

proximity networks and semi-metric behavior



in bioinformatics, social networks, and recommendation systems

luis m. rocha

Indiana university

school of informatics

1900 East Tenth Street, Bloomington IN 47406

and

Instituto Gulbenkian de Ciência

Apartado 14, 2781-901 Oeiras, Portugal

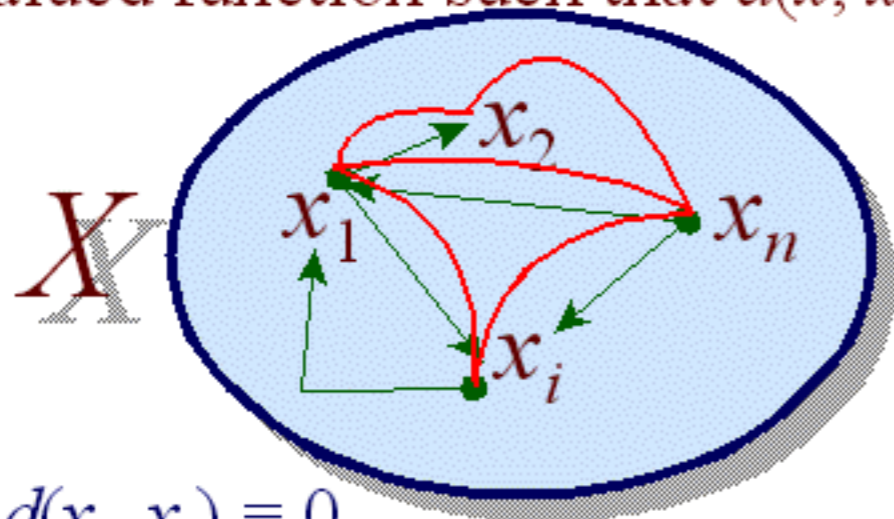
rocha@indiana.edu
<http://informatics.indiana.edu/rocha>



**INDIANA
UNIVERSITY**

Measured from associative “knowledge” graphs

d is a distance function on set X if it is a nonnegative, symmetric, real-valued function such that $d(x, x) = 0$ (Shore & Sawyer 1993)



$$d(x_i, x_i) = 0$$

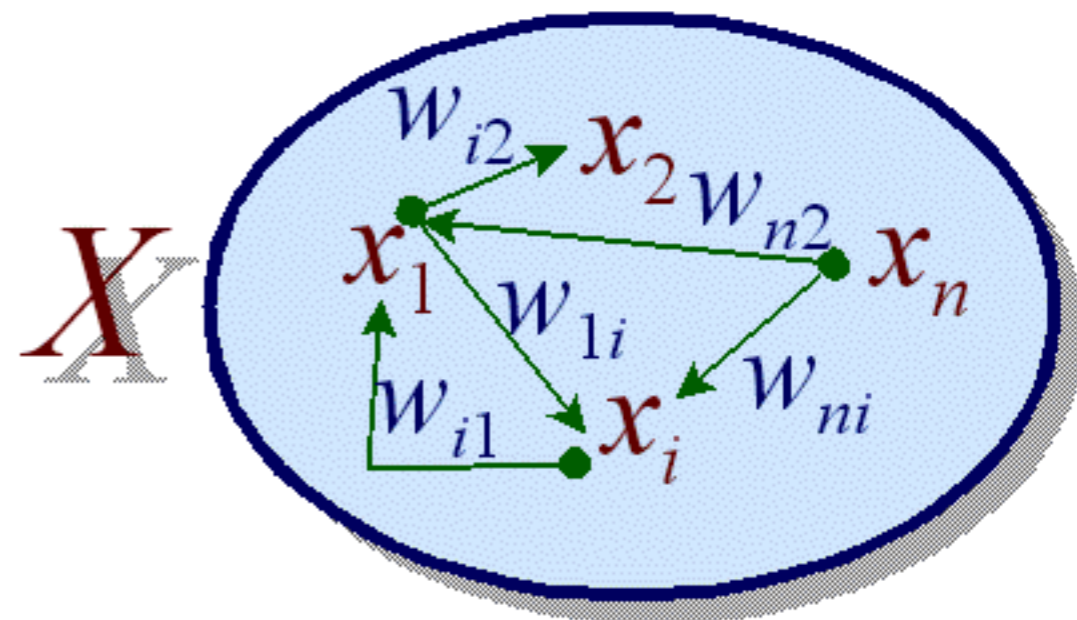
$$d(x_i, x_j) = 1, \text{ if there is an edge}$$

$$d(x_i, x_k) = d(x_i, x_j) + \dots + d(x_l, x_k) \quad 1, \text{ if there is a path}$$

Due to the symmetry requirement, distance functions yield non-directed distance graphs

$$d(x_1, x_2) \leq d(x_1, x_3) + d(x_3, x_2)$$

Metric: the smallest distance between nodes is always the most direct path



In real-valued weighted graphs, derived distance functions can be semi-metric

$$d(k_1, k_2) > d(k_1, k_3) + d(k_3, k_2)$$

Semi-metric

In graphs used to store “knowledge”, what does it mean?

operations

$$A(x) : X \rightarrow [0, 1]$$

Standard Fuzzy Operations

$$\bar{A}(x) = 1 - A(x)$$

$$(A \cap B)(x) = \min[A(x), B(x)]$$

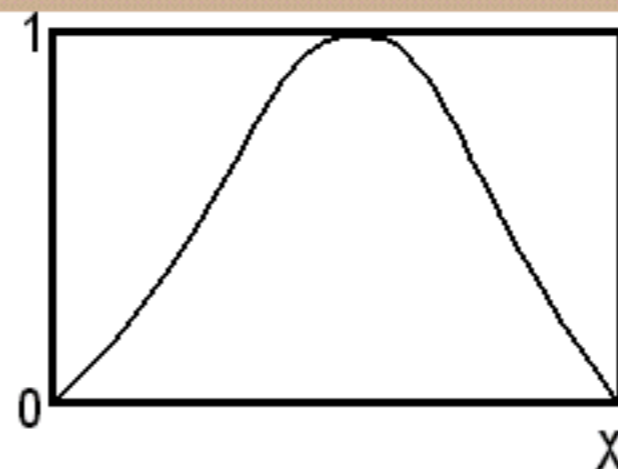
$$(A \cup B)(x) = \max[A(x), B(x)]$$

- Follows

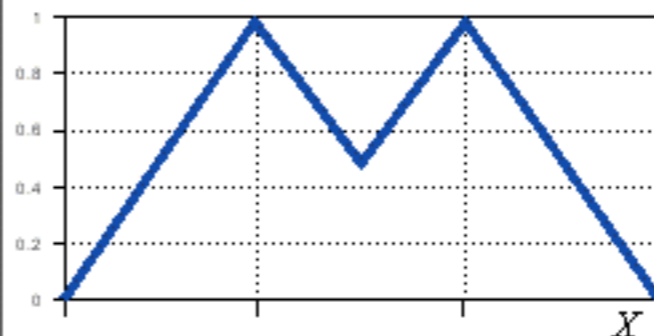
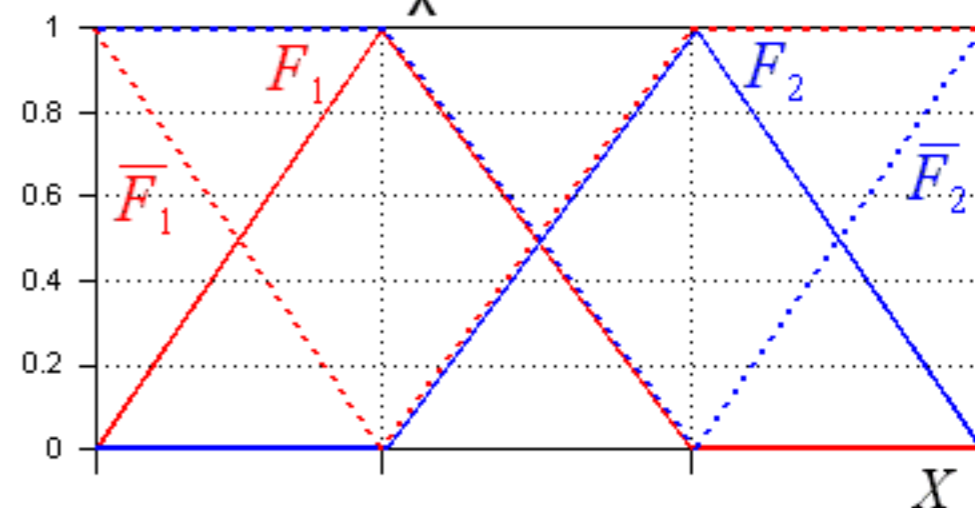
- Involution, commutativity, associativity, distributivity, Identity De Morgan's Laws, etc

- Does not Follow

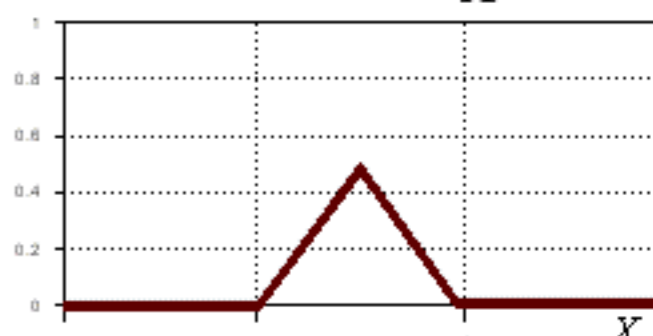
- Laws of contradiction and excluded middle

*Fuzziness*

Degree of Membership/Truth



Union



Intersection

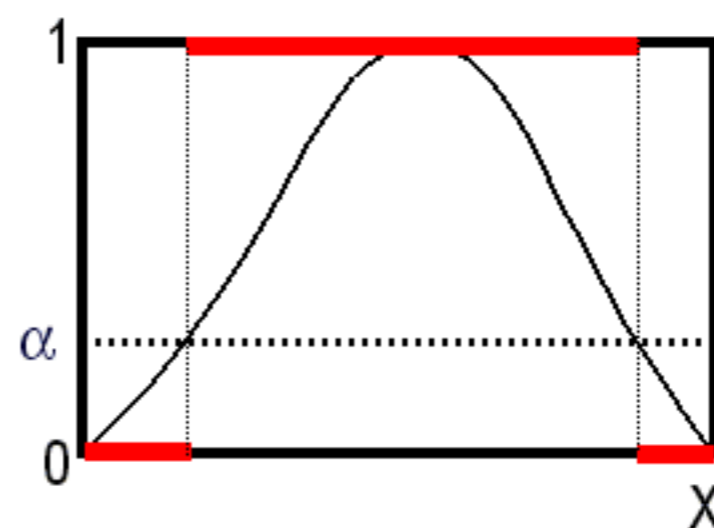
$$A \cap A \neq \emptyset$$

$$A \cup \bar{A} \neq X$$

α -cut

Fuzzy Sets:

$$A(x) : X \rightarrow [0, 1]$$



α -cut: Crisp set at threshold α

De Morgan's Laws

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$i(a, b) = ab$$

$$i(a, b) = \frac{ab}{a + b - ab}$$

$$u(a, b) = a + b - ab$$

$$u(a, b) = \frac{a + b - 2ab}{1 - ab}$$

■ Complement, Intersection and Union that follow De Morgan's Laws plus

► Complement

- Boundary Conditions: $c(0)=1$ and $c(1)=0$
- Monotonicity: if $a \leq b$ then $c(a) > c(b)$
- Continuity
- Involution: $c(c(a)) = a$

► Intersection (T-Norm)

- Boundary condition: $i(a, 1) = a$
- Monotonicity: if $b \leq d$ then $i(a, b) \leq i(a, d)$
- Comutativity: $i(a, b) = i(b, a)$
- Associativity: $i(a, i(b, d)) = i(i(a, b), d)$
- Continuity
- Strict Monotonicity: if $a_1 < a_2$ and $b_1 < b_2$ then $i(a_1, b_1) < i(a_2, b_2)$
- Subidempotency: $i(a, a) \leq a$

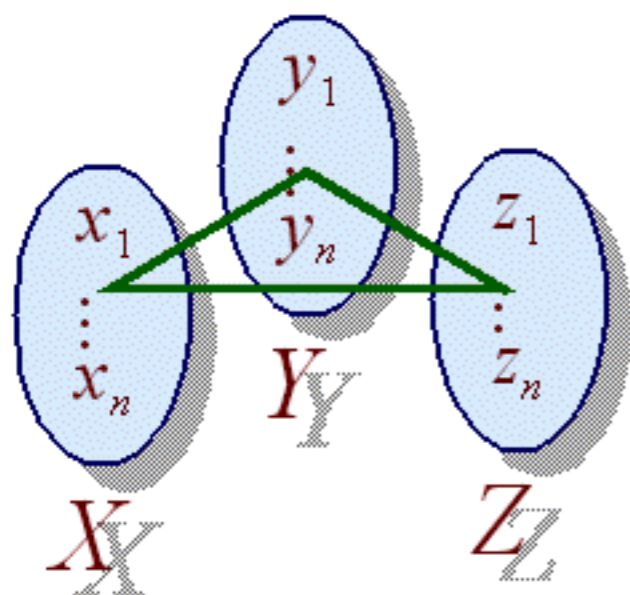
► Union (T-Conorm)

- Boundary condition: $u(a, 0) = a$
- Monotonicity: if $b \leq d$ then $u(a, b) \leq u(a, d)$
- Comutativity: $u(a, b) = u(b, a)$
- Associativity: $u(a, u(b, d)) = u(u(a, b), d)$
- Continuity
- Strict Monotonicity: if $a_1 < a_2$ and $b_1 < b_2$ then $u(a_1, b_1) < u(a_2, b_2)$
- Superidempotency: $u(a, a) > a$

$$c_\lambda(a) = \frac{1 - a}{1 + \lambda a}$$

Sugeno
Complement:
 $\lambda \in (-1, \infty)$

Represent the presence or absence of association, interaction or interconnectedness between the elements of two or more sets.

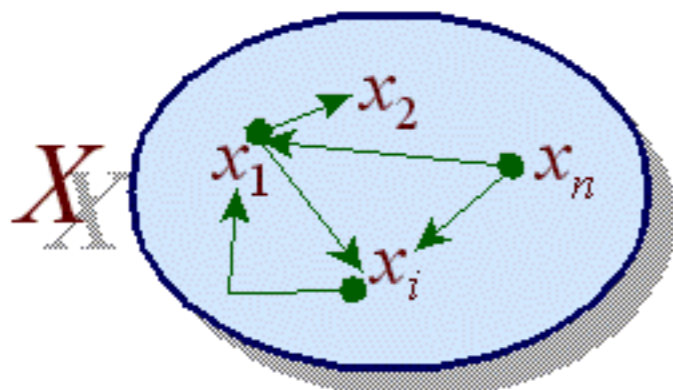
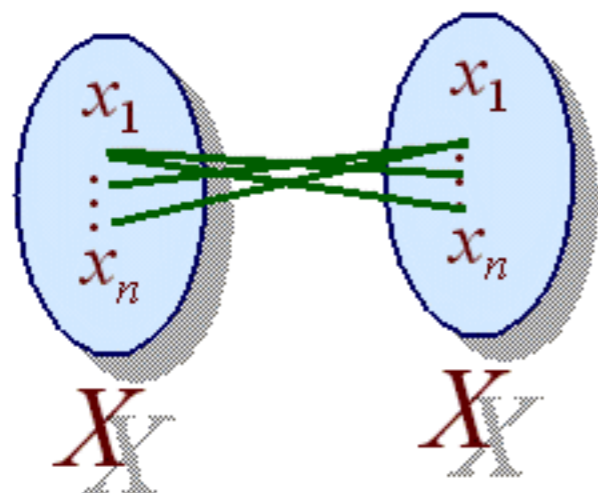
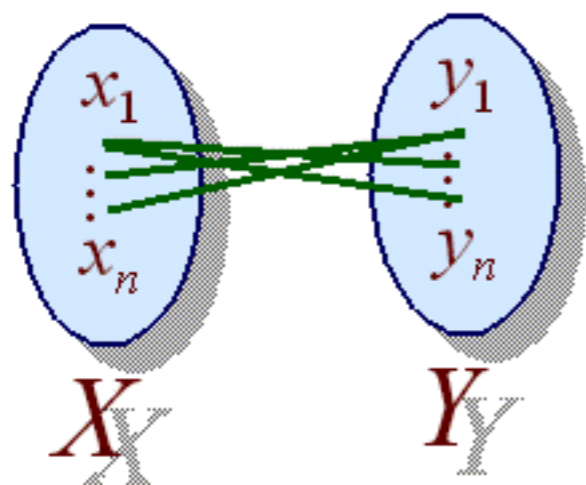


$$r(x_i, y_j, z_k) = 1$$

- a relation R between sets X_1, X_2, \dots, X_n is a subset of the Cartesian product of these sets: $R(X_1, X_2, \dots, X_n) \subseteq X_1 \times X_2 \times \dots \times X_n$.
 - ▶ Traditional logical operations between sets can be used to modify relations

$$R(x_1, x_2, \dots, x_n) = \begin{cases} 1 & \text{iff } (x_1, x_2, \dots, x_n) \in R \\ 0 & \text{otherwise} \end{cases}$$

$R(\mathbf{x}) \in [0, 1], \quad \forall \mathbf{x} \in \mathbf{X}$ **Fuzzy: Degree of Relation or association**

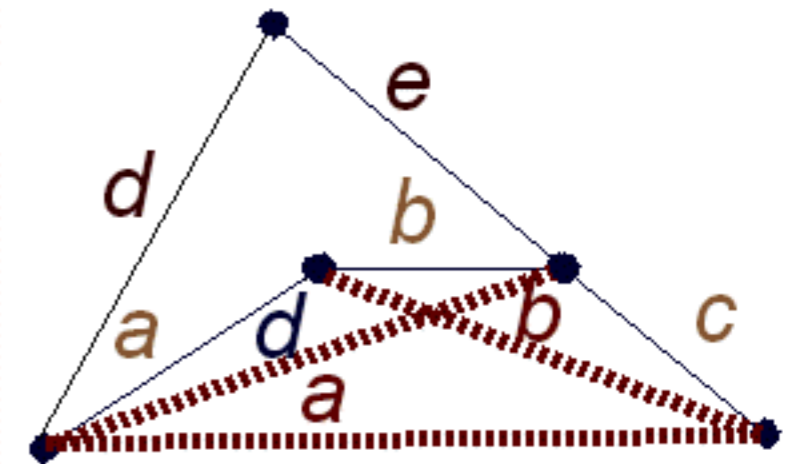


- Binary fuzzy relations are a generalization of real functions
 - ▶ Two or more elements of Y may relate to an element of X
 - ▶ Easily represented by matrices of dimension $n \times m$

- Graphs are binary relations defined on a single set: $R(X, X)$.
 - ▶ Degrees of association between elements of the same set
 - ▶ If symmetric, R represents a non-directed graph

properties

- Reflexive
 - ▶ iff $R(x, x) = 1$ for all $x \in X$
 - every element of X is maximally associated with itself
- Symmetric
 - ▶ iff $R(x, y) = R(y, x)$ for all $x, y \in X$
 - Matrices require only $(n^2 - n)/2$ elements to be defined
- (Max-Min) Transitive
 - ▶ iff $R(x, z) \geq \max_{y \in X} \min[R(x, y), R(y, z)]$ for all $x, z \in X$
 - For each indirect connection between x and z through some y , the weight of the connection is the smallest of each connection (x to y and y to z). Finally, the weight of the connection between x and z , is the largest of all indirect connections through all y (strongest path defined by *weakest link*)



Max-Min Transitivity

 $a < b < c$ $a < d < e$

Max-Min Composition: $R \circ R = \max_k \min(r_{ik}, r_{kj}) = r'_{ij}$
 where r_{ij} denotes $R(x_i, x_j)$

The max-min composition of matrices is performed in the same way as the numerical counterpart, except that *multiplication* and *summation* are substituted by the *Min* (\cap) and *Max* (\cup) operations respectively.

■ Transitive closure of a relation $R(X, X)$

- ▶ The relation that is transitive, contains $R(X, X)$, and whose elements have the smallest possible membership weights that still allow the first two requirements.
 - It yields a relation where all pairs of elements which were directly or indirectly related in the original relation, are now directly related
 - 1. $R' = R \cup (R \circ R)$; 2. If $R' \neq R$, make $R = R'$ and go back to step 1; 3. Stop: $R_T = R'$

Generic Composition:

$$R \circ R = \bigcup_k \bigcap (r_{ik}, r_{kj}) = r'_{ij}$$

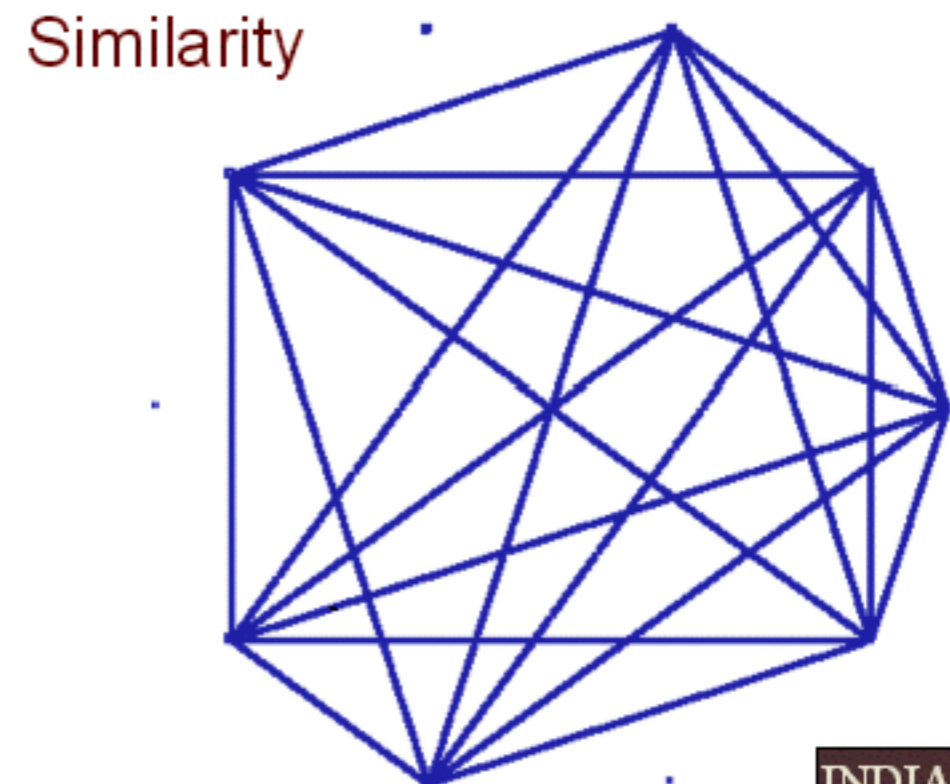
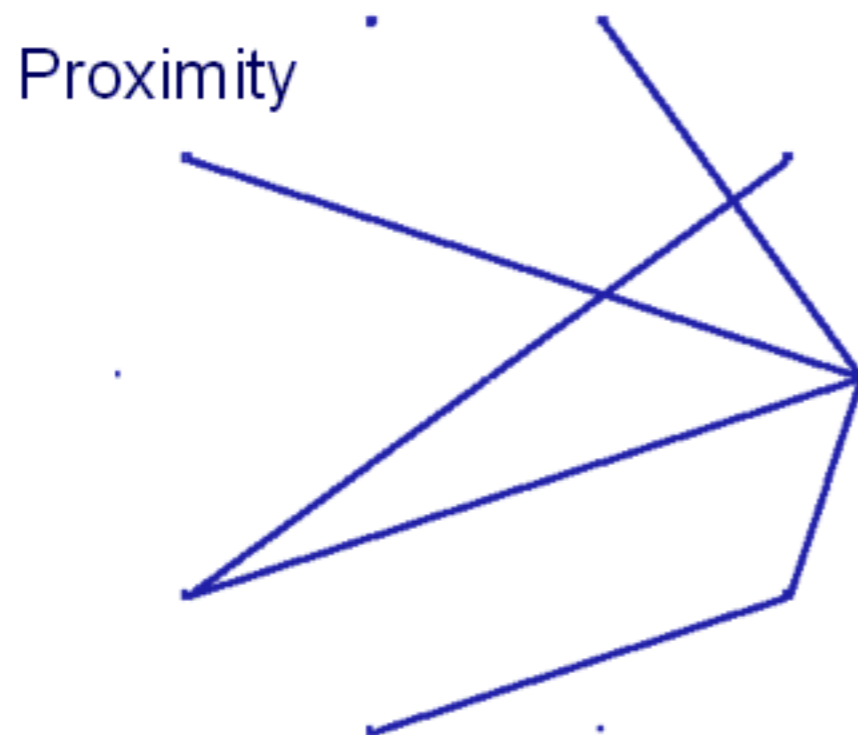


■ Similarity Relation

- ▶ A reflexive, symmetric, and transitive binary fuzzy relation
 - Also known as an equivalence relation.

■ Proximity Relation

- ▶ A reflexive and symmetric binary fuzzy relation
 - Also known as a compatibility relation
 - The transitive closure of a proximity relation is a similarity relation.





ACTIVE RecoMmenDation Project

Luis M. Rocha, Andreas Rechtsteiner,
Tiago Simas, Chien-Feng Huang, Judith
Cohn, Eugene Gavrilov, Johan Bollen

<http://arp.lanl.gov>

Building Adaptive Webs that co-evolve with user communities

- Extraction of co-occurrence (associative) networks
 - ▶ Represent associative knowledge of information resources and users
- Identification of implicit associations in networks
 - ▶ Discovery of relevant items
 - ▶ Identify Communities of Users
- Conversation amongst information resources
 - ▶ driven by uncertainty reduction
 - ▶ Produce context-specific, proactive recommendations
- Collective Adaptation of network architecture
 - ▶ Evolving knowledge organization

from document relations

- Document × Keyterms
 - Keyterm Co-Occurrence
- Document × Document
 - Co-Citation or Hyperlink structure
- Document × Author
 - Co-Authorship (Collaboration Network)
- Genes × MeSH Keyterms
 - Gene/keyterm Co-Occurrence

	X (Keywords)
Y (Documents)	$R: X \times Y$

Given a binary relation R between sets X and Y we extract two proximity relations: $XYP(x_i, x_j)$ is the probability that both x_i and x_j are related in R to the same element $y \in Y$. Conversely, $YXP(y_i, y_j)$ is the probability that both y_i and y_j are related in R to the same element $x \in X$.

$$XYP(x_i, x_j) = \frac{\sum_{k=1}^m (r_{i,k} \wedge r_{j,k})}{\sum_{k=1}^m (r_{i,k} \vee r_{j,k})}; \quad YXP(y_i, y_j) = \frac{\sum_{k=1}^n (r_{k,i} \wedge r_{k,j})}{\sum_{k=1}^n (r_{k,i} \vee r_{k,j})}$$

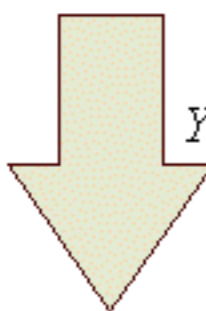
With some support constraint

produce associative (probabilistic) networks

	X (Keywords)
Y (Documents)	$R: X \times Y$

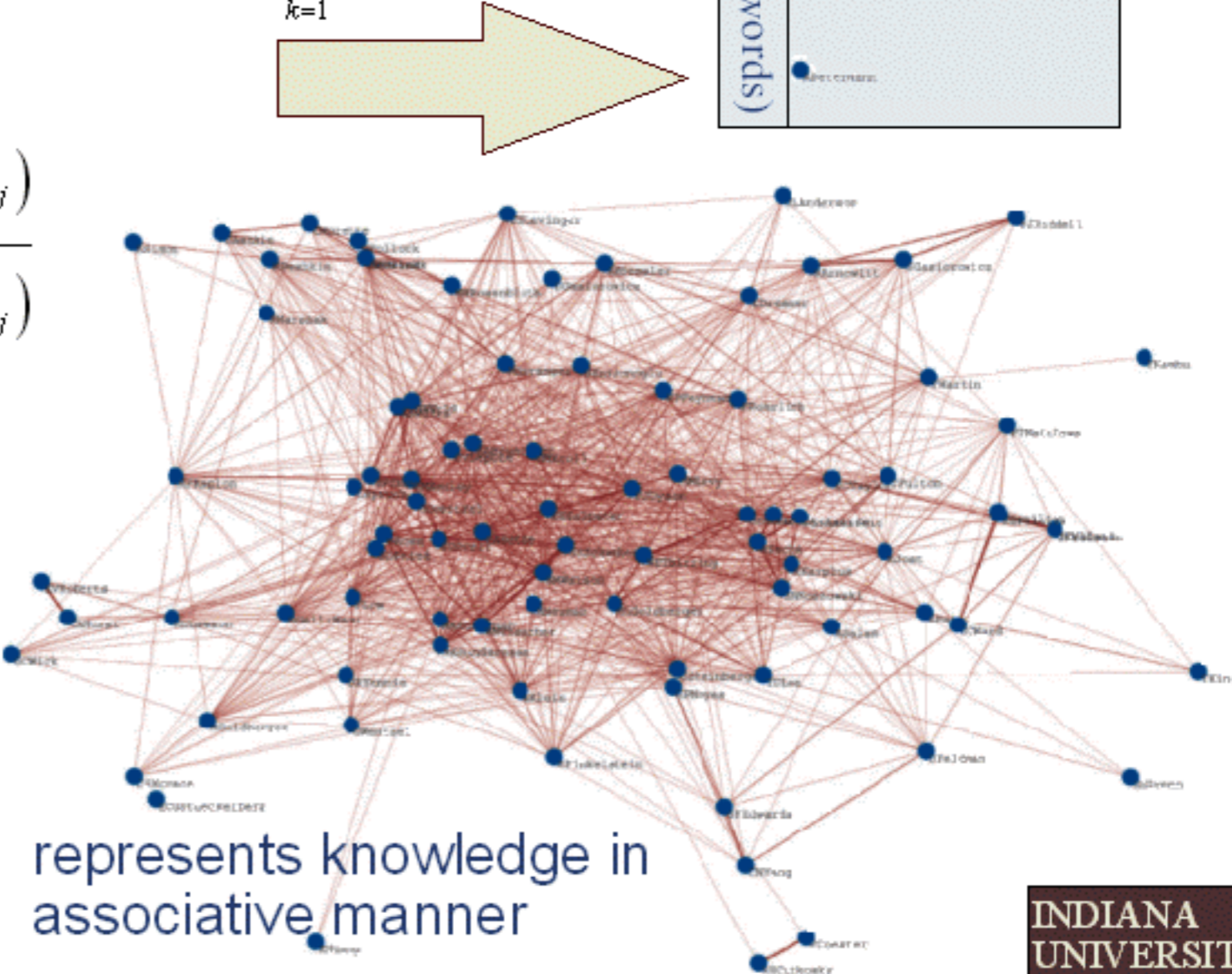
$$XYP(x_i, x_j) = \frac{\sum_{k=1}^m (r_{i,k} \wedge r_{j,k})}{\sum_{k=1}^m (r_{i,k} \vee r_{j,k})}$$

	X (Keywords)
X (Keywords)	$XYP: X \times X$



$$YXP(y_i, y_j) = \frac{\sum_{k=1}^n (r_{k,i} \wedge r_{k,j})}{\sum_{k=1}^n (r_{k,i} \vee r_{k,j})}$$

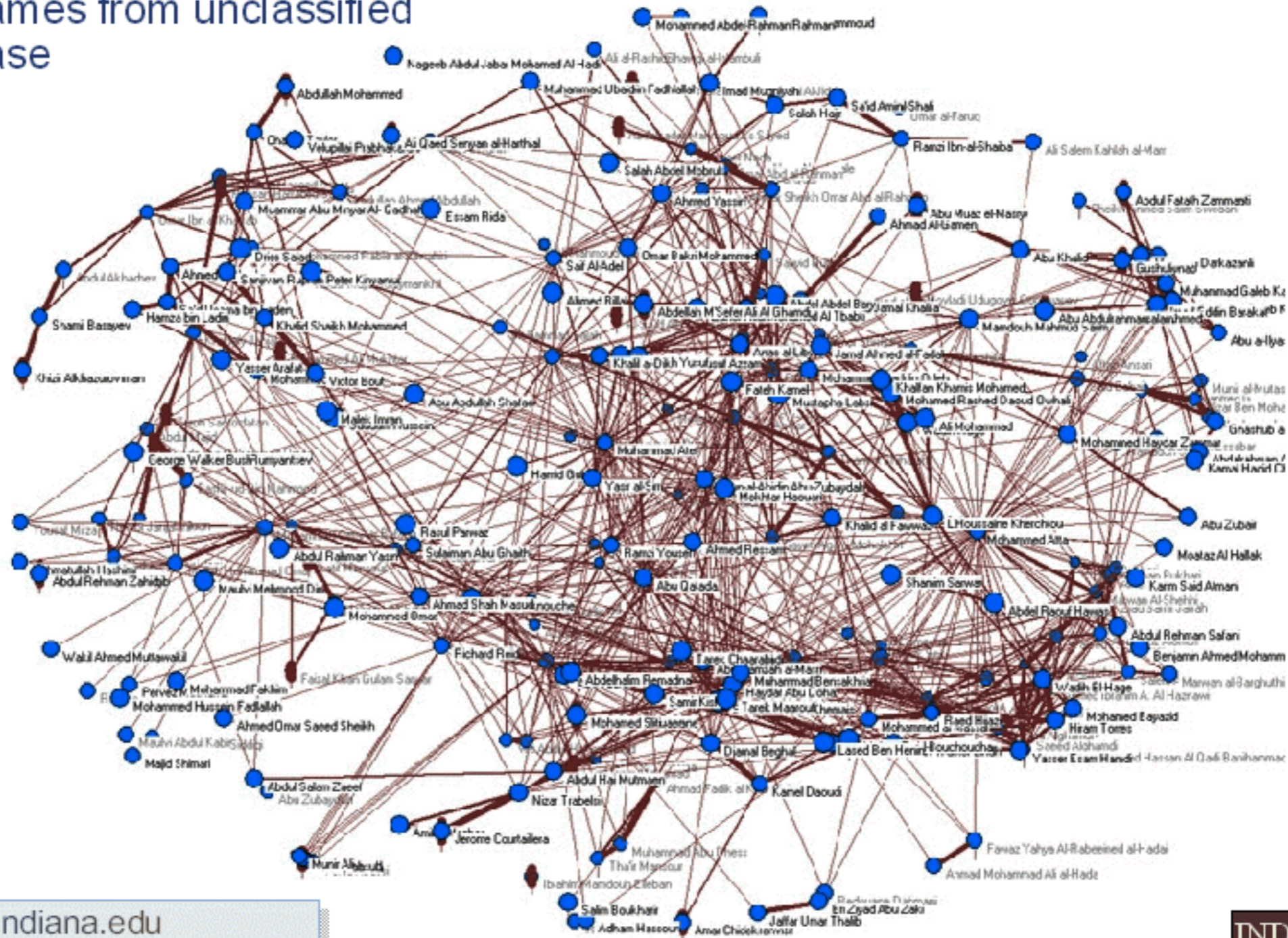
	Y (Documents)
Y (Documents)	$YXP: Y \times Y$



represents knowledge in associative manner

PDP2

318 names from unclassified database

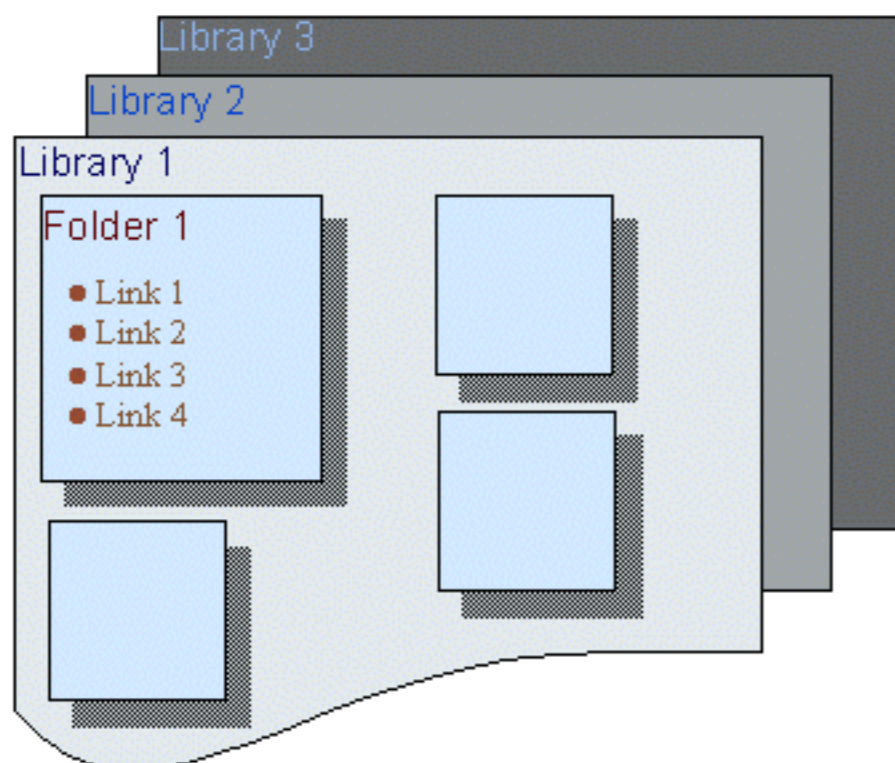


rocha@indiana.edu
<http://informatics.indiana.edu/rocha>



informatics
luis rocha 2005

architecture

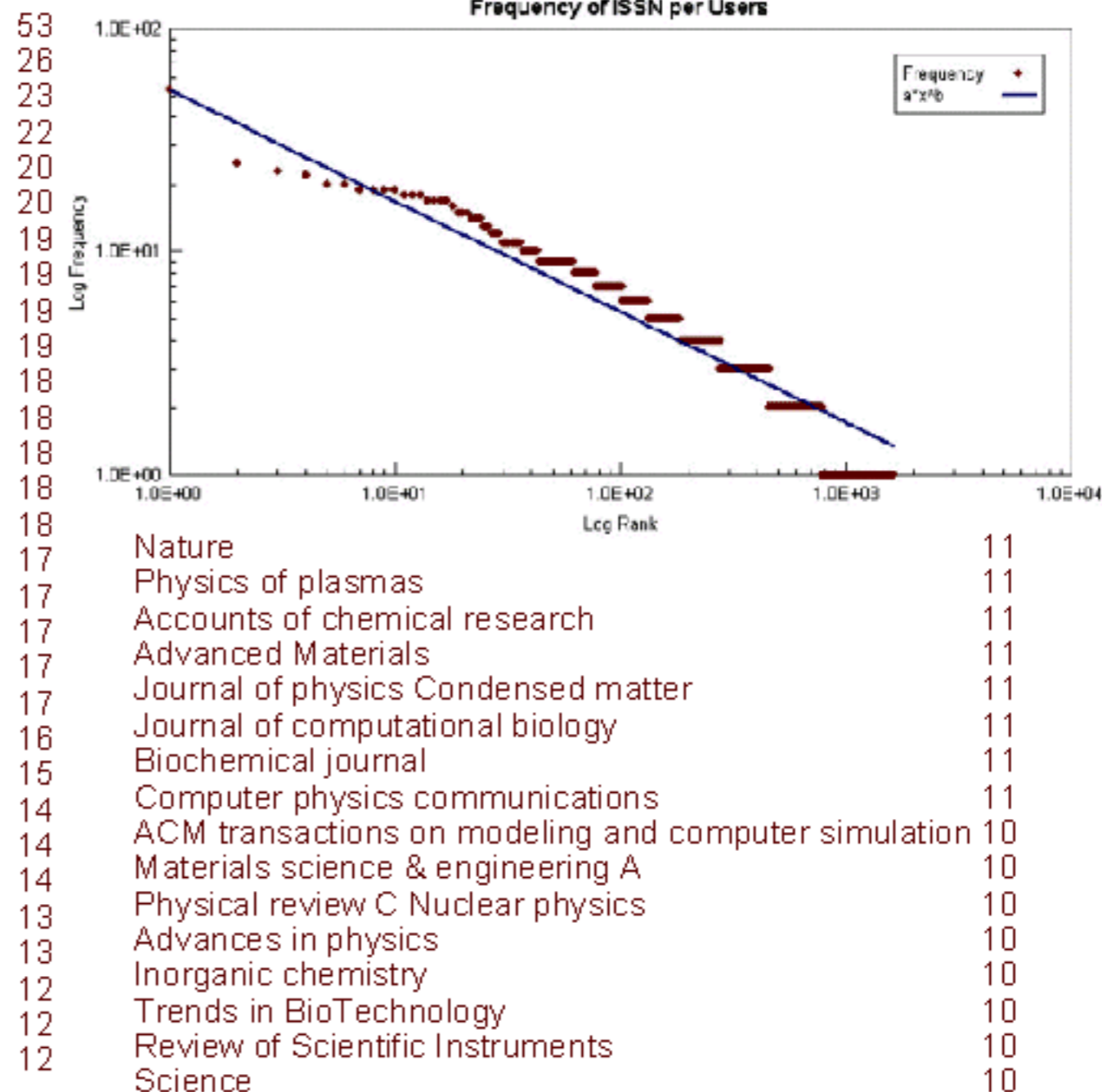


■ Three nested entities:

- ▶ Libraries \Rightarrow Folders \Rightarrow Links
 - A *library/personality* is associated with a given area of interest and consists of one or more folders.
 - A *folder* contains related types of links within a library
 - A *link* is a URL.

occurrences in personalities

Physical review letters
 Physical Review B
 Physical review E
 Physical review A General physics
 Journal of physical chemistry B
 Computers & geosciences
 Scientific American
 Journal of the American Chemical Society
 Journal of Chemical Physics
 Reviews of modern physics
 Bioinformatics
 IEEE trans. on geoscience and remote sensing
 PNAS
 Journal of computational physics
 Advances in water resources
 Journal of applied geophysics
 Applied geochemistry
 APL
 Journal of physical chemistry A
 Phil. mag. B Physics of condensed matter
 Bul. of Environmental Contamination and Toxicology
 Journal of applied physics
 American journal of physics
 Analytical chemistry
 DLib
 Chemical physics
 NIM
 Chemical physics letters
 Physics reports
 Physical review A



rocha@indiana.edu
<http://informatics.indiana.edu/rocha>

from co-occurrence in mylibrary.lanl.gov

	ISSN
Personality	$A: P \times I$

326 personalities with at least one ISSN
 253 users with at least one ISSN
 623 ISSN occurring at least twice

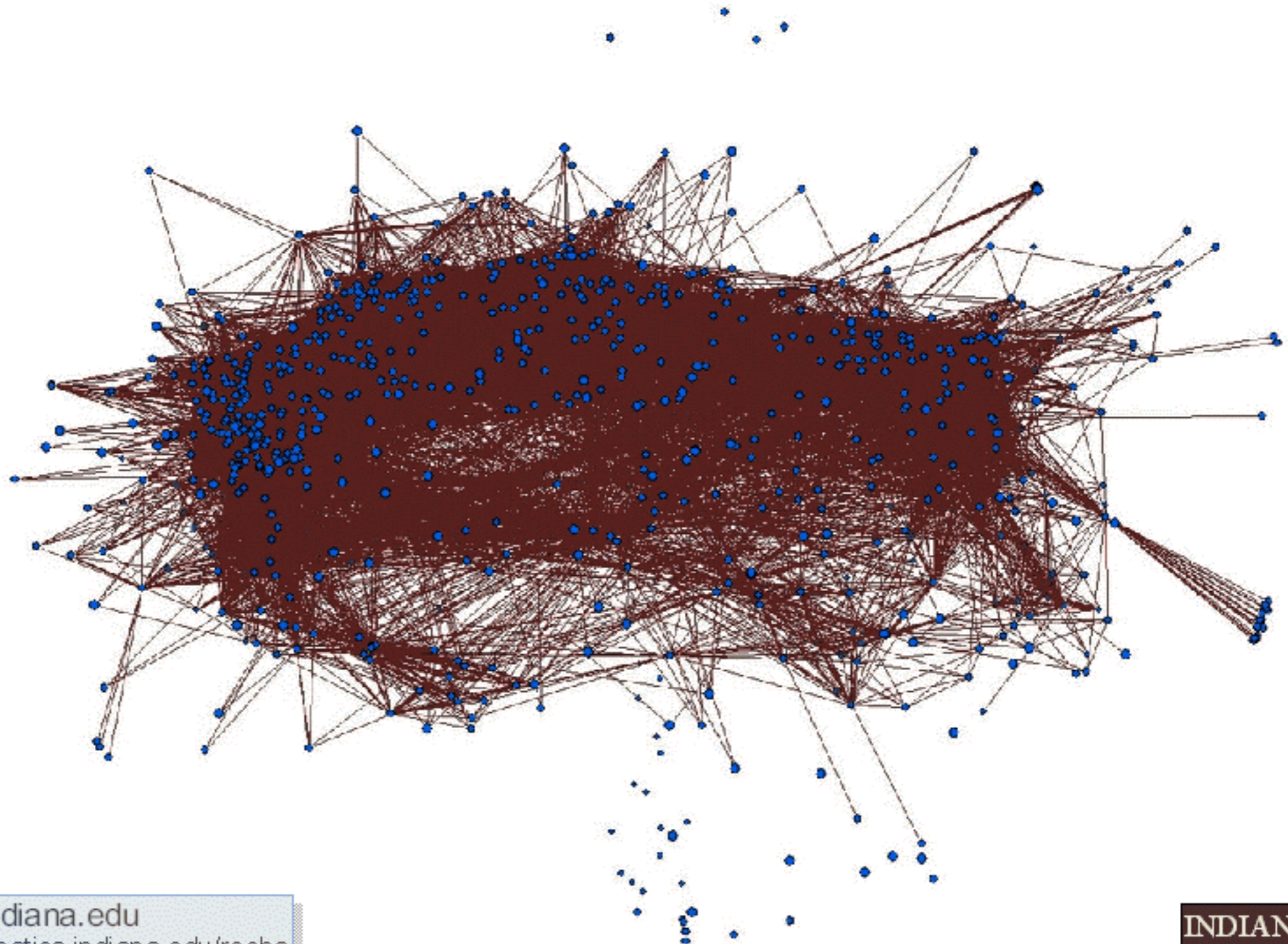
Given a binary relation A between sets of Personalities P and ISSN I we extract two proximity relations: $PIP(p_s, p_t)$ is the probability that both personalities p_s and p_t link to the same ISSN $i \in I$. Conversely, $IPP(i_s, i_t)$ is the probability that both ISSN i_s and i_t co-occur in the same personality (given that one of them occurs) $p \in P$.

$$pip(p_s, p_t) = \frac{\sum_{k=1}^m (a_{i,k} \wedge a_{j,k})}{\sum_{k=1}^m (a_{i,k} \vee a_{j,k})} = \frac{N_{\cap}(p_s, p_t)}{N_{\cup}(p_s, p_t)} \quad ipp(i_s, i_t) = \frac{\sum_{k=1}^m (a_{i,k} \wedge a_{j,k})}{\sum_{k=1}^m (a_{i,k} \vee a_{j,k})} = \frac{N_{\cap}(i_s, i_t)}{N_{\cup}(i_s, i_t)}$$

(Personality ISSN Proximity)

(ISSN Personality Proximity)

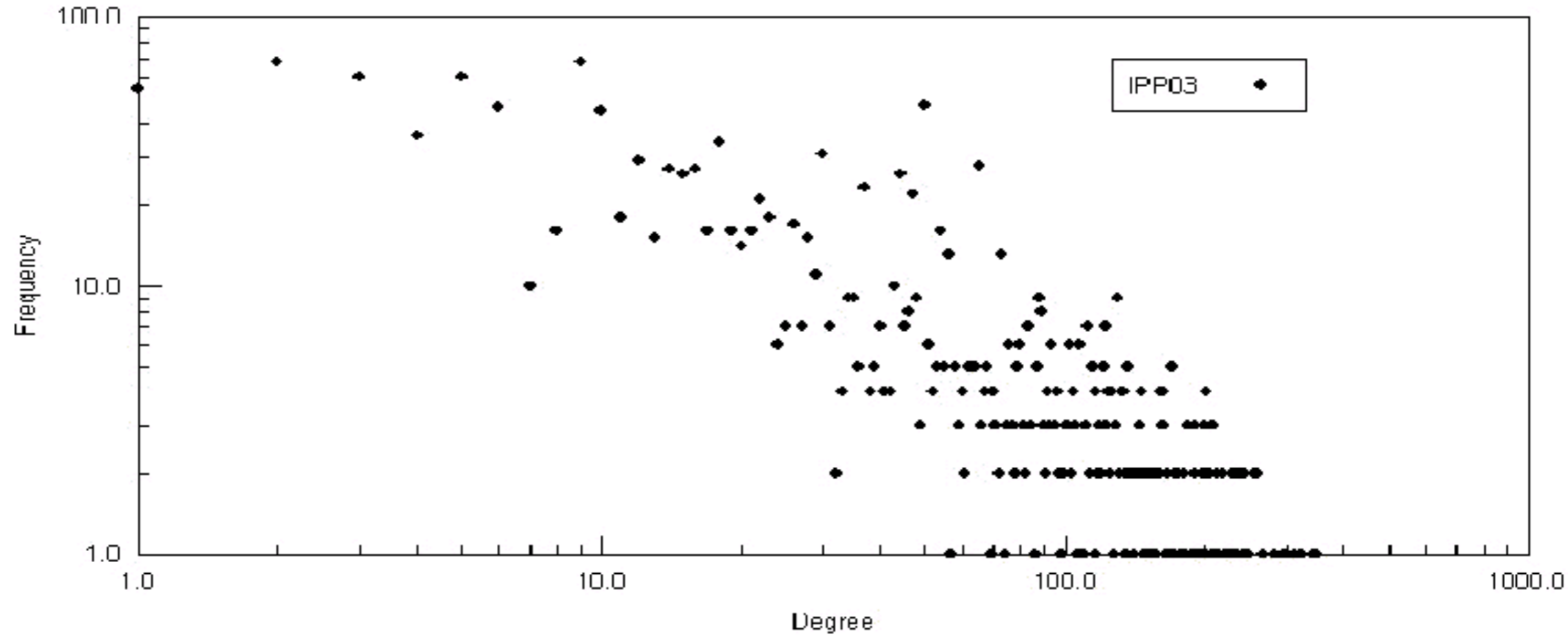
from co-occurrence in user personalities in mylibrary.lanl.gov: IPP



rocha@indiana.edu
<http://informatics.indiana.edu/rocha>



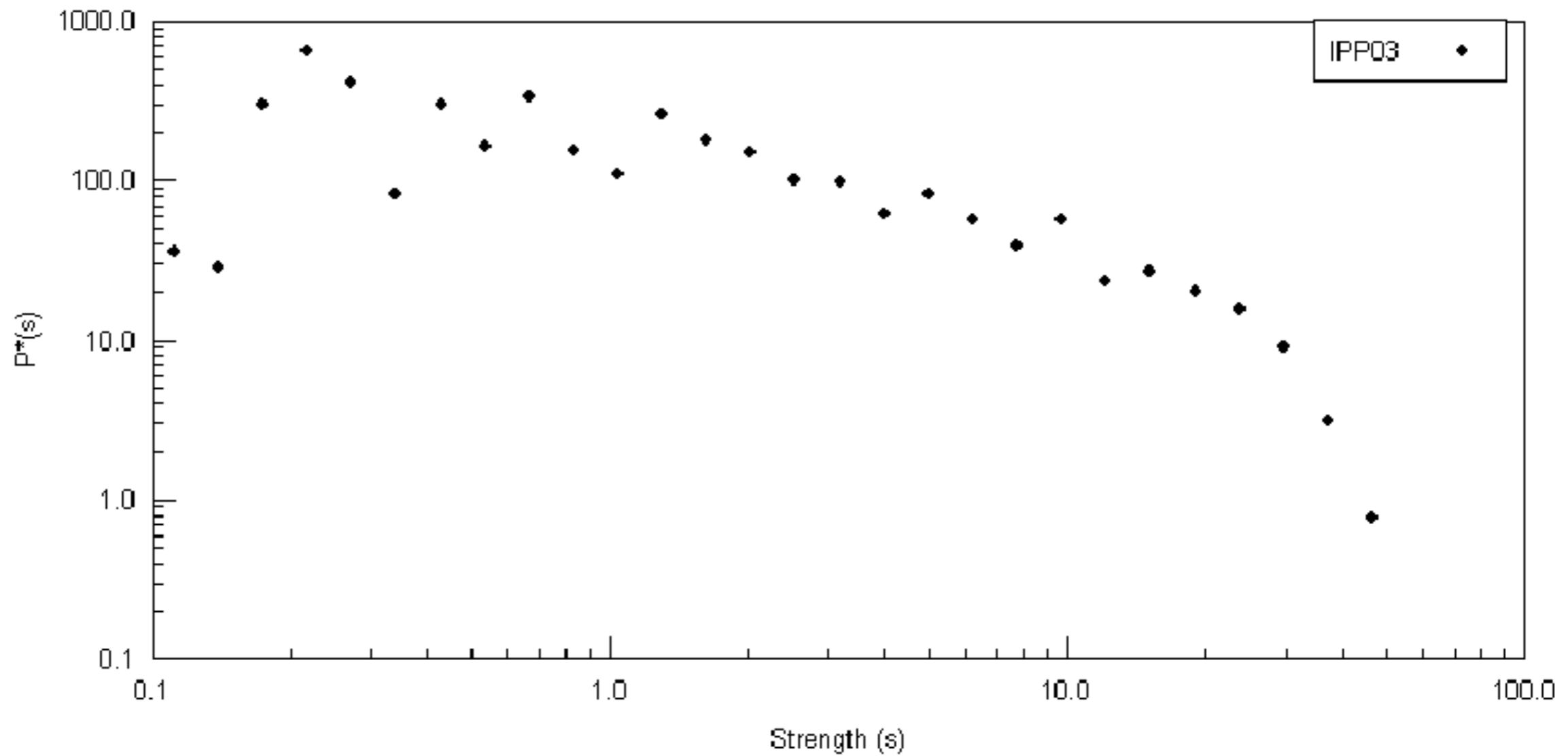
All weights



informatics
luis rocha 2005

cumulative strength distribution (binned)

all weights



informatics
luis rocha 2005

rocha@indiana.edu
<http://informatics.indiana.edu/rocha>

INDIANA
UNIVERSITY

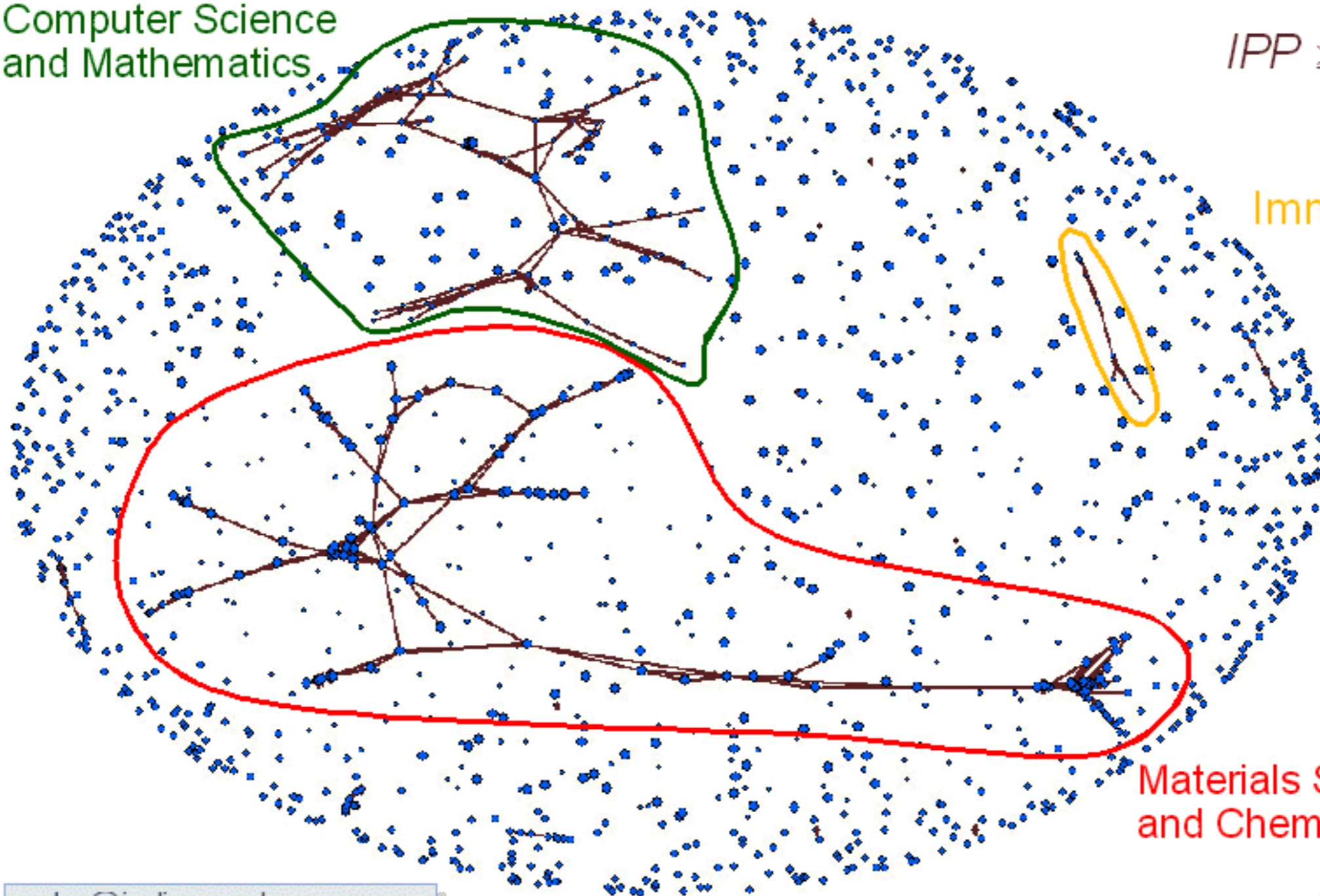
from co-occurrence in user personalities in mylibrary.lanl.gov: IPP

Computer Science
and Mathematics

$IPP \geq 0.5$

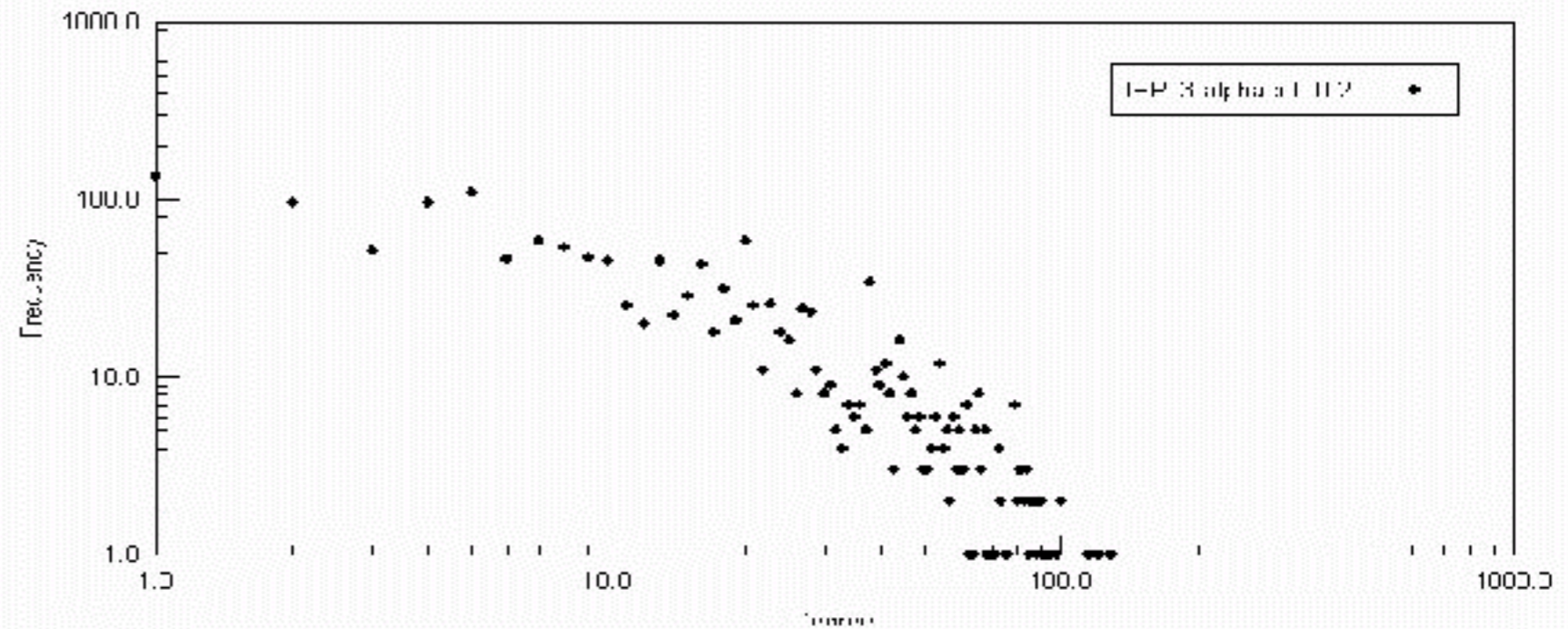
Immunology

Materials Science
and Chemistry

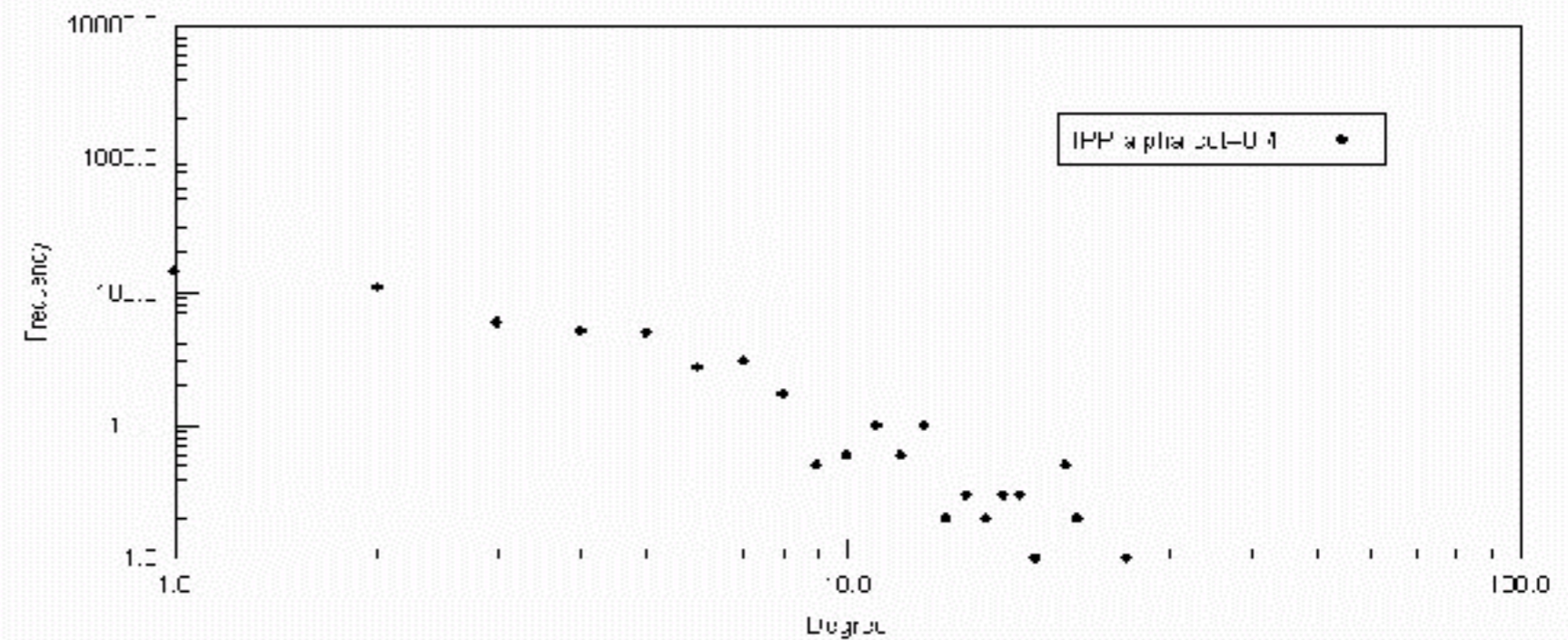


cumulative degree distribution

α -cut = 0.2



α -cut = 0.4

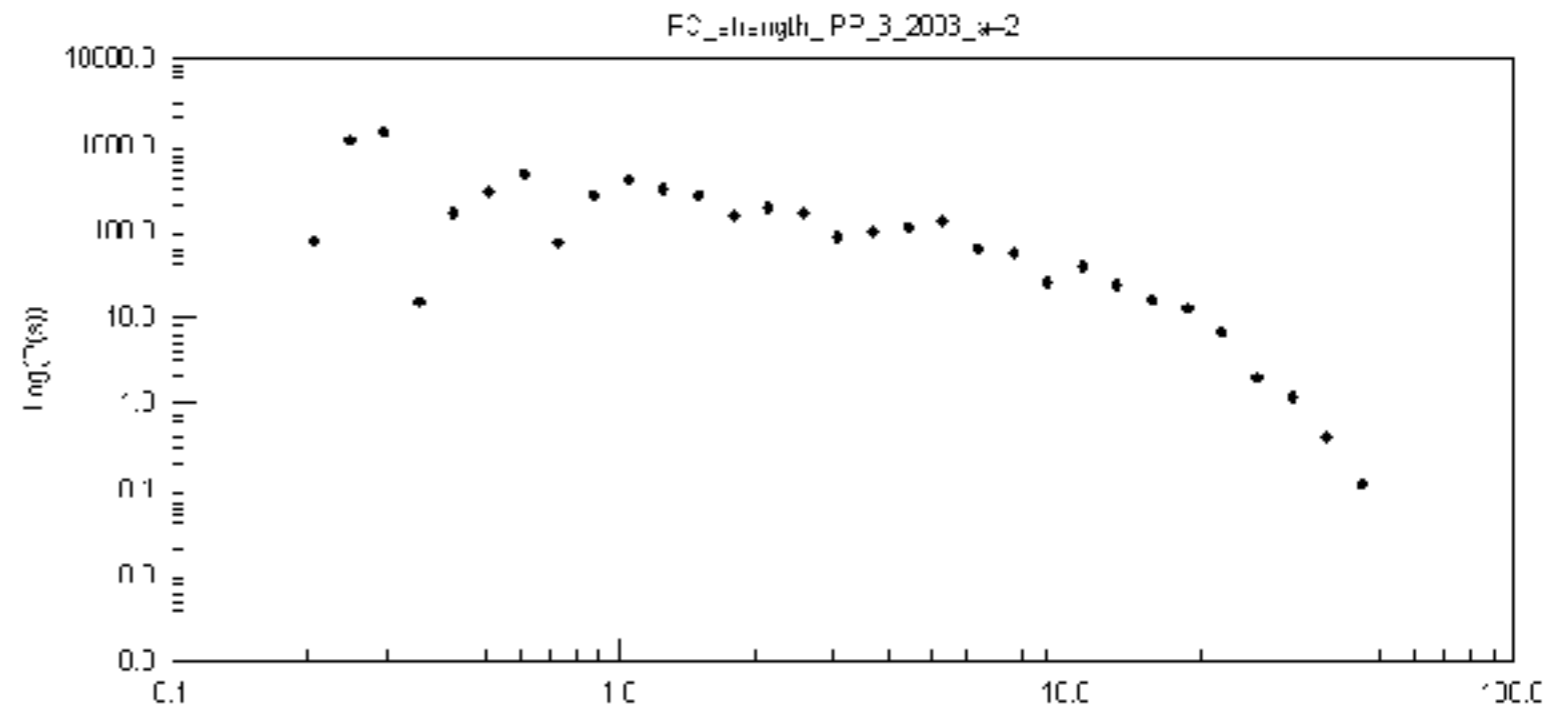


informatics
luis rocha 2005

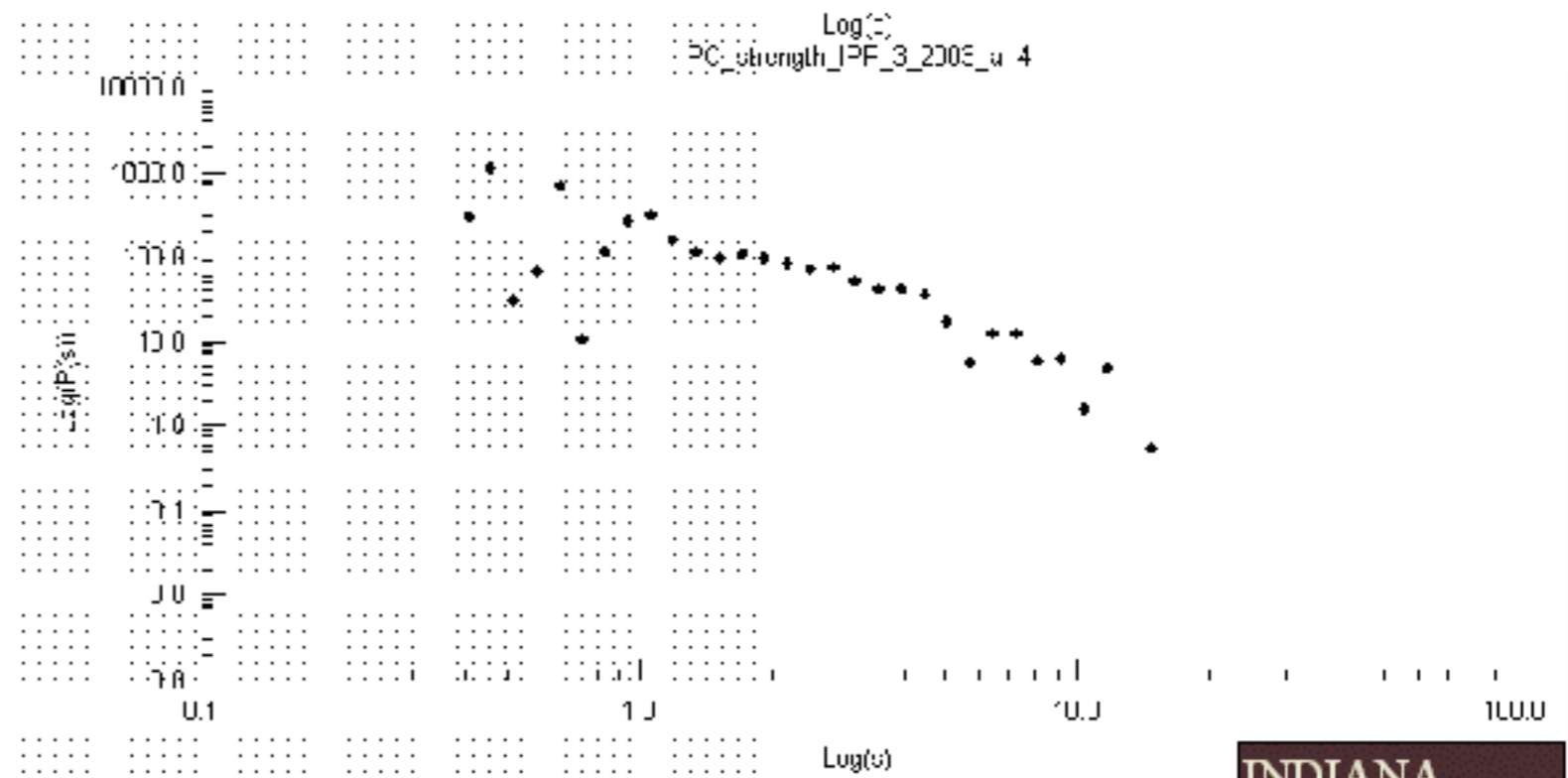


cumulative strength distribution (binned)

α -cut = 0.2



α -cut = 0.4



top 5 most frequent ISSN and their neighbors

$$ipp \geq 0.3$$

$$ipp(i_s, i_t) = \frac{N_{\cap}(i_s, i_t)}{N_{\cup}(i_s, i_t)} \geq 3$$

0031-9007--Physical review letters

1095-3787 --Physical review E: 0.4074

0556-2805--Physical Review B: 0.3621

0034-6861--Reviews of modern physics: 0.3333

0556-2791--Physical review A General physics: 0.3636

0556-2805--Physical Review B

0031-9007--Physical review letters: 0.3621

0370-1573--Physics reports: 0.3103

0921-4526--Physica B Condensed matter: 0.3462

0921-4534--Physica C Superconductivity: 0.3462

0034-6861--Reviews of modern physics: 0.3235

0556-2791--Physical review A General physics: 0.3333

1434-6036--European physical journal B: 0.3077

1095-3787--Physical review E

0031-9007--Physical review letters: 0.4074

0556-2791 --Physical review A General physics

0031-9007--Physical review letters: 0.3636

0370-1573--Physics reports: 0.3077

0556-2805--Physical Review B: 0.3333

1089-490x --Physical review C Nuclear physics: 0.3913

0034-6861--Reviews of modern physics: 0.4643

1434-6036--European physical journal B: 0.3043

1089-5647 --Journal of physical chemistry B

0002-7863--Journal of the American Chemical Society: 0.3000

0021-9606--Journal of Chemical Physics: 0.6250

1089-5639--Journal of physical chemistry A: 0.7619

0009-2614--Chemical physics letters: 0.6000

0301-0104--Chemical physics: 0.5714

0743-7463--Langmuir: 0.3810

mylibrary.lanl.gov

■ IPP

- ▶ Recommendations of ISSN based on co-occurrence in Personalities
 - Users who linked to this journal, also linked to...

■ PIP

- ▶ Recommendations of other users' personalities: collaboration
 - These personalities are similar to yours
- ▶ Recommendations of specific links in close personalities
 - Users who read many of the same journals where interested in these links

Bollen, Johan, Luis M. Rocha [2000]. "An Adaptive Systems Approach to the Implementation and Evaluation of Digital Library Recommendation Systems." In: *Research and Advanced Technology for Digital Libraries: 4th European Conference, ECDL 2000. Lectures Notes in Computer Science*, Springer-Verlag, pp.356-359.

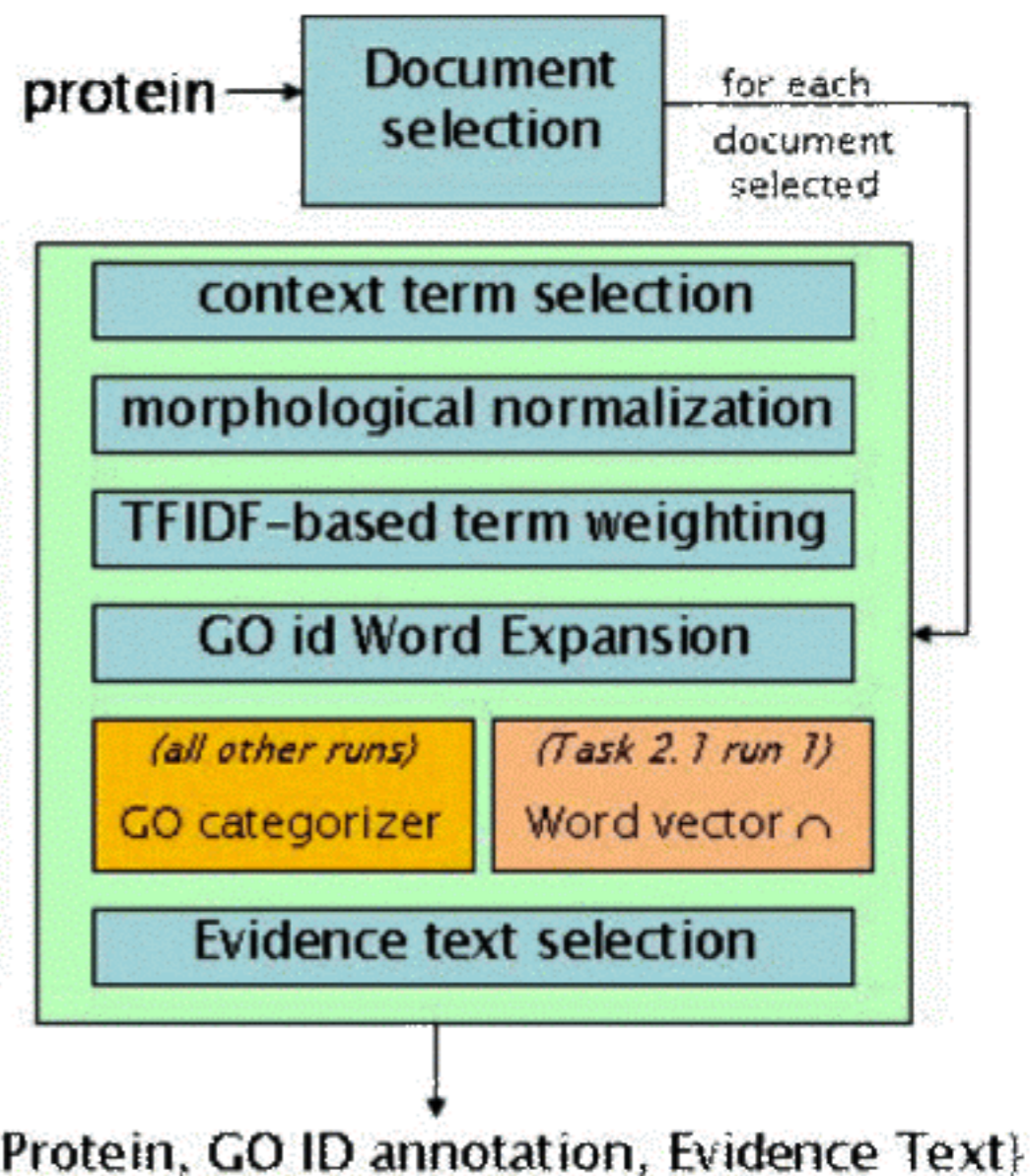
Rocha, Luis M. and Johan Bollen [2001]. "Biologically Motivated Distributed Designs for Adaptive Knowledge Management". In: *Design Principles for the Immune System and other Distributed Autonomous Systems*. L. Segel and I. Cohen (Eds.) Santa Fe Institute Series in the Sciences of Complexity. Oxford University Press, pp. 305-334.

Rocha, Luis M. [2002]. "Combination of Evidence in Recommendation Systems Characterized by Distance Functions". In: *Proceedings of the 2002 World Congress on Computational Intelligence: FUZZ-IEEE'02*. Honolulu, Hawaii, May 2002. IEEE Press, pp. 203-208.

Biocreative competition (EMBO Workshop)

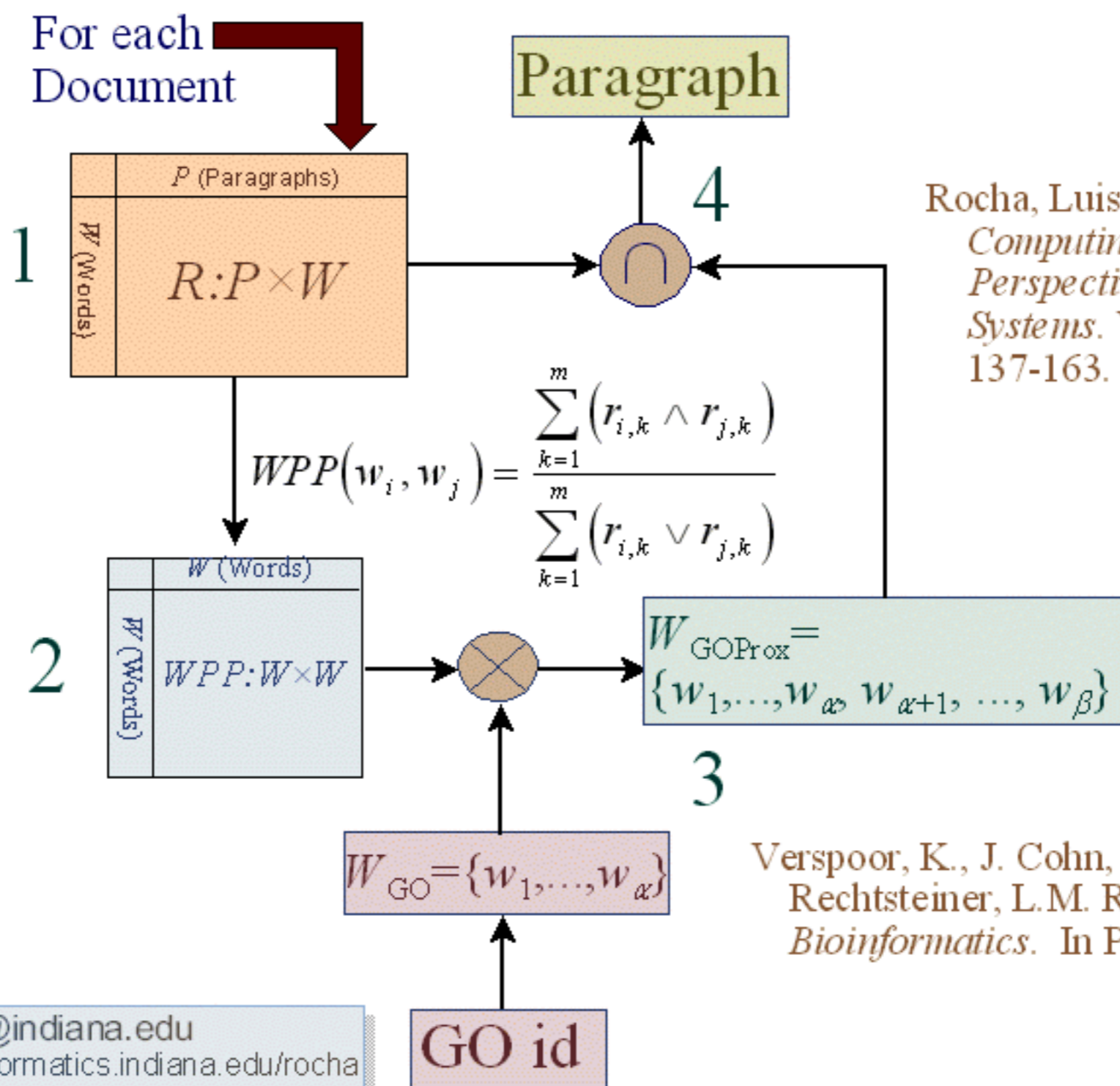
a critical assessment of text mining methods in molecular biology

- Task 2: Given a document, discover the portion of text most appropriate to annotate the protein's function, and produce appropriate Gene Ontology node for annotation
 - ▶ Learning set: triplets (protein, document, GO id)
 - ▶ Test set: same



Verspoor, K., J. Cohn, C. Joslyn, S. Mniszewski, A. Rechtsteiner, L.M. Rocha, T. Simas [2004]. *BMC Bioinformatics*. In Press.

based on proximity measure

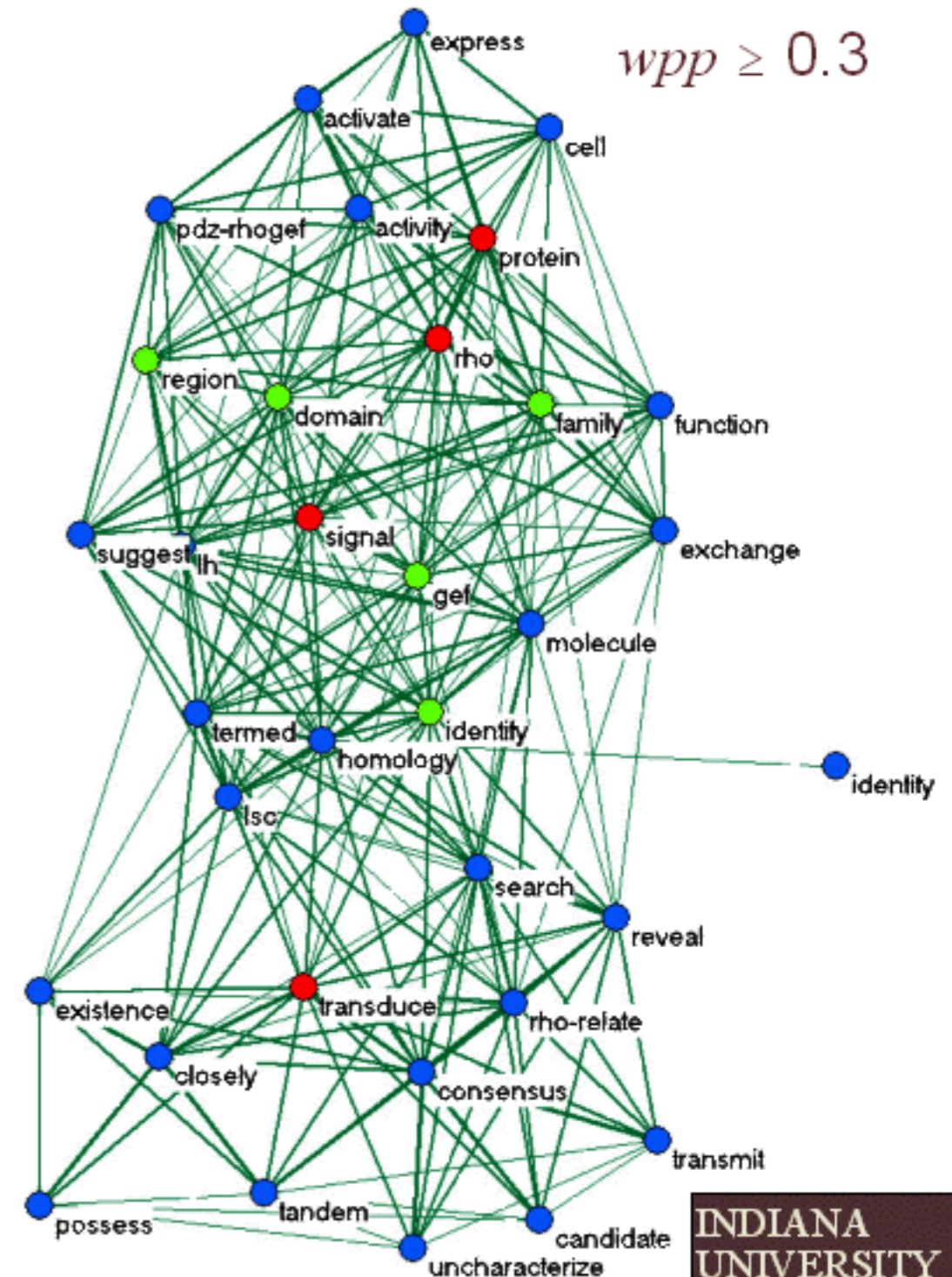


Rocha, Luis M. [2002]. In: *Soft Computing Agents: A New Perspective for Dynamic Information Systems*. V. Loia (Ed.) IOS Press, pp. 137-163.

Verspoor, K., J. Cohn, C. Joslyn, S. Mniszewski, A. Rechtsteiner, L.M. Rocha, T. Simas [2004]. *BMC Bioinformatics*. In Press.

example document

- document bc005868
 - ▶ *WPP* contains 1102 words
 - ▶ Subgraph of 34 words
 - Red nodes: words removed from the respective GO annotation (0007266): Rho, protein, signal, transducer).
 - Blue nodes: words that co-occur very frequently ($wpp > 0.5$) with at least one of the red nodes
 - Green nodes: additional words recommended with largest average proximity to all input words (red nodes)



Verspoor, K., J. Cohn, C. Joslyn, S. Mniszewski, A. Rechtsteiner, L.M. Rocha, T. Simas [2004]. *BMC Bioinformatics*. In Press.



Task 2.1 Results


 Proximity-based run

User, Run	"perfect"	"generally"	cumulative
7, 1	25.28%	14.31%	39.59%
14, 1	22.16%	6.41%	34.57%
20, 1	27.97%	5.30%	33.27%
4, 1	24.91%	6.88%	31.78%
20, 2	26.02%	5.58%	31.60%
20, 3	22.21%	5.48%	27.70%
5, 2	15.43%	8.36%	23.79%
5, 1	15.43%	7.16%	22.58%
5, 3	14.31%	7.99%	22.30%
15, 2	11.62%	6.41%	18.03%
9, 1	11.62%	1.21%	12.83%
7, 3	6.13%	3.72%	9.85%
17, 1	7.71%	1.77%	9.48%
15, 1	5.48%	2.60%	8.09%
7, 2	4.00%	3.72%	7.71%
10, 3	4.65%	0.37%	5.02%
9, 3	3.81%	0.65%	4.46%
10, 2	4.18%	0.19%	4.37%
10, 1	3.35%	0.28%	3.62%
9, 2	3.07%	0.46%	3.53%
17, 2	0.65%	0.00%	0.65%

Verspoor, K., J. Cohn, C. Joslyn,
S. Mniszewski, A. Rechtsteiner,
L.M. Rocha, T. Simas [2004].
BMC Bioinformatics. In Press.

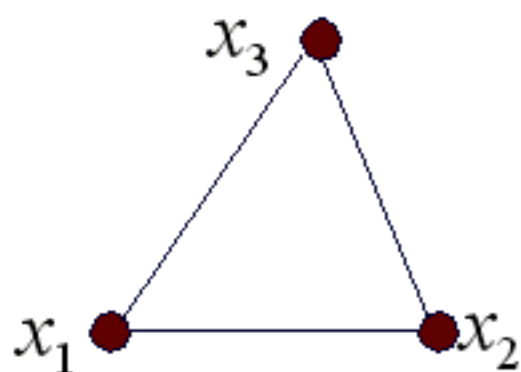
rocha@indiana.edu
<http://informatics.indiana.edu/rocha>

identification of implicit associations in networks

semi-metric behavior

$$d_X(x_i, x_j) = \frac{1}{XYP(x_i, x_j)} - 1; \quad d_Y(y_i, y_j) = \frac{1}{YXP(y_i, y_j)} - 1$$

d is a distance function because it is a nonnegative, symmetric, real-valued function such that $d(k, k) = 0$



Distance from a Proximity Graph is semi-Metric
Distance from a Similarity Graph is Metric

$$d(x_1, x_2) \leq d(x_1, x_3) + d(x_3, x_2)$$

Metric

$$d(x_1, x_2) > d(x_1, x_3) + d(x_3, x_2)$$

Semi-metric

Evolution

3.89

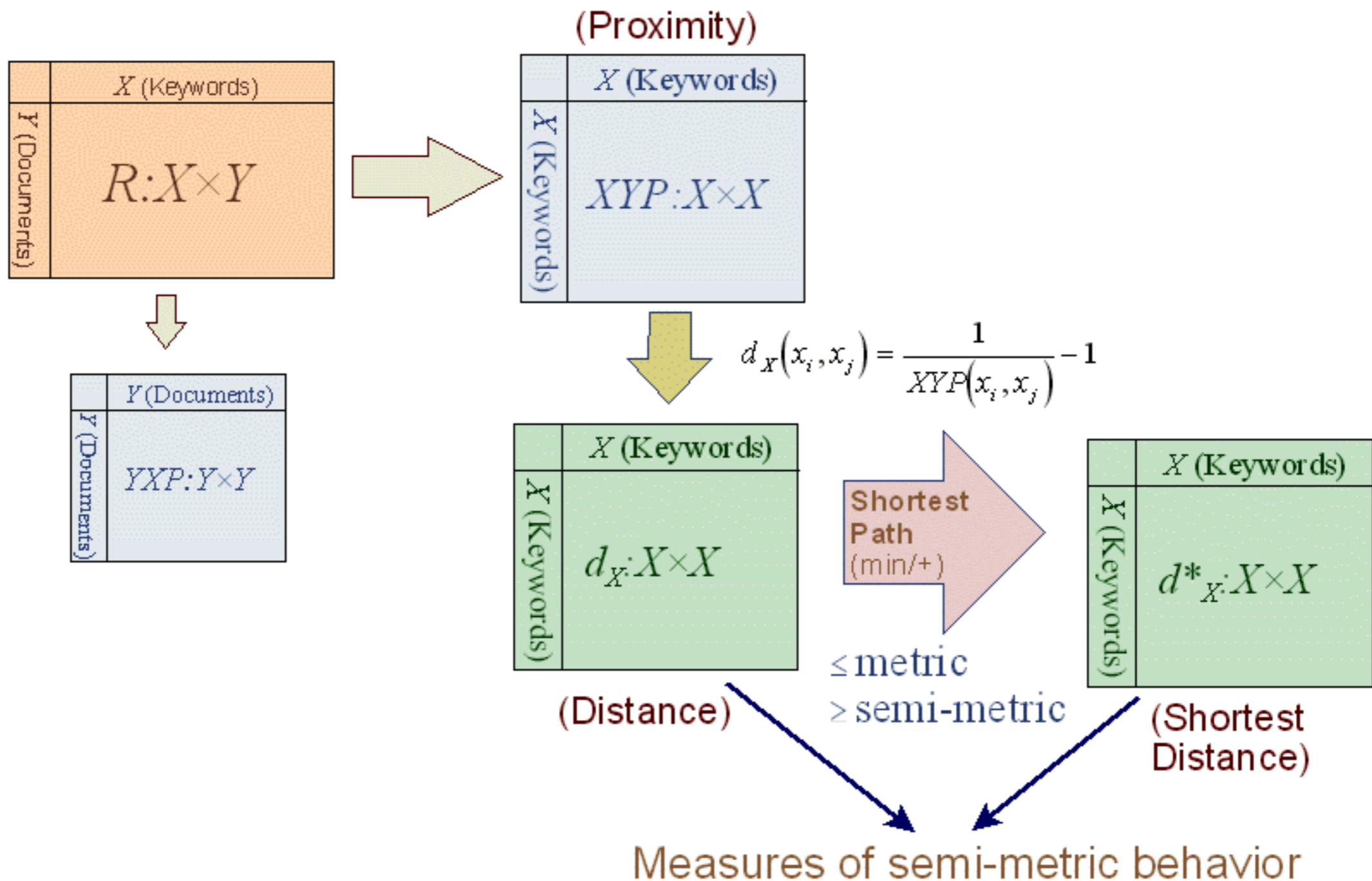
Adaptive
Systems

6.89

Cognition

44

Semi-metric ratio: 6.3861



Semi-metric Measures

■ Semi-metric ratio

- ▶ Absolute measure of indirect distance reduction

$$s(x_i, x_j) = \frac{d_{direct}(x_i, x_j)}{d_{shortest}(x_i, x_j)}$$

■ Relative Semi-metric ratio

- ▶ Distance reduction against maximum contraction

$$rs(x_i, x_j) = \frac{d_{direct}(x_i, x_j) - d_{shortest}(x_i, x_j)}{d_{max} - d_{min}}$$

■ Below Average Ratio

- ▶ Captures semi-metric distance reductions which contract to below the average distance for a given node. Captures some of the cases of initial ∞ distance

$$b(x_i, x_j) = \frac{\overline{d_{x_i}}}{d_{shortest}(x_i, x_j)}$$

what do semi-metric edges imply?

- Pairs with larger semi-metric behavior denote a *latent association*
 - ▶ Not grounded on direct evidence provided by the relation R , but rather implied by the overall network of associations in this relation.
 - ▶ Meaning depends on the semantics of the application
 - In graphs of keyword co-occurrence in documents: associated with novelty and can be used to identify trends.
 - In social networks it may identify pairs of people, groups, etc. for which we do not have direct evidence, in the available documents, that a real association exists, but who could easily be indirectly associated.
 - ▶ In recommendation system for journals now at LANL

Rocha, Luis M. [2002]. "Semi-metric Behavior in Document Networks and its Application to Recommendation Systems". In: *Soft Computing Agents: A New Perspective for Dynamic Information Systems*. V. Loia (Ed.) International Series Frontiers in Artificial Intelligence and Applications. IOS Press, pp. 137-163.

Rocha, Luis M. [2002]. "Combination of Evidence in Recommendation Systems Characterized by Distance Functions". In: *Proceedings of the 2002 World Congress on Computational Intelligence: FUZZ-IEEE'02*. Honolulu, Hawaii, May 2002. IEEE Press, pp. 203-208.

catching strong indirect associations in mylibrary.lanl.gov

0020-1669--Inorganic chemistry 0031-9007--Physical review letters
 0031-9007--Physical review letters 0743-7463--Langmuir
 0003-2700--Analytical chemistry 0031-9007--Physical review letters
 0096-3003--Applied mathematics and computation 0031-9007--Physical review letters
 0031-9007--Physical review letters 0022-3115--Journal of nuclear materials
 1049-3301--ACM transactions on modeling and computer simulation 0031-9007--Physical review letters
 1364-548X--Chemical communications 0031-9007--Physical review letters
 1064-8275--SIAM journal on scientific computing 0031-9007--Physical review letters
 0965-5425--Computational mathematics and mathematical physics 0031-9007--Physical review letters
 0031-9007--Physical review letters 1359-6454--Acta materialia
 0003-7028--Applied spectroscopy 0031-9007--Physical review letters
 0031-9007--Physical review letters 0022-2461--Journal of materials science
 0031-9007--Physical review letters 1359-6462--Scripta materialia
 0031-9007--Physical review letters 0022-4596--Journal of solid state chemistry
 0031-9007--Physical review letters 0021-8898--Journal of applied crystallography

IPP_3: parameter *rs*

1097-6256--Nature neuroscience 1065-9471--Human Brain MApping
 1097-6256--Nature neuroscience 0278-0062--IEEE transactions on medical imaging
 1097-6256--Nature neuroscience 1053-8119--NeuroImage
 1063-7796--Physics of particles and nuclei 0218-3013--International journal of modern physics E Nuclear physics
 1053-8119--NeuroImage 1065-9471--Human Brain MApping
 0031-9007--Physical review letters 0743-7463--Langmuir

IPP_3: parameter *b*

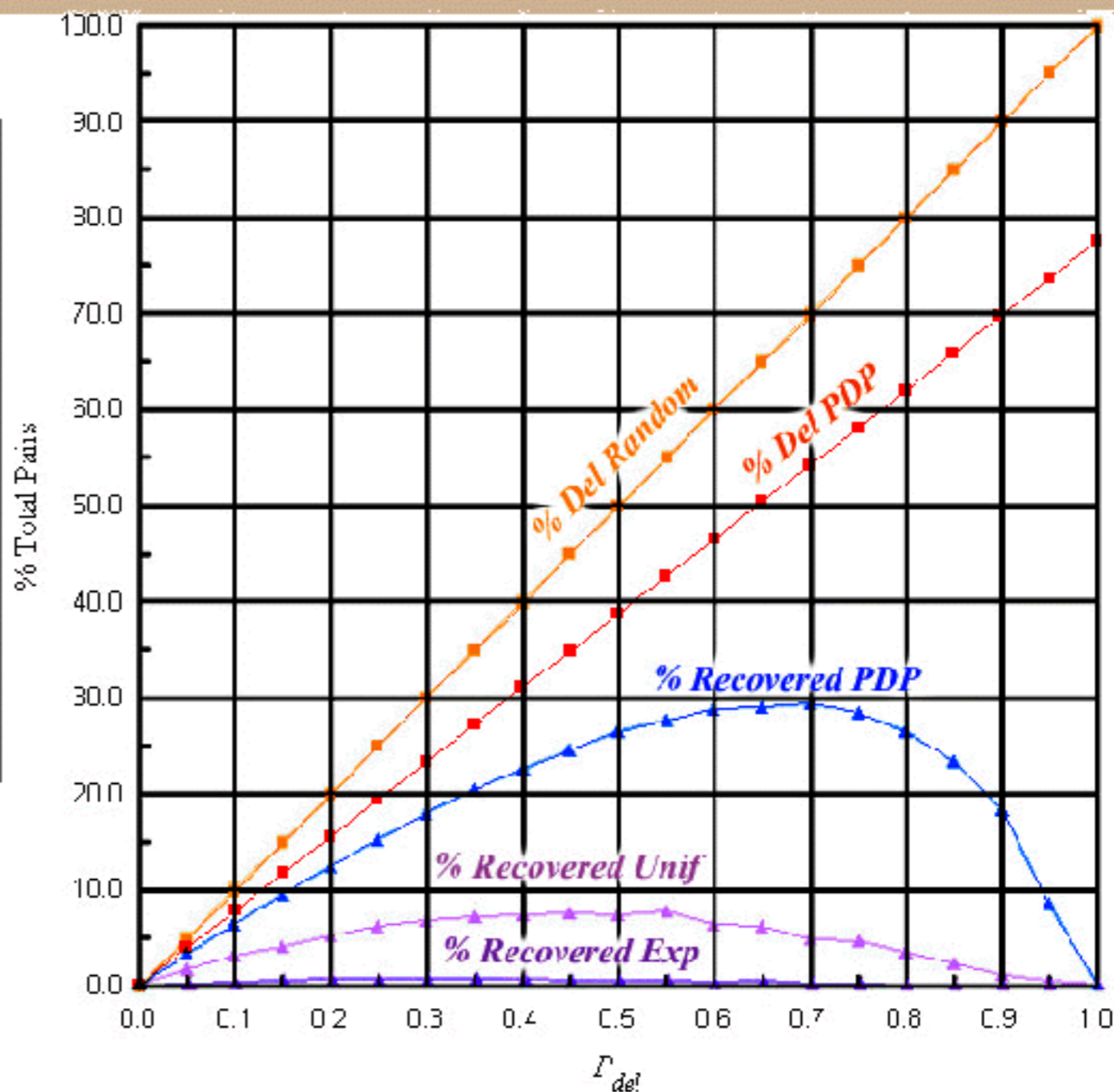
0031-9007--Physical review letters 0020-1669--Inorganic chemistry
 0031-9007--Physical review letters 0141-1594--Phase transitions
 0031-9007--Physical review letters 0928-1045--Journal of computeraided materials design
 0031-9007--Physical review letters 0042-207X--Vacuum
 1097-6256--Nature neuroscience 0031-9155--Physics in medicine & biology
 1097-6256--Nature neuroscience 0096-3518--IEEE transactions on acoustics speech and signal processing
 1097-6256--Nature neuroscience 0740-7467--IEEE ASSP magazine
 1097-6256--Nature neuroscience 1070-9908--IEEE signal processing letters
 0022-5355--Journal of vacuum science and technology 0734-2101--Journal of vacuum science & technology A Vacuum surfaces
 and films

random deletion experiments

- Perfect Knowledge
 - ▶ Transitive Closure of real graph
 - ▶ Metric Distance Graph
- Incomplete Knowledge
 - ▶ Each positive association is deleted with probability p_{del}
 - ▶ 100 graphs for each value of p_{del}

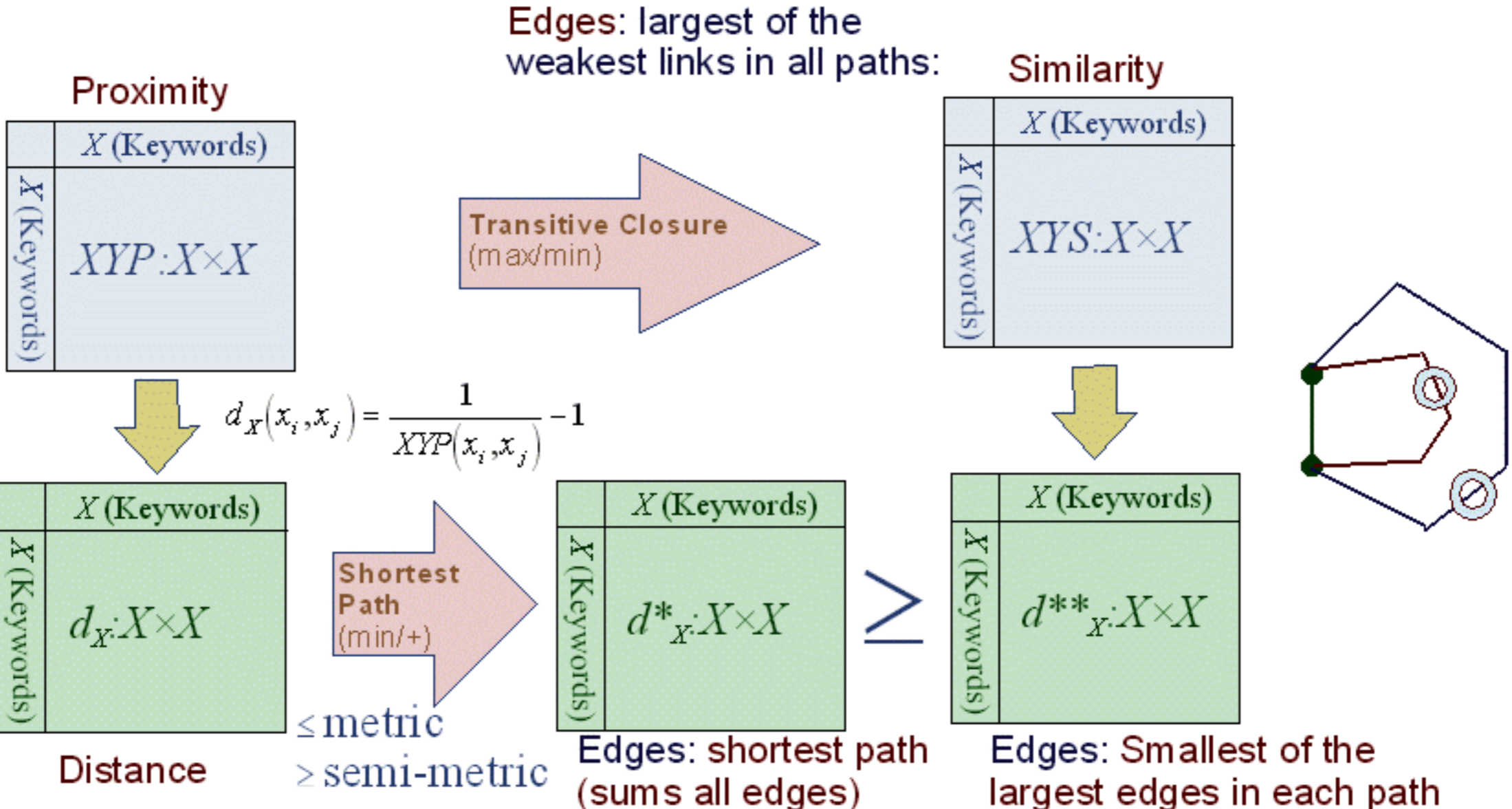
Full Deletion

Recovery via parameter b



shortest paths or weakest links

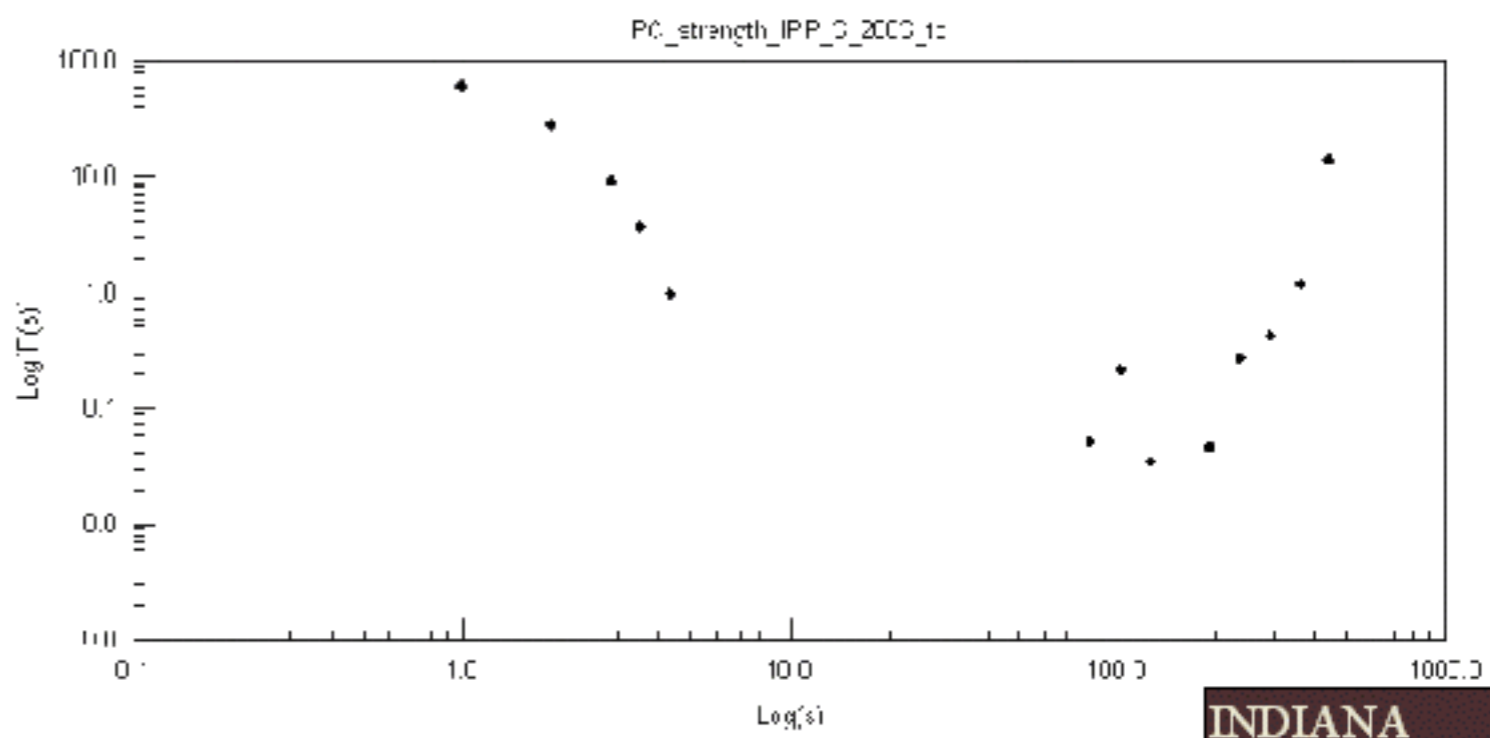
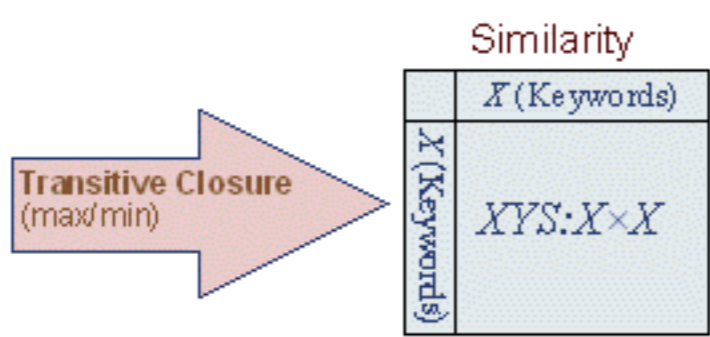
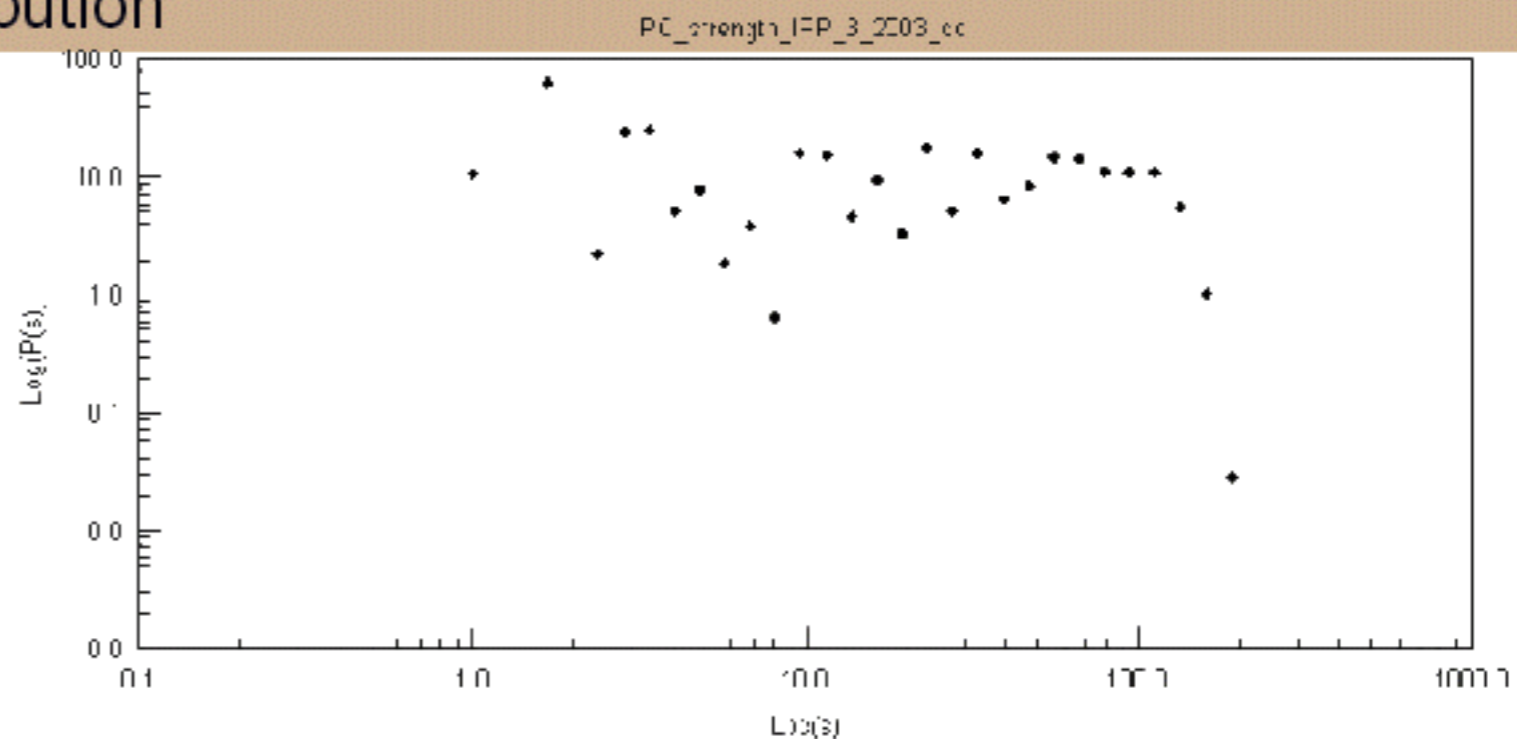
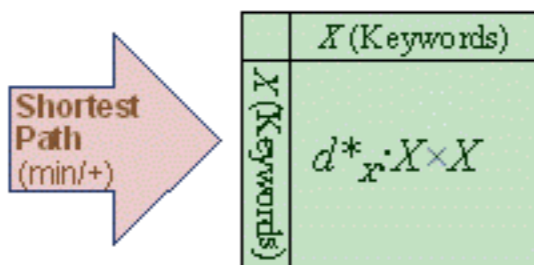
informatics
luis rocha 2005



For any monotonic increasing distance function

comparison of the two closures

cumulative strenght distribution



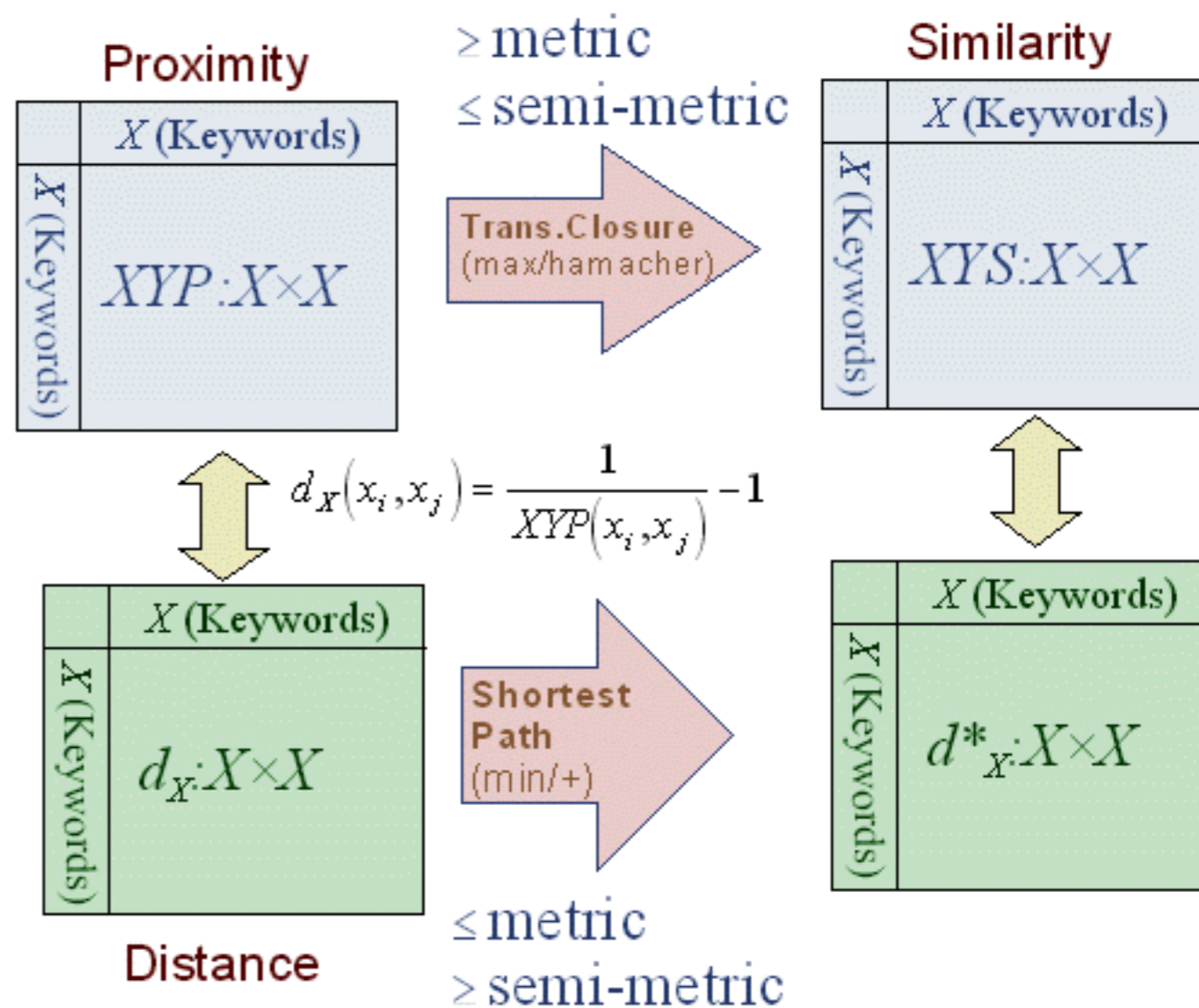
Hammacher function

If we use :

$$i(a, b) = \frac{ab}{a + b - ab}$$

$$u(a, b) = \max[a, b]$$

With Tiago Simas



Current work

$$i(a,b) = \frac{ab}{a + b - ab}$$

Are not dual: no complement can satisfy de Morgan's laws!

$$u(a,b) = \max[a,b]$$

- Can fuzzy graphs represent distance graphs?
- What fuzzy intersection/union comes closest to distance graphs?
- What captures the semi-metric behavior best?
 - ▶ Shortest paths on distance graphs?
 - ▶ Some transitive closure?

scientific community working on feynman diagrams

as published in *Physical Review*, 1949-54

	$P(\text{author names})$
$P(\text{author names})$	$C: P \times P$

- Collaboration Relation: C
 - ▶ Who wrote a paper with whom
- Acknowledgment Relation: A
 - ▶ Who acknowledged, or informally received information from whom

	$P(\text{author names})$
$P(\text{author names})$	$A: P \times P$

$$CP(p_i, p_j) = \frac{\sum_{k=1}^m (c_{i,k} \wedge c_{j,k})}{\sum_{k=1}^m (c_{i,k} \vee c_{j,k})}$$

76 Authors

$CP(p_i, p_j)$ is a **co-collaboration probability**: the probability that two authors have collaborated with the same authors

$$AP(p_i, p_j) = \frac{\sum_{k=1}^m (a_{i,k} \wedge a_{j,k})}{\sum_{k=1}^m (a_{i,k} \vee a_{j,k})}$$

91 Authors

$AP(p_i, p_j)$ is a **co-acknowledgment probability**: the probability that two authors have acknowledged or have been acknowledged by the same authors

semi-metric analysis

- *CP* is almost metric

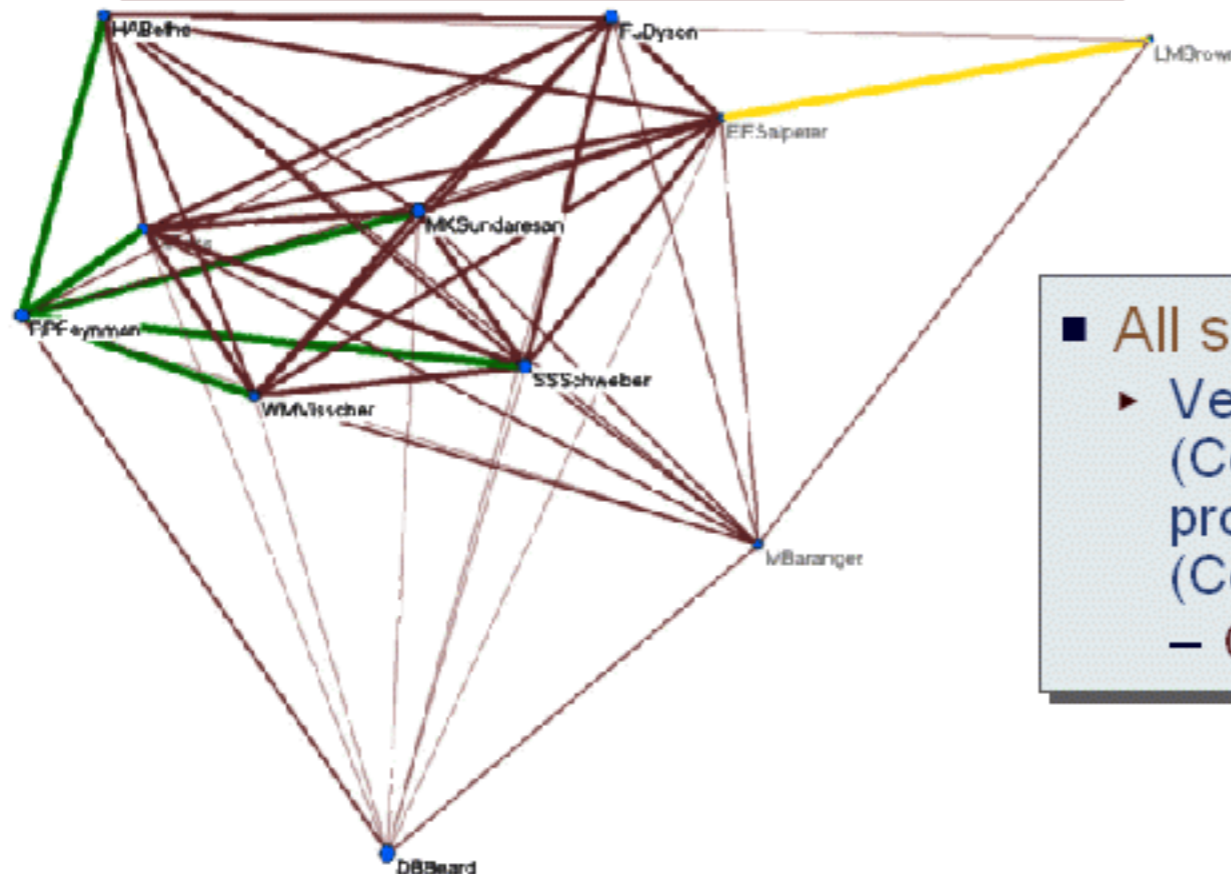
- ▶ 139 papers, 76 authors
- ▶ Percentage of pairs with positive semi-metric ratios (r s and s parameters): 0.667%
- ▶ Percentage of pairs with indirect distances smaller than the average distance of direct edges to either node (b parameter): 0.439%
- ▶ Very few implicit associations



5 most semi-metric pairs (rs and b parameters)

HABethe	RPFeynman
RPFeynman	WMVisscher
MRoss	RPFeynman
RPFeynman	SSSchweber
MKSundaresan	RPFeynman

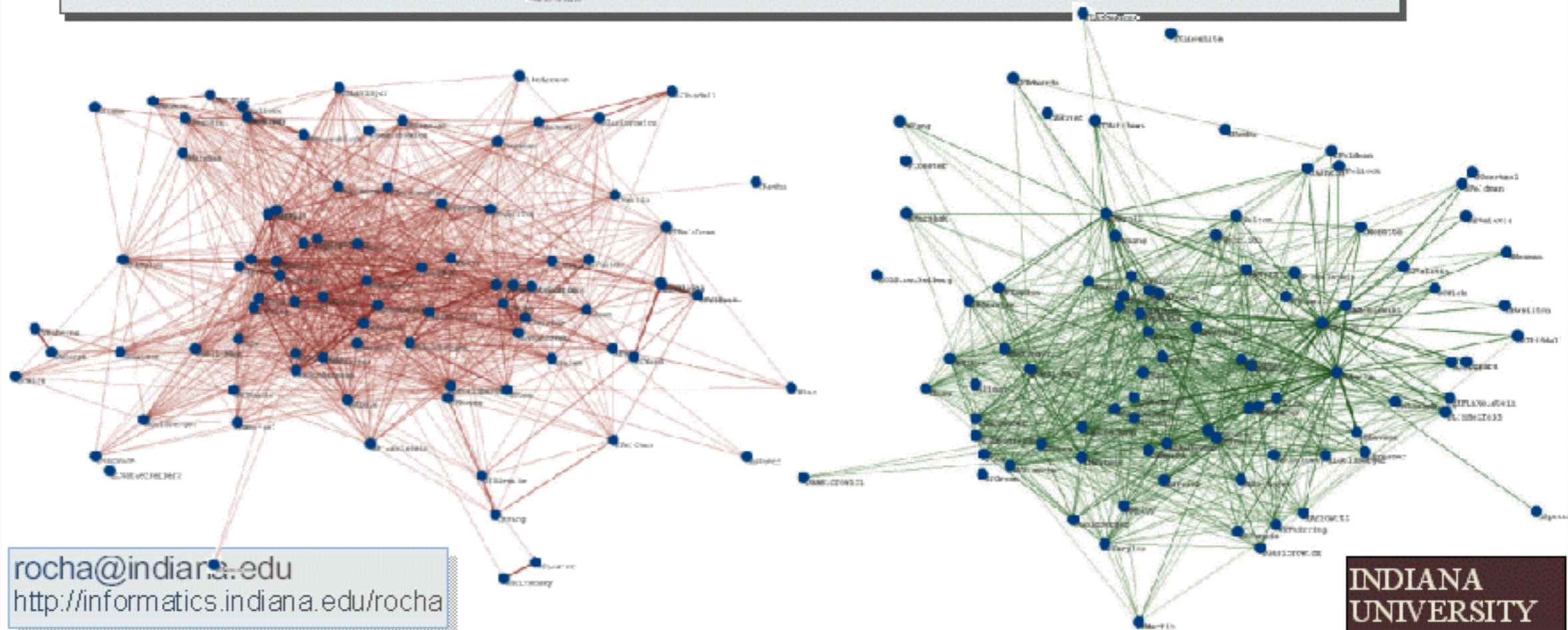
- Weak co-collaboration (*CP*), but strong co-acknowledgment (*AP*).
 - While they have not co-collaborated much in PR, they have acknowledged or have been acknowledged by many of the same people



- All six authors in top pairs
 - Very strong proximities to *EESalpeter* (Cornell) and *KMWatson* (postdoc at IAS, prof at Indiana) in *AP* and with to *EESalpeter* (Cornell) and *FJDyson* (Cornell, IAS) in *CP* – Cornell and Institute of Advanced Studies

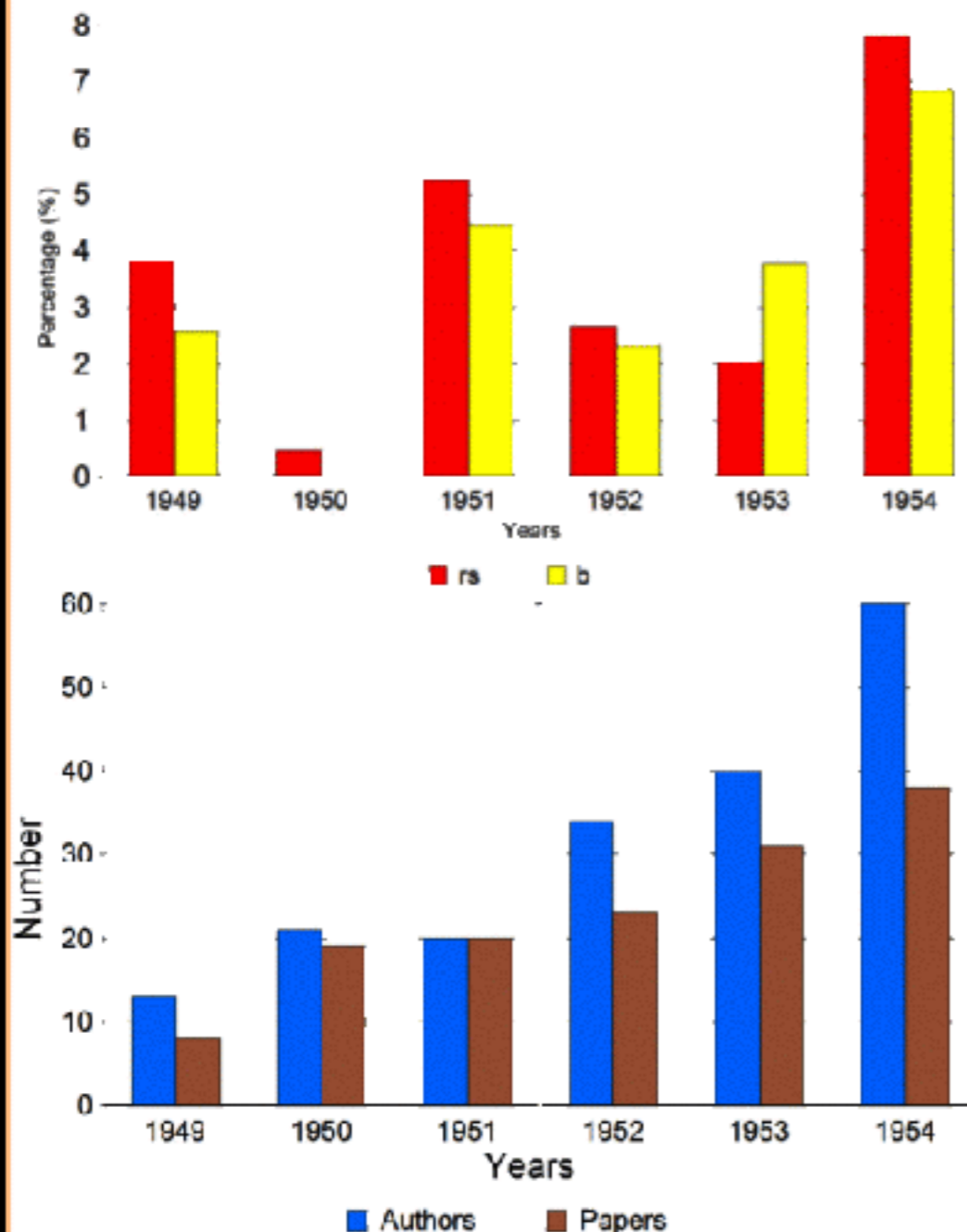
semi-metric analysis

- *AP* is very semi-metric
 - ▶ 139 papers, 91 authors
 - ▶ Percentage of pairs with positive semi-metric ratios (r s and s parameters): 18.3%
 - ▶ Percentage of pairs with indirect distances smaller than the average distance of direct edges to either node (b parameter): 30.8%
 - ▶ Many strong implicit associations



rocha@indiana.edu
<http://informatics.indiana.edu/rocha>

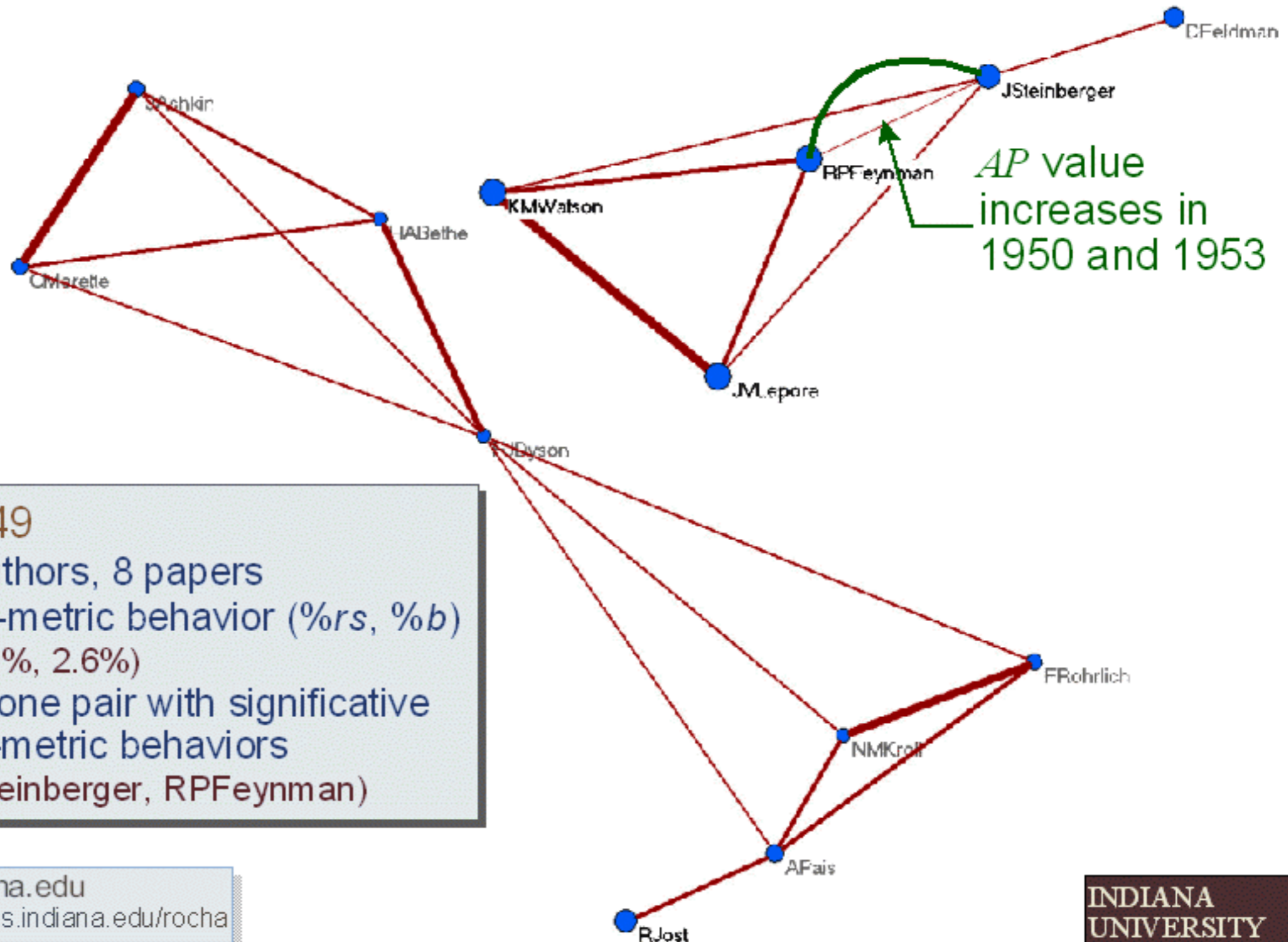
semi-metric analysis



- AP computed for every individual year between 1949 and 1954
 - ▶ Papers, Authors
 - (8, 13); (19, 21); (20, 20); (23, 34); (31, 40); (38, 60)
 - ▶ Compared with the global AP , the individual years are more metric
 - 1950 is almost completely metric
 - ▶ Semi-metric behavior ($\%rs$, $\%b$)
 - 1949: (3.9%, 2.6%)
 - 1950: (0.5%, 0.0%)
 - 1951: (5.3%, 4.5%)
 - 1952: (2.7%, 2.3%)
 - 1953: (2.1%, 3.8%)
 - 1954: (7.8%, 6.9%)
 - ▶ Can semi-metric pairs uncover latent and future associations?

dynamics of co-acknowledgment network

1949



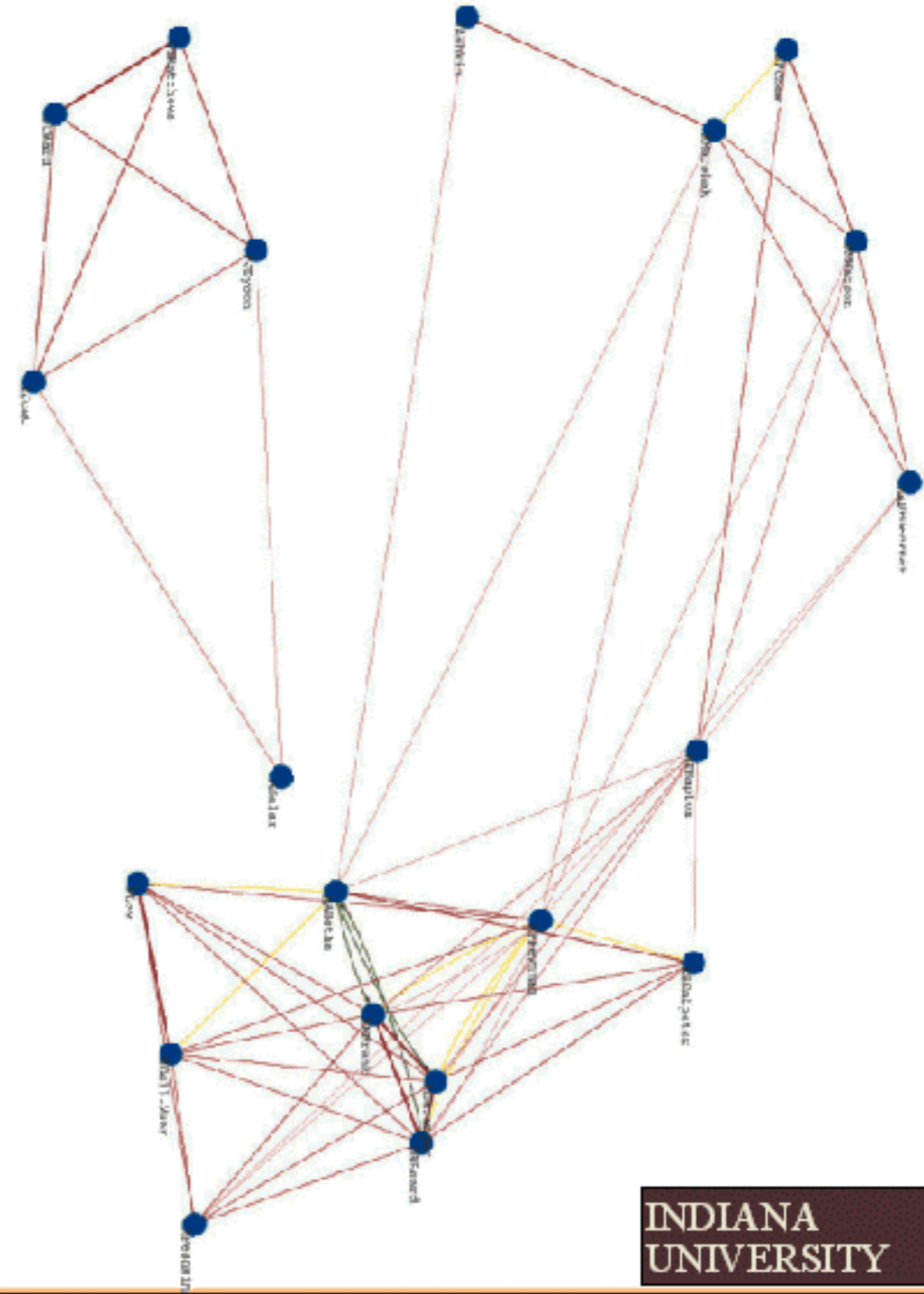
■ AP 1949

- ▶ 13 authors, 8 papers
- ▶ Semi-metric behavior ($\%rs$, $\%b$)
 - (3.9%, 2.6%)
- ▶ Only one pair with significant semi-metric behaviors
 - (Jsteinberger, RPFeynman)

1951

■ AP 1951

- ▶ Not very strong semimetric behavior
- ▶ (HABethe, RMFrank): both r_s and b
 - Advisor/ Student at Cornell
 - David Kaiser suspects Bethe learned about the diagrams via Frank
- ▶ (DBBeard, HABethe): both r_s and b
 - Wrote paper together in 1951
- ▶ (HABethe, Mbaranger): both r_s and b
 - Wrote paper together in 1953; high value of proximity in co-collaboration network (CP); value of AP increases in 1952 and 1953
- ▶ (RPFeynman, EESalpeter): b
 - High proximity in co-collaboration network. No link in AP 1951, but AP increases in 1952 and 1953.
- ▶ (HABethe, Flow) : b
 - No link in AP 1951, but AP increases in 1952 and 1953.
- ▶ (HABethe, Mgell-Mann): b
 - No link in AP 1951, but AP increases in 1954.



second stage of adaptive webs

- Extraction of co-occurrence (associative) networks
 - ▶ Represent associative knowledge of information resources and users
- Identification of implicit associations in networks
 - ▶ Discovery of relevant items, missing information, trends, associations with higher future probability of occurring
 - ▶ Identify Communities of Users
 - ▶ Applications
 - Recommendation systems, social networks, bioinformatics
 - ▶ Complex systems methodology: network analysis and knowledge integration