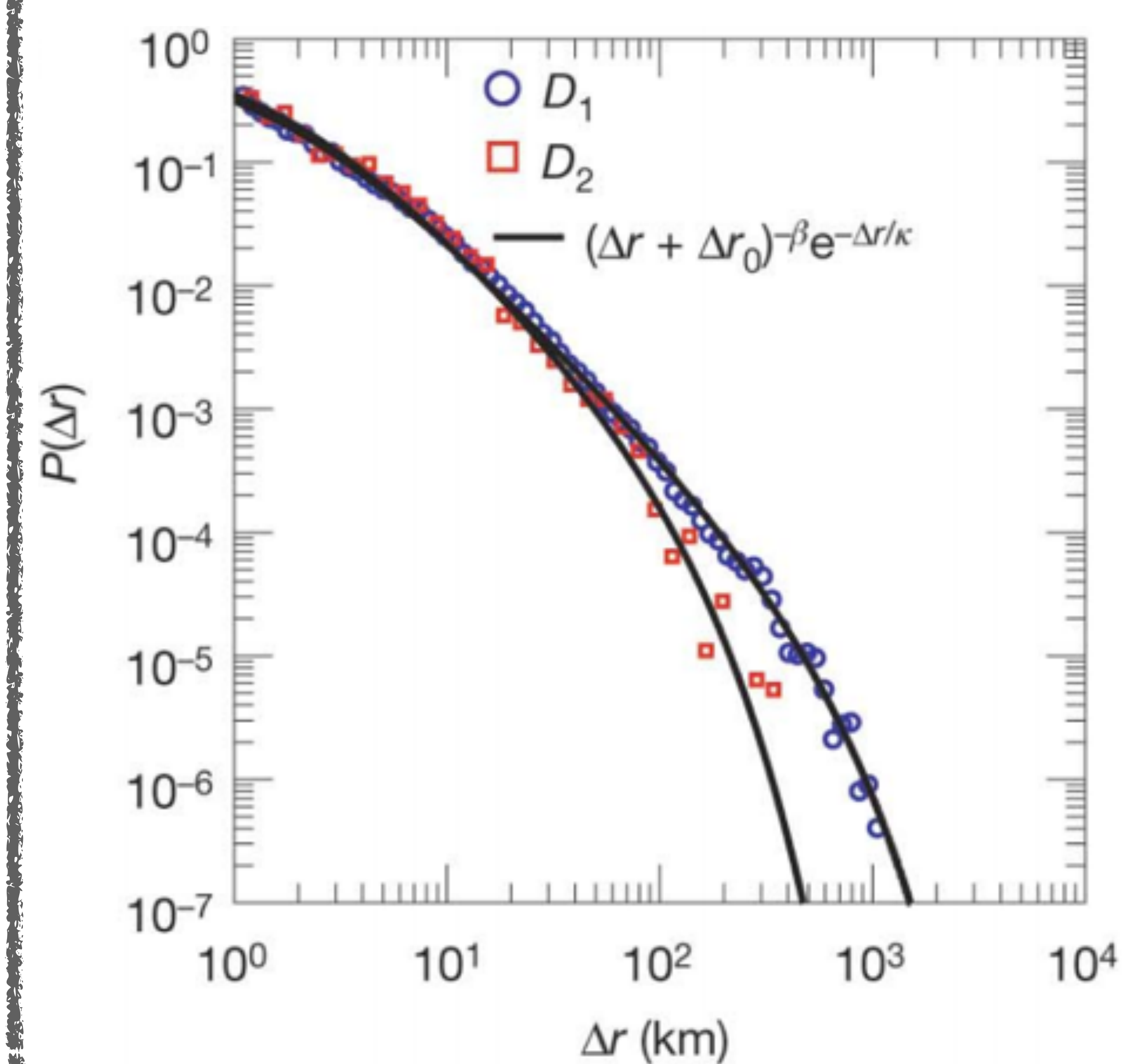


Human mobility patterns

mobile phone tracking

Gonzalez MC, et al., Nature 453, 779-782 (2008)



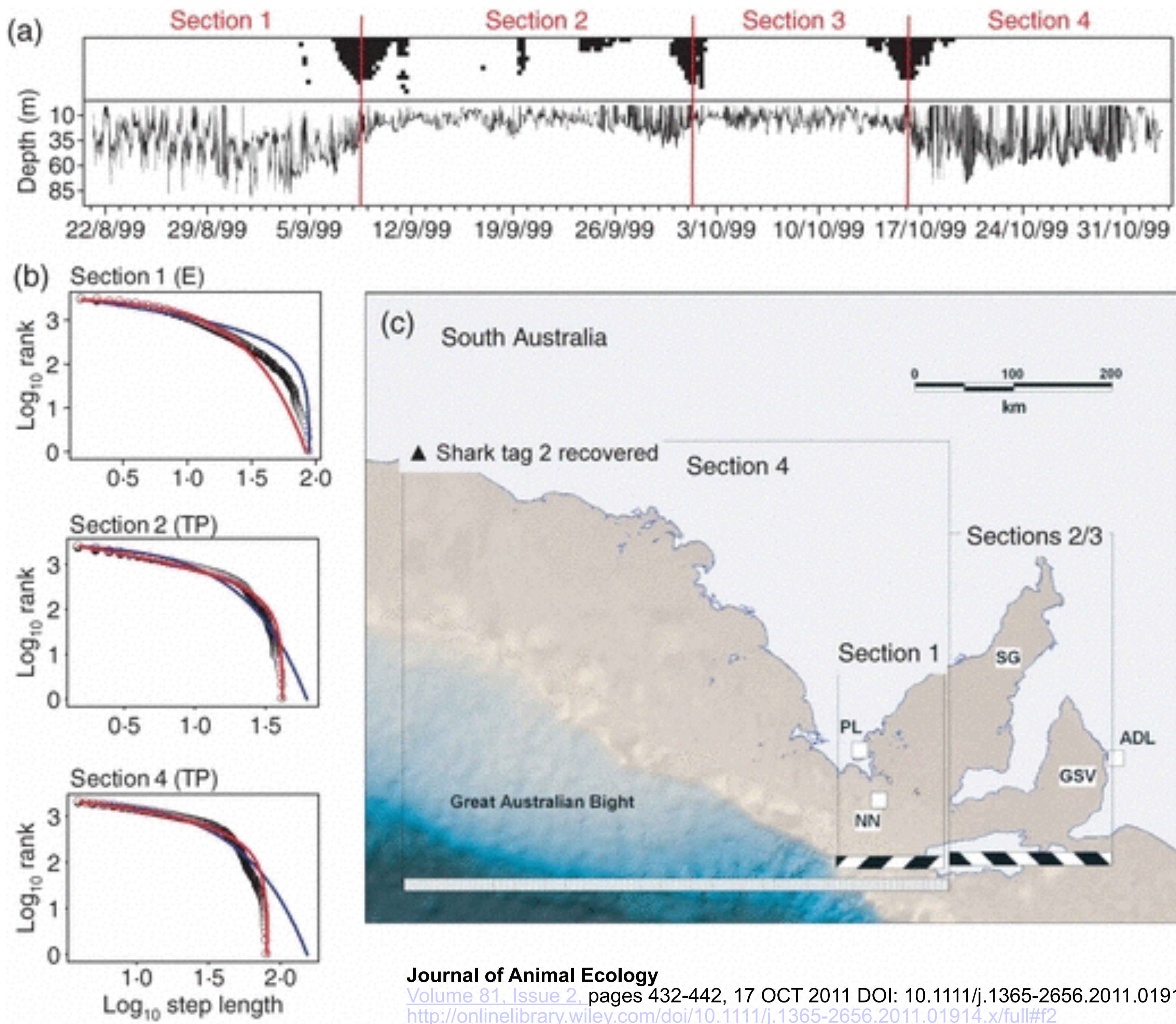


Lévy flights: a universal description for movement patterns of animals and humans

from www.christineberrie.com

Problems

Results for animals are controversial



Journal of Animal Ecology
Volume 81, Issue 2, pages 432-442, 17 OCT 2011 DOI: 10.1111/j.1365-2656.2011.01914.x
http://onlinelibrary.wiley.com/doi/10.1111/j.1365-2656.2011.01914.x/full#f2

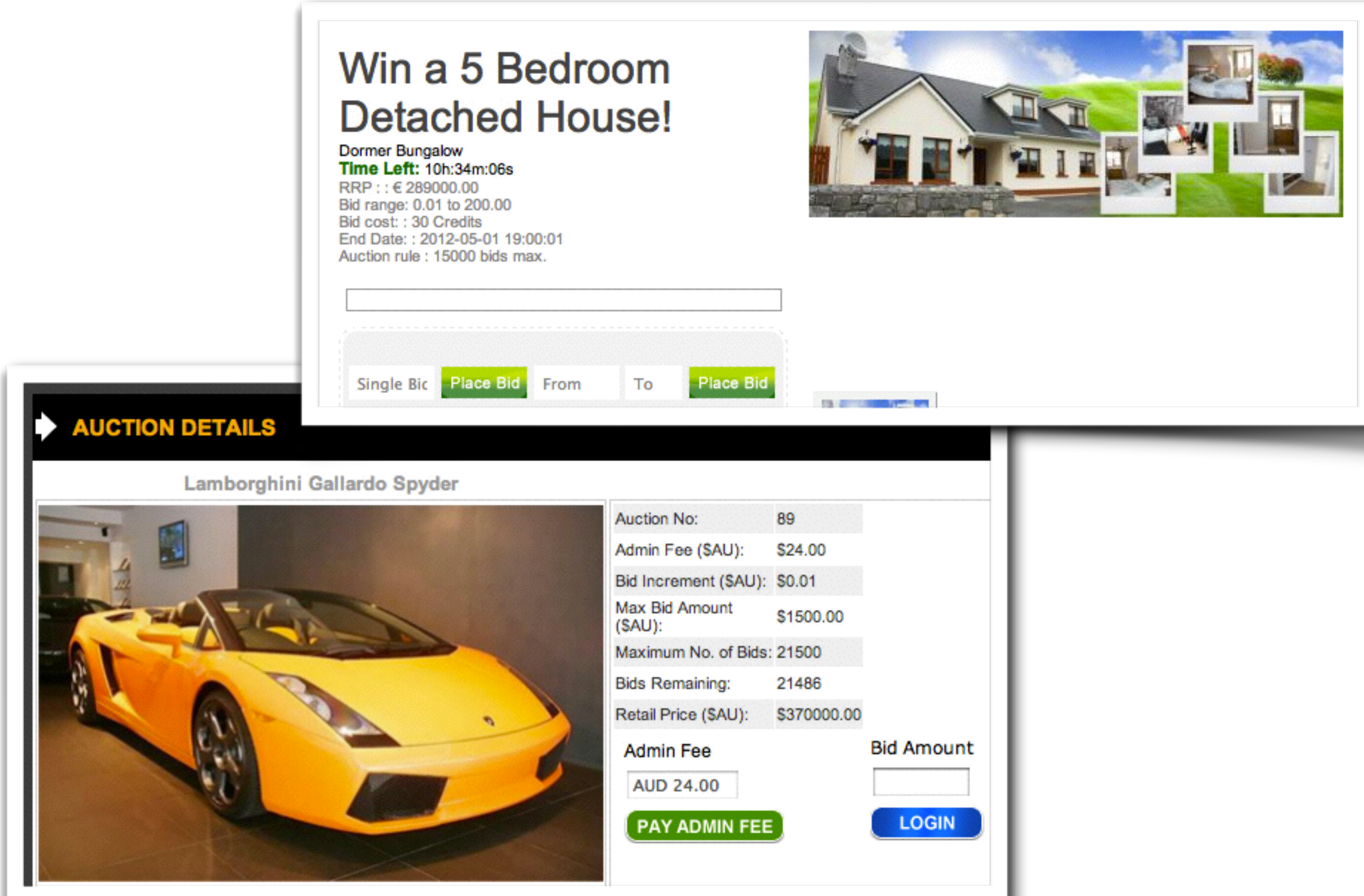
Problems

1) Data are aggregated over many “different” individuals. Are we measuring heterogeneity at the individual or at the population level?

2) Three orders of magnitude represent a “luxury”. Is the scaling compatible with a power law?

3) Do modern humans move in the space for search purposes?

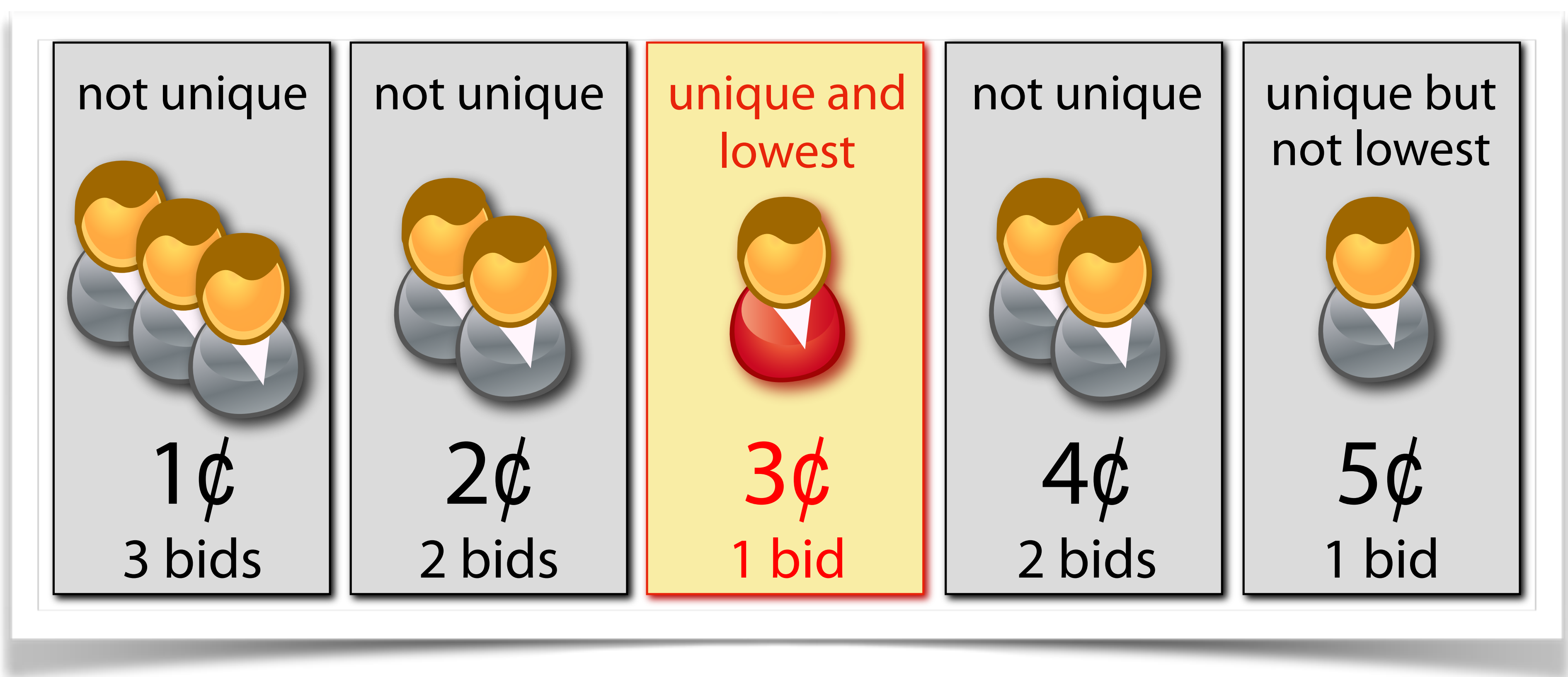
Lowest unique bid auctions



Lowest unique bid auctions

game rules

- **high-valued** goods are put up for auction $V = 10^3\ \$ - 10^5\ \$$
- agents **independently** explore the bid space in search for the winning bid $M = 10^1\ \$ - 10^3\ \$$
- each bid **costs** $c = 10^0\ \$ - 10^1\ \$$



the winning bid is the lowest and unmatched bid

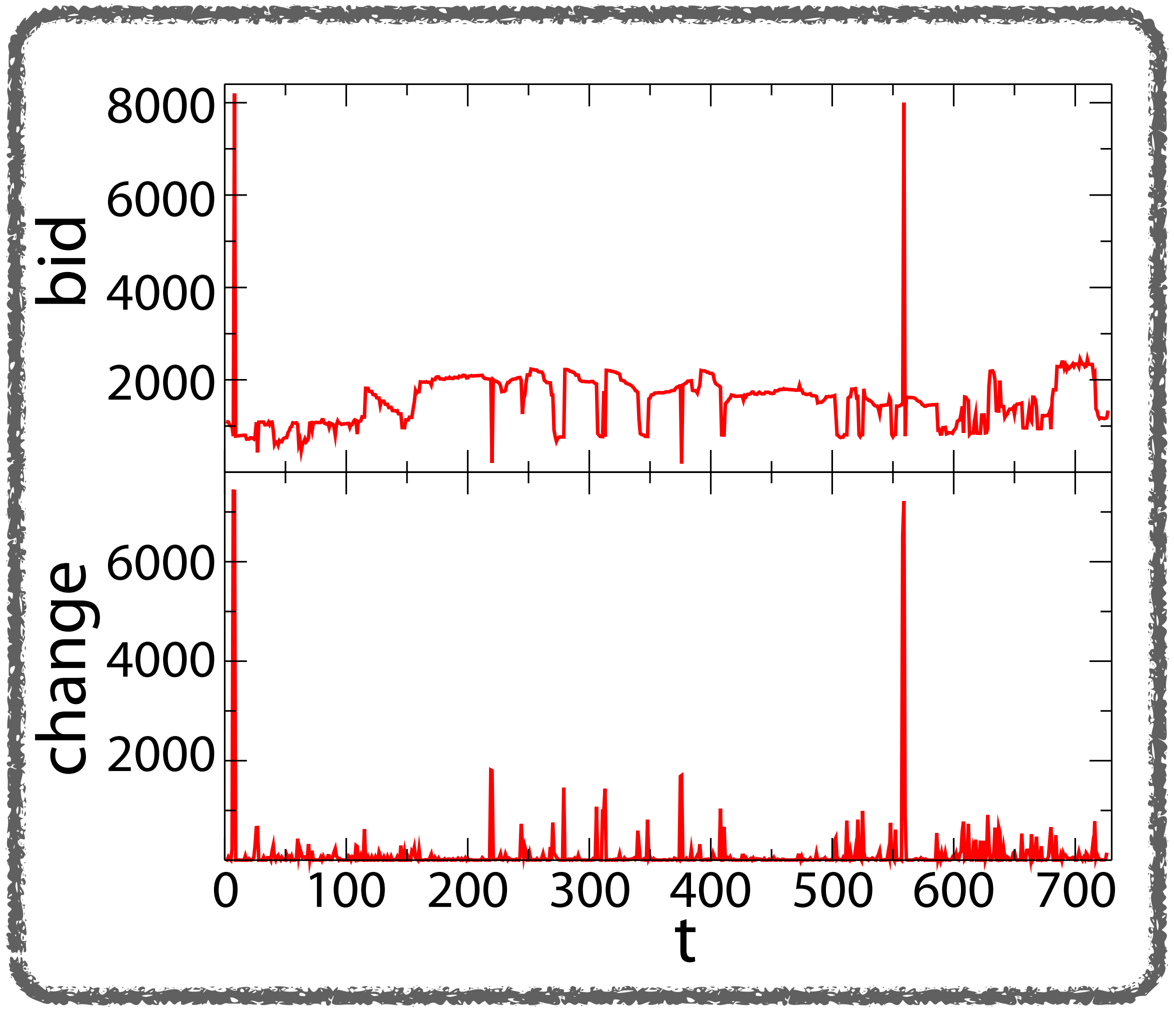
Lowest unique bid auctions

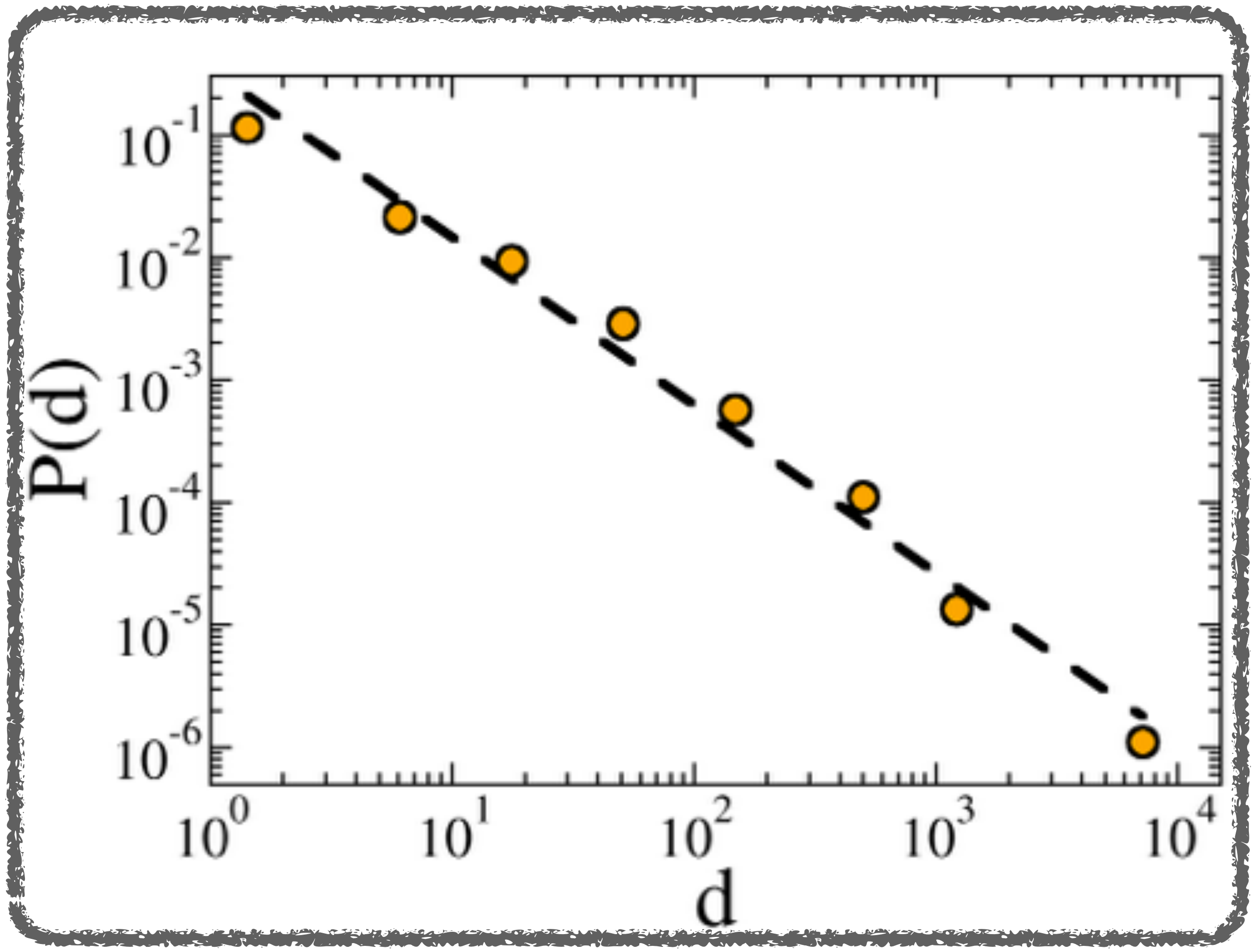
data sources

Bidding History for Auction #100 "Vespa LX 50"

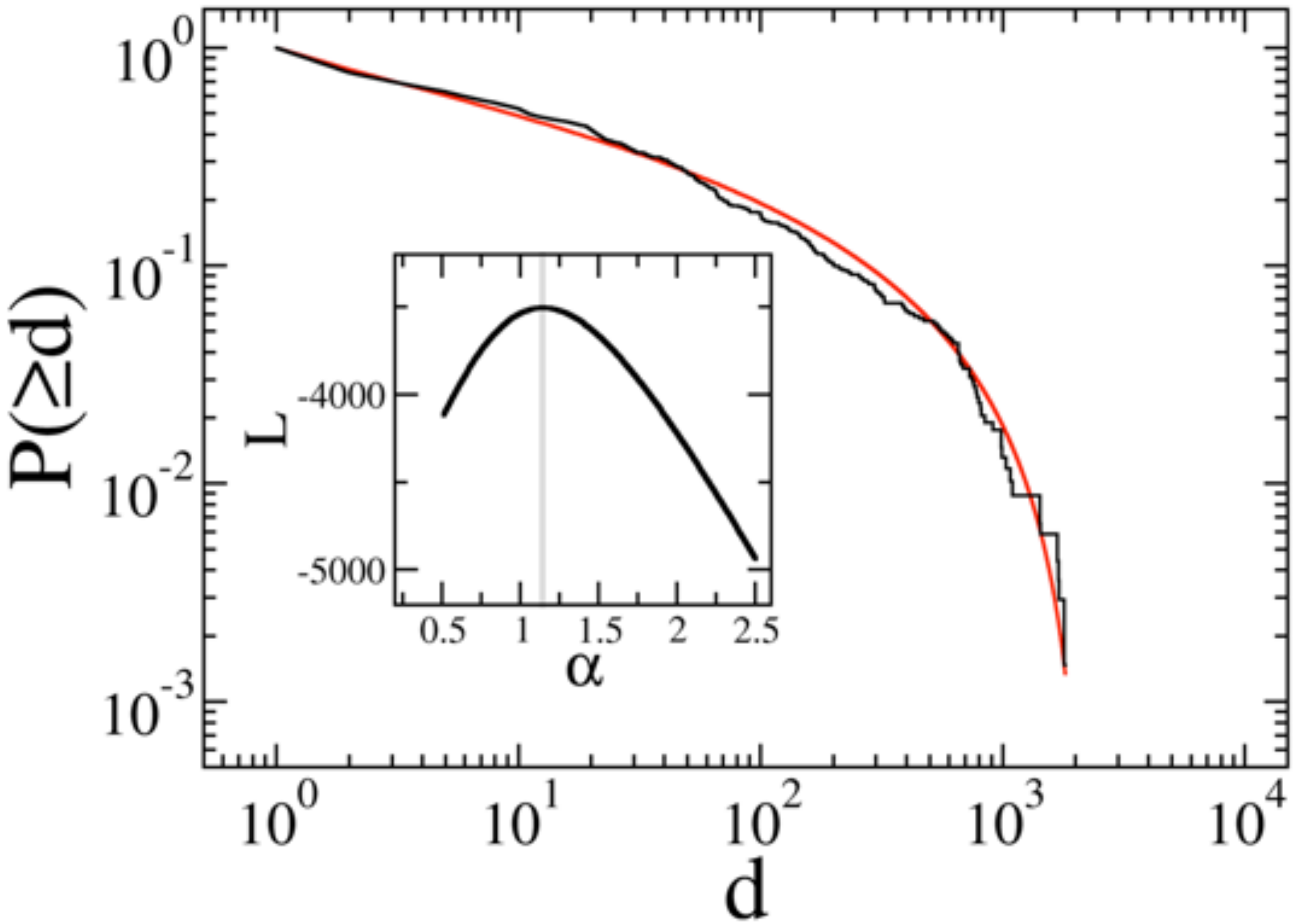
Date	Member #	Bid Amount
22/12/07 14:44	11134	$0.04
22/12/07 14:52	13822	$0.09
22/12/07 18:49	13718	$0.03
23/12/07 08:22	13836	$0.03
23/12/07 08:23	13836	$0.07
23/12/07 11:08	11406	$0.55
23/12/07 21:48	6986	$1.97
23/12/07 21:49	6986	$1.95
23/12/07 21:49	6986	$1.93
23/12/07 21:49	6986	$1.91
23/12/07 21:50	6986	$1.89
23/12/07 21:50	6986	$1.87
23/12/07 21:50	6986	$1.85
23/12/07 21:50	6986	$1.00
23/12/07 21:51	6986	$1.83
23/12/07 21:51	6986	$1.81
23/12/07 21:51	6986	$1.79
23/12/07 21:52	6986	$1.77
23/12/07 21:52	6986	$1.75
23/12/07 21:52	6986	$1.73
23/12/07 21:53	6986	$1.71
23/12/07 21:53	6986	$1.69
23/12/07 22:02	6986	$0.31
23/12/07 22:02	6986	$0.33
23/12/07 22:03	6986	$0.35
23/12/07 22:03	6986	$0.37
23/12/07 22:03	6986	$0.39
23/12/07 22:03	6986	$0.41
23/12/07 22:04	6986	$0.43
23/12/07 22:04	6986	$0.45
23/12/07 22:04	6986	$0.47

Patterns of bid space exploration





Using the model to fit the data



$$\left(Q_\alpha\right)_{ji} = \frac{|i-j|^{-\alpha}\ \left[1-\delta\left(i-j\right)\right]}{m_j\left(\alpha\right)}$$

transition matrix

$$p\left(b_1, b_2, \ldots, b_T \,|\alpha\right) = \left(Q_\alpha\right)_{0,b_1}\ \left(Q_\alpha\right)_{b_1,b_2}\ \left(Q_\alpha\right)_{b_2,b_3}\ \cdots\ \left(Q_\alpha\right)_{b_{T-1},b_T}$$

fitting data by maximizing the likelihood

Testing the validity of the model

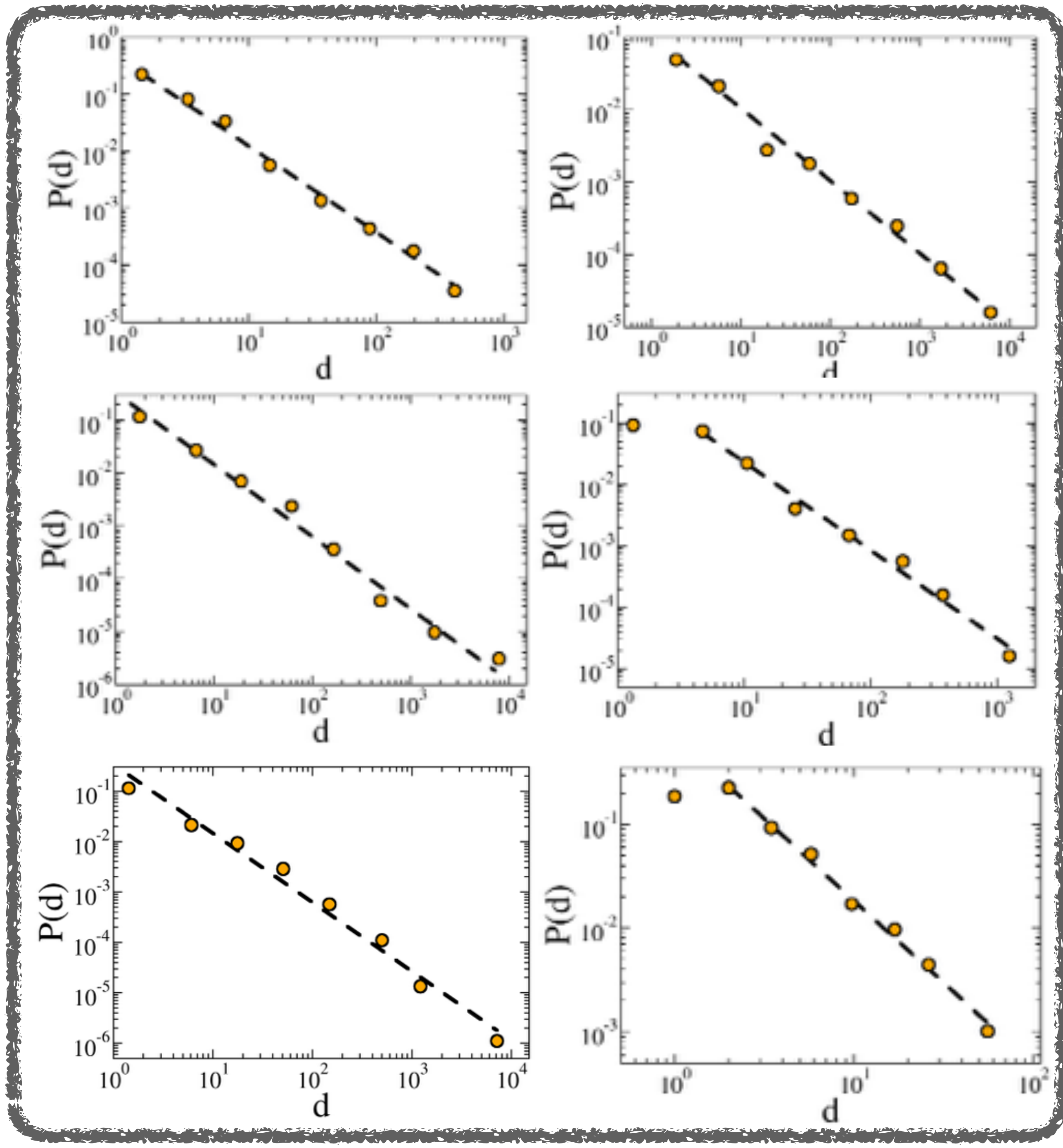
$$P_{\alpha'}(d) = \left[\sum_{i,j} (Q_{\alpha'})_{ij}\, \delta(d - |i-j|)\right] \left[\sum_{d}\sum_{i,j} (Q_{\alpha'})_{ij}\, \delta(d - |i-j|)\right]^{-1}$$

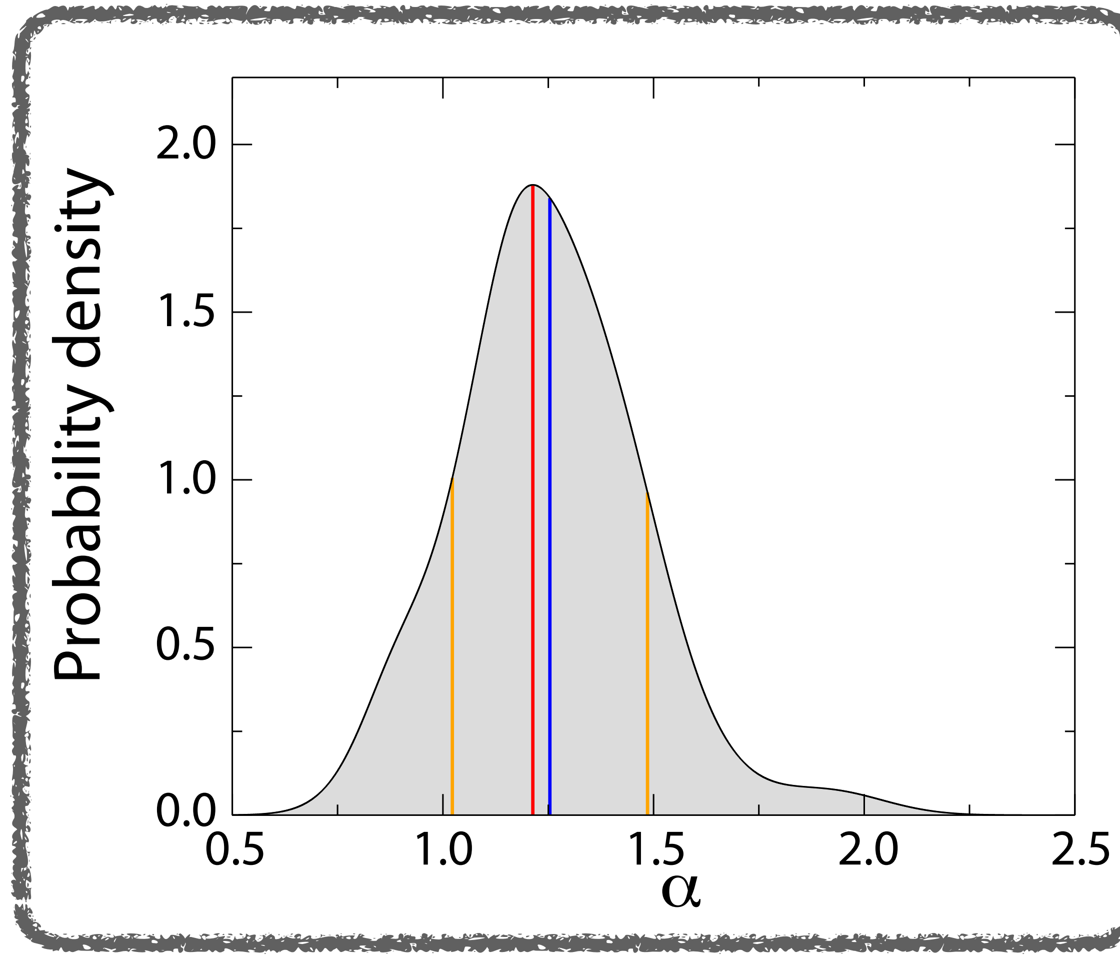
goodness of fit

a	u	α	α'	M	p
1	23	1.6(1)	1.4(1)	490	0.00
1	81	1.2(1)	1.2(1)	2220	0.26
100	1715	1.6(2)	1.0(1)	150	0.15
100	81	1.5(2)	1.3(1)	480	0.01
104	3093	1.7(3)	1.0(1)	70	0.51
108	134	1.7(1)	1.3(1)	70	0.09
14	134	1.2(1)	1.1(1)	530	0.40
14	81	1.9(2)	1.3(1)	870	0.01
15	423	1.6(4)	1.2(1)	120	0.53
179	3663	1.8(1)	1.3(2)	70	0.35
19	1	1.3(1)	1.3(1)	3050	0.29
19	1313	1.2(1)	1.1(1)	1650	0.50
19	134	1.2(1)	1.0(1)	2320	0.19
19	1433	1.1(1)	0.9(1)	1840	0.99
19	1448	1.4(1)	1.1(1)	2480	0.08
19	1558	1.1(1)	1.2(1)	2290	0.93
19	1576	1.0(1)	0.9(1)	9580	0.34
19	1601	1.4(1)	1.2(1)	480	0.09
19	1632	1.4(1)	1.1(1)	2210	0.13
19	1640	1.3(1)	1.2(1)	3080	0.00

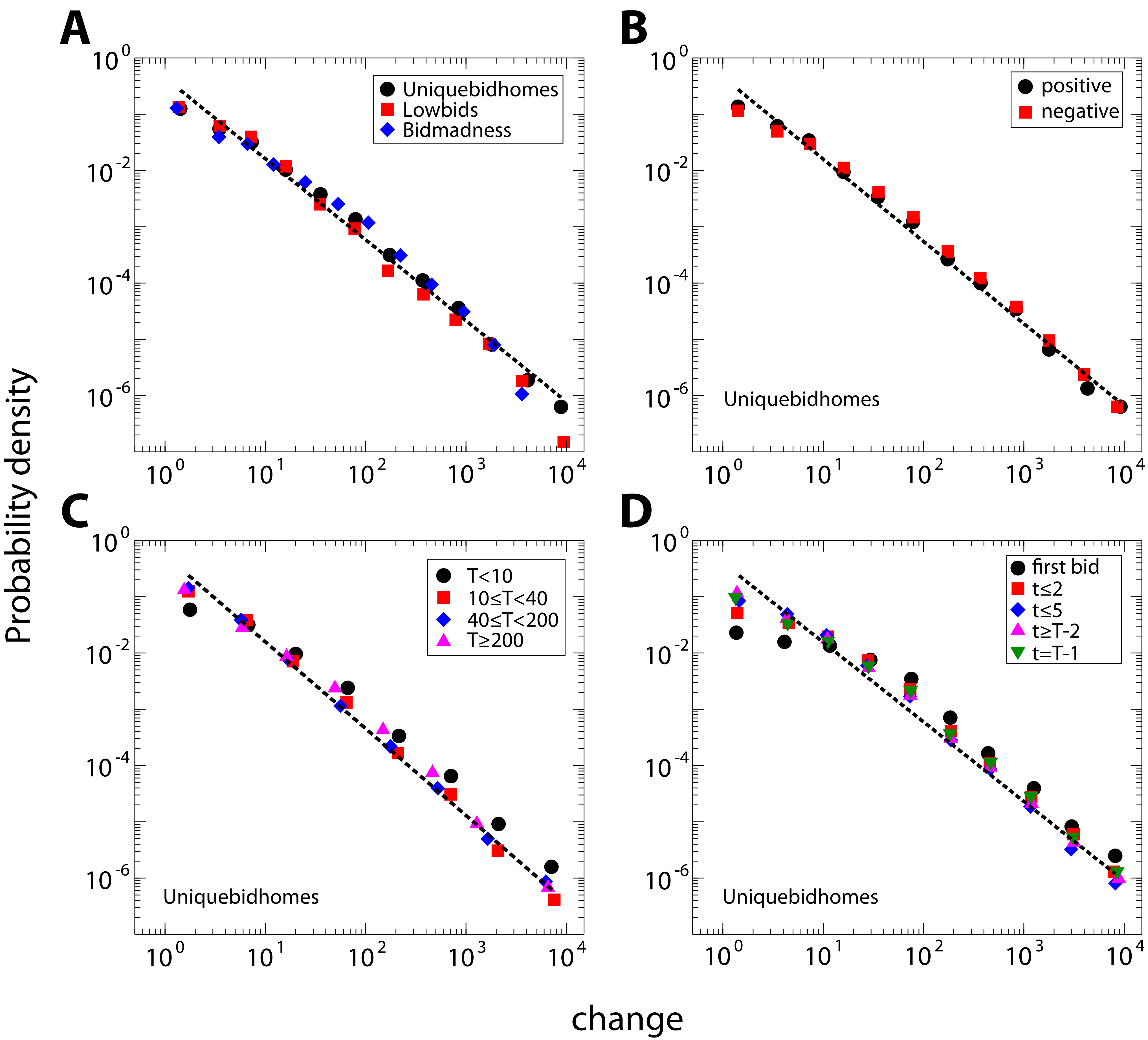
a	u	α	α'	M	p
19	1642	1.3(1)	1.2(1)	3200	0.45
19	1644	1.4(1)	1.2(1)	3010	0.00
19	1645	1.4(1)	1.1(1)	1750	0.03
19	3	1.2(1)	1.3(1)	3310	0.91
19	363	1.2(1)	0.9(1)	890	0.20
19	434	1.1(1)	1.1(1)	1750	0.77
19	438	1.5(1)	0.9(1)	2120	0.43
20	617	1.6(2)	1.3(1)	200	0.09
22	134	1.2(1)	1.3(1)	1000	0.96
44	433	1.1(1)	1.6(1)	3340	0.08
46	2003	1.3(3)	1.4(1)	110	0.03
5	128	1.6(1)	1.4(1)	1830	0.01
62	2392	1.3(3)	1.3(1)	130	0.52
71	324	1.6(2)	1.4(1)	860	0.00
73	1640	1.5(2)	1.4(1)	1920	0.27
73	1715	1.5(1)	1.2(1)	140	0.68
79	134	1.6(2)	1.3(1)	120	0.21
91	1715	1.5(2)	1.3(1)	70	0.67
97	1715	1.6(2)	1.2(1)	80	0.19

Patterns of bid space exploration





Patterns of bid space exploration



Modeling lowest unique bid auctions

single auction

$$p_{\gamma}\left(i\right) = i^{-\gamma} / m\left(\gamma\right) \qquad m\left(\gamma\right) = \sum_{j=1}^{M} j^{-\gamma}$$

prob. that the agent bids on value i

$$P_{\gamma}\left(\{n\}\right) = N! \prod_{k=1}^{M} \frac{\left[p_{\gamma}\left(k\right)\right]^{n_k}}{n_k!}$$

prob. to observe a configuration

$$\{n\} = \left(n_1, n_2, \ldots, n_k, \ldots, n_M\right)$$

N = Number of agents
T = Number of bids per agent
M = Max bid value
γ = LF exponent of the population
β = LF exponent of the agent
c = Fee value
V = Value of the good

Modeling lowest unique bid auctions

single auction

$$u_\gamma(i) = P_\gamma(n_i = 1) = N p_\gamma(i)\left[1 - p_\gamma(i)\right]^{N-1}$$

prob. that a bid on value i is unique

Focus on the agent with strategy β

$$l_{\beta,\gamma}(v) = p_\beta(v)\ \left[1 - p_\gamma(v)\right]^N \prod_{k<v} \left[1 - u_\gamma(k)\right]$$

prob. that the agent places a unique and lowest bid on value v

$$w_{\beta,\gamma} = \sum_{v=1}^{M} l_{\beta,\gamma}(v)$$

prob. that the agent wins the auction

$$\langle v \rangle_{\beta,\gamma} = \sum_{v=1}^{M} v\, l_{\beta,\gamma}(v)$$

average value of her winning bid

Modeling lowest unique bid auctions

multiple auctions

$G =$ Total number of auctions
$g =$ Total number of wins

$$P_{\beta,\gamma}(g) = \binom{G}{g} (w_{\beta,\gamma})^g \; (1 - w_{\beta,\gamma})^{G-g}$$

prob. that the agent wins g auctions

$$R_{\beta,\gamma}(I \,|g\,) = \sum_{v_1+v_2+\ldots+v_g=I} l_{\beta,\gamma}(v_1)\; l_{\beta,\gamma}(v_2) \cdots l_{\beta,\gamma}(v_g)$$

prob. that the sum of her winning bids is I in g wins

$$r_{\beta,\gamma}(g) = (gV - I)\,/G$$

economic return of the agent

Modeling lowest unique bid auctions

multiple auctions

$$R_{\beta,\gamma}(I) = \sum_g P_{\beta,\gamma}(g)\; R_{\beta,\gamma}(I\,|g\,)$$

prob. that the sum of her winning bids is I

In the limit $G \gg 1$

$$\langle g \rangle = G\, w_{\beta,\gamma}$$

average number of wins

$$I = \langle g \rangle \langle v \rangle_{\beta,\gamma} = G\, w_{\beta,\gamma}\, \langle v \rangle_{\beta,\gamma}$$

sum of the winning bids

$$r_{\beta,\gamma} = w_{\beta,\gamma}\,\left(V - \langle v \rangle_{\beta,\gamma}\right)$$

economic return of the agent

$r_{\beta,\gamma} > c$ positive return ↑

$r_{\beta,\gamma} < c$ negative return ↓

Modeling lowest unique bid auctions

multiple bids

$$q_\alpha^{(1)}(i) = i^{-\alpha}/m(\alpha)$$

first bid

$$(Q_\alpha)_{ji} = \frac{|i-j|^{-\alpha}\,[1-\delta(i-j)]}{m_j(\alpha)}$$

transition matrix

$$q_\alpha^{(t)}(i) = \sum_{j=1}^{M} (Q_\alpha)_{ji}\; q_\alpha^{(t-1)}(j)$$

next bids

$$s_\alpha^{(T)}(i) = 1 - \prod_{t=1}^{T}\left[1 - q_\alpha^{(t)}(i)\right]$$

probability to visit a given site

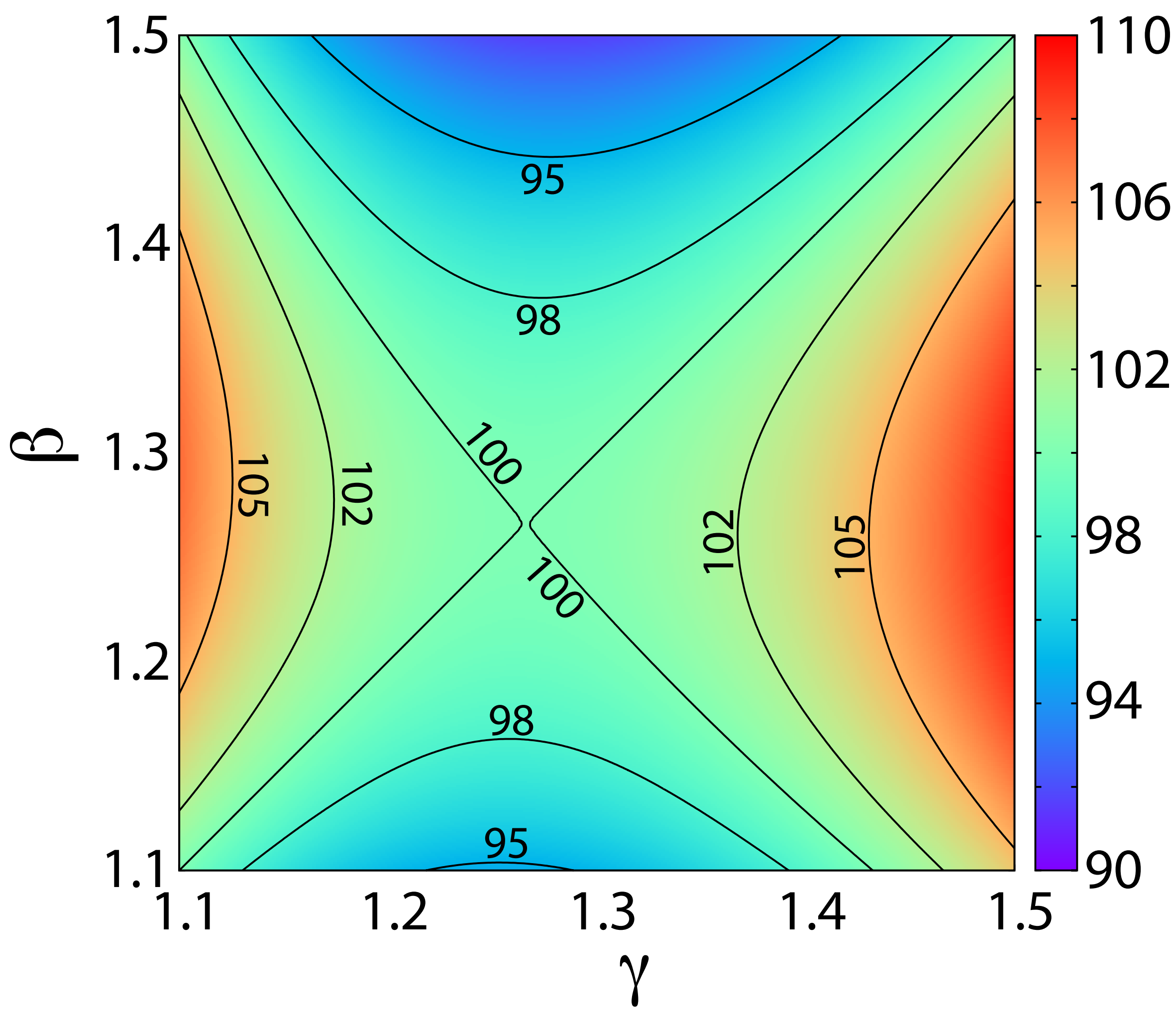
$$u_{\beta,\gamma}^{(T)}(i) = N\, s_\gamma^{(T)}(i)\left[1 - s_\gamma^{(T)}(i)\right]^{N-1}\left[1 - s_\beta^{(T)}(i)\right]$$

prob. that the agent makes on value i a unique bid

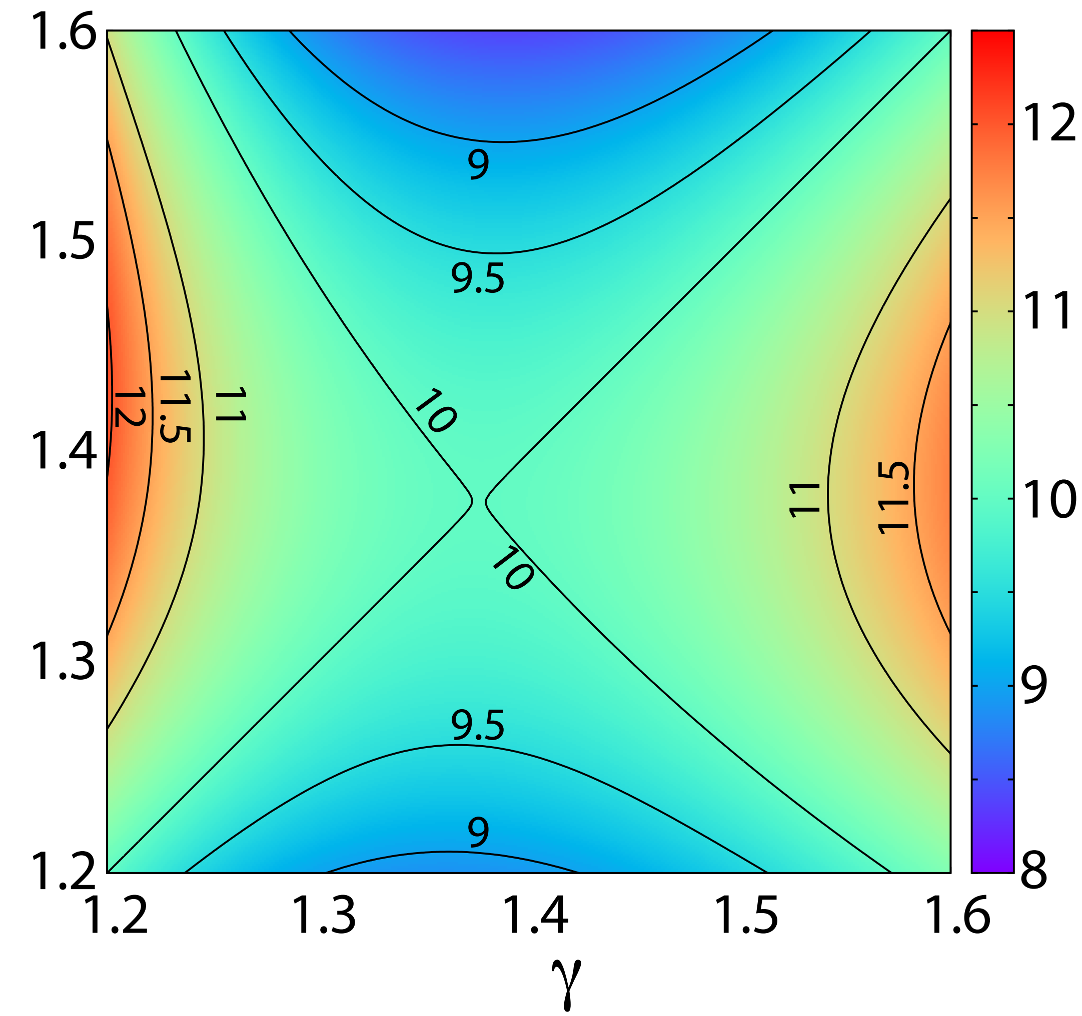
$$l_{\beta,\gamma}^{(T)}(v) = s_\beta^{(T)}(v)\left[1 - s_\gamma^{(T)}(v)\right]^{N}\prod_{k<v}\left[1 - u_{\beta,\gamma}^{(T)}(v)\right]$$

prob. that the agent makes on value i a lowest and unique bid

Model predictions



single bid, T = 1



multiple bids, T = 10

parameters of real auctions

Data set	Tot. Auctions	Tot. Agents	Tot. Bids	$\langle M \rangle$	$\langle c \rangle$	$\langle N \rangle$	$\langle B \rangle$
UBH	189	3 740	55 041	362	437	50	6
LB	55	445	3 740	1 284	478	13	6
BM	336	3 719	127 275	504	174	40	14

Evolutionary model for the auctions

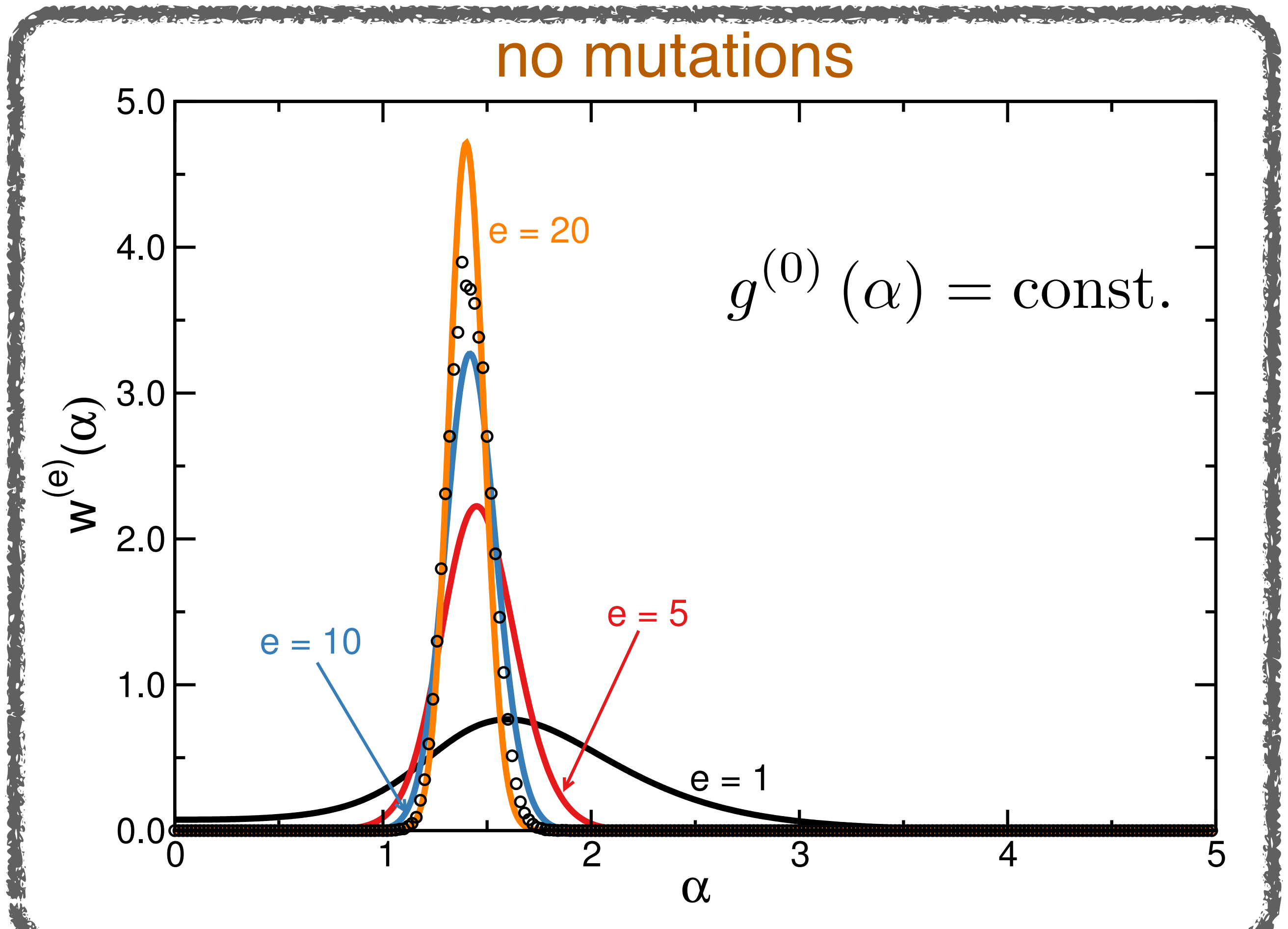
A Moran-like model

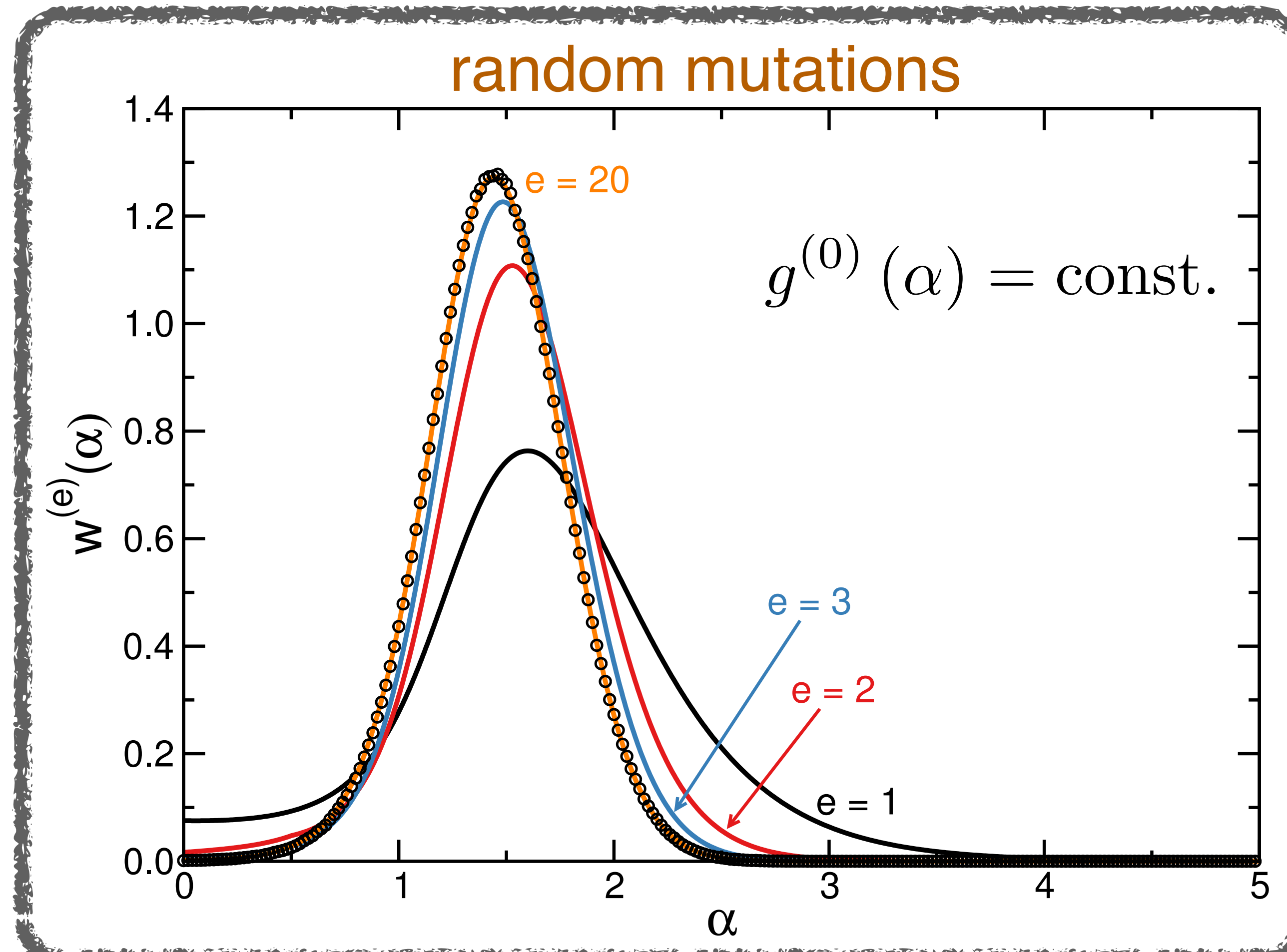
At generation e=0, agents take their strategies form an arbitrary pdf $g^{(0)}(\alpha)$

1) they play the game.

2) a losing agent copies the strategy of the winner plus some random noise.

3) e -> e + 1/N. Go back to point 1.



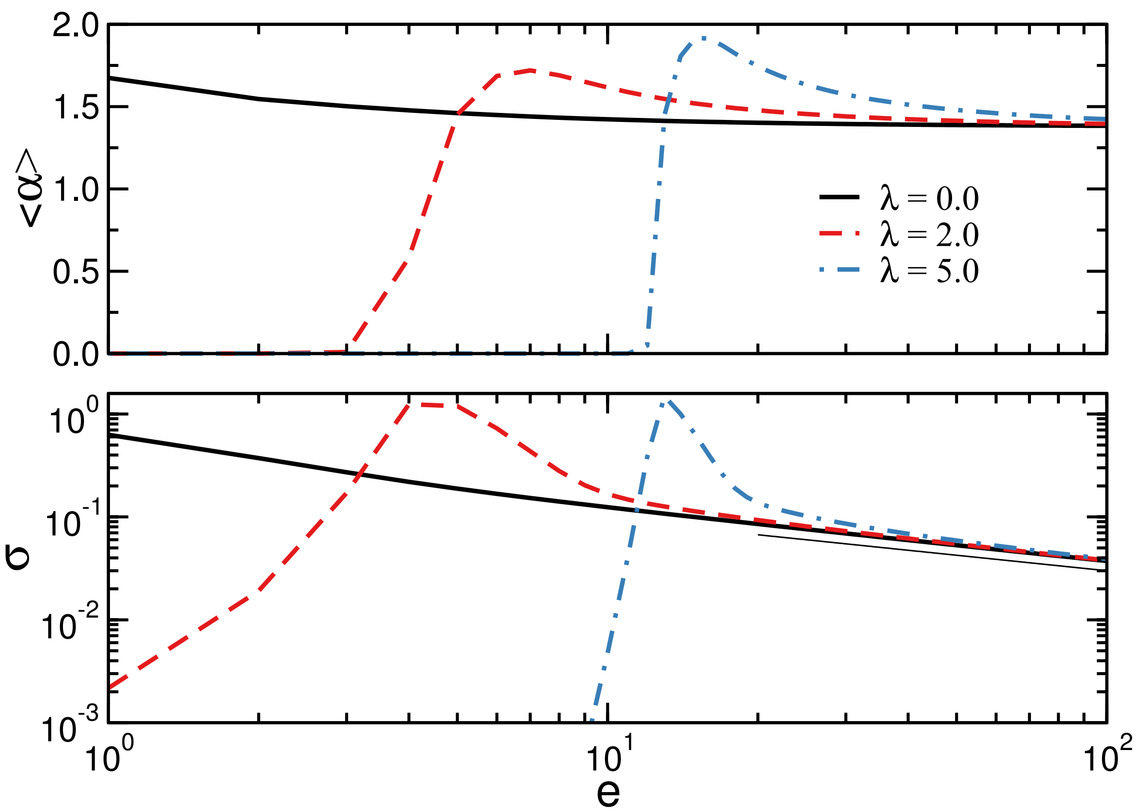


$$w^{(e)}(\alpha) = \text{prob. that the winning strategy is } \alpha$$

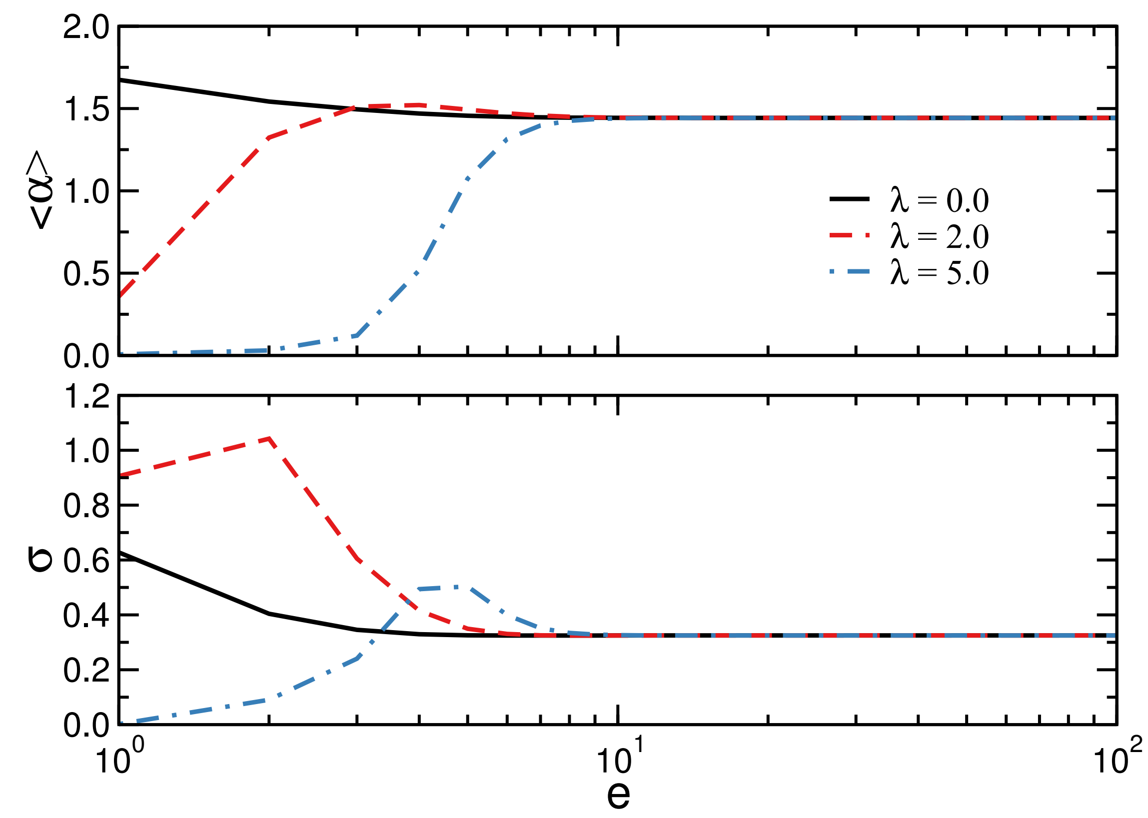
Evolutionary model for the auctions

A Moran-like model

no mutations

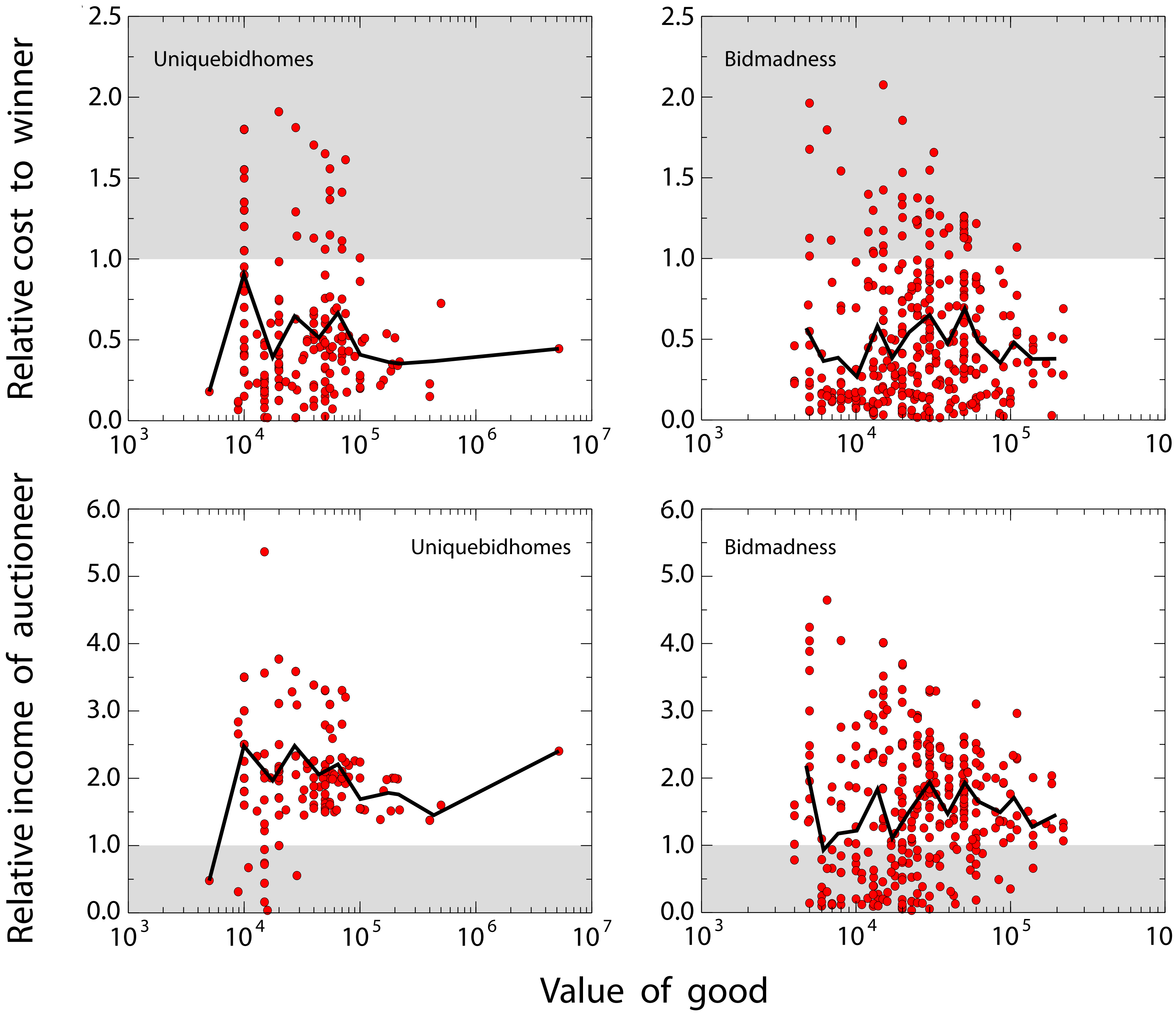


random mutations



$$g^{(0)}(\alpha) \sim \alpha^{-\lambda}$$

Agents bid rationally, but ...



$$r_{\beta_s,\gamma_s} < w_{\beta_s,\gamma_s} V \leq \frac{V}{N+1} < T\,c$$

the economic return is always negative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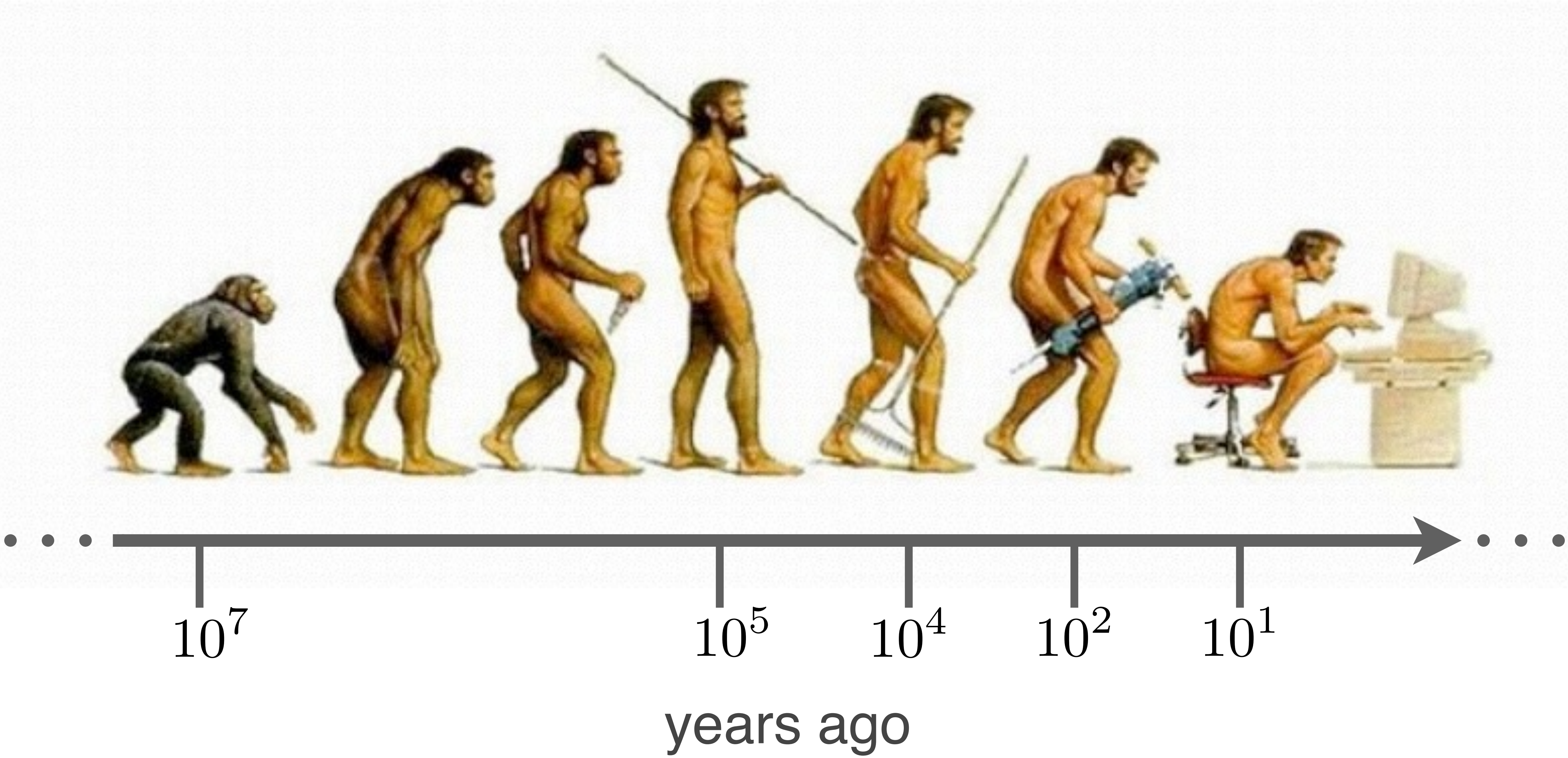
www.uniquebidhomes.com
Safari can't find the server
Safari can't open the page "www.uniquebidhomes.com" because Safari can't find the server "www.uniquebidhomes.com".
www.lowbids.com.au
s working on your cPanel® and WHM™ Server
SmartStore.com.au
www.smartstore.com.au
smartstore.com.au
CATEGORIES
Bedding
Entertainment
Kitchen & Household
Kitchen Appliances
New Arrivals
Outdoors
Personal Care
Vacuum Cleaners
NEWSLETTER
NEW HOT ITEMS
No New Hot Items at this time
FREE DEALS
No Free Deals at this time
up to 50% off
SALE
End of Financial Year Sale
CART
YOUR DISCOUNT CLEARANCE SUPERSTORE
Over 500,000 successful transactions and deliveries Australia-wide. PICKUP AVAIL for all Products from Caulfield, VIC. Please contact us to arrange.
- Next day dispatch means you get your order faster
- Safe Online Shopping with Secure Payment
- Over 700,000 Happy Customers
- Top Class Customer Service
Tell People ABOUT Smartstore
follow @ facebook

Do humans search as animals?



We are running some experiments...

http://cgi.soic.indiana.edu/~i601levy/index.php

Preliminary results

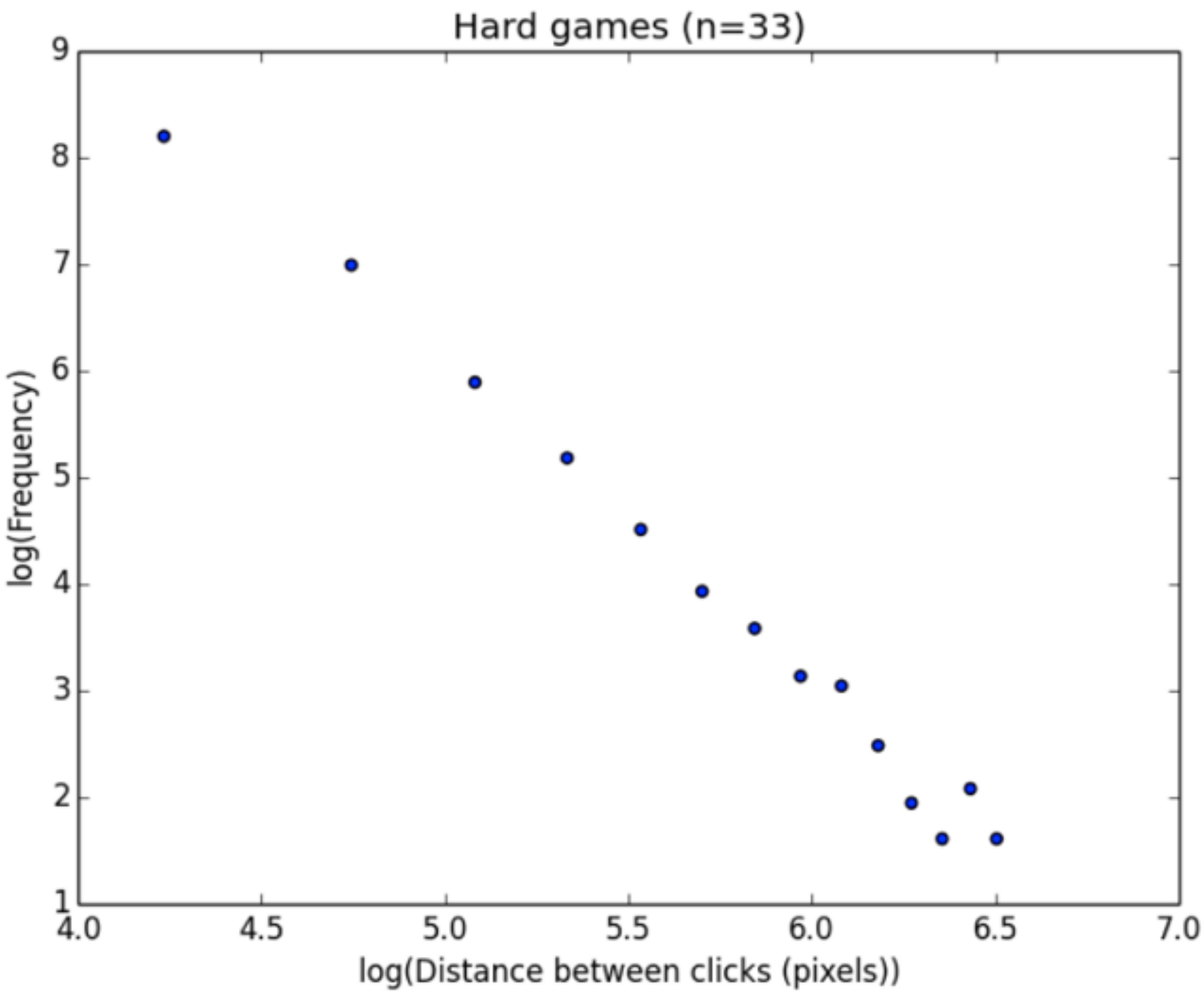


Figure 9 - Distance between clicks for hard games

Thanks

L.A.N. Amaral, Northwestern University

A. Baronchelli, City University London

References

Evolution of optimal Lévy-flight strategies in human mental searches
F. Radicchi and A. Baronchelli
Phys. Rev. E **85**, 061121 (2012)

Rationality, irrationality and escalating behavior in lowest unique bid auctions
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Lévy flights in human behavior and cognition
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