

NETWORK COMMUNITY ANALYSIS

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SCHOOL OF INFORMATICS
AND COMPUTING

INDIANA UNIVERSITY

Bloomington

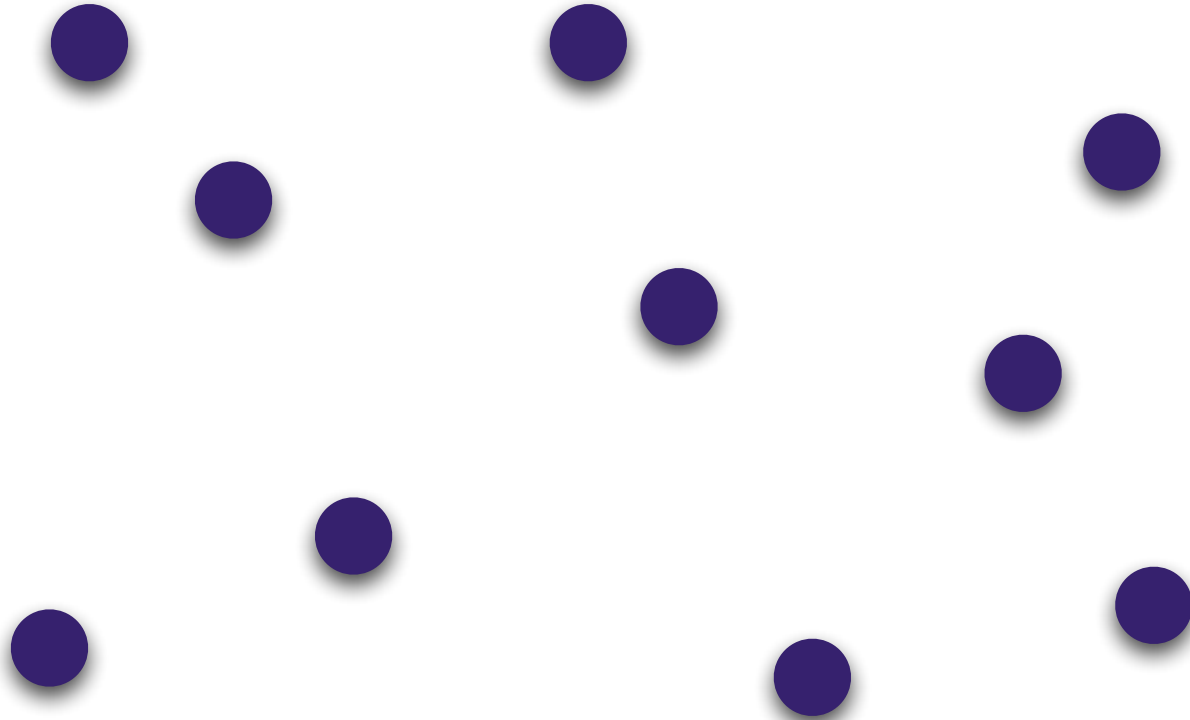
Disclaimer:

We're just scratching the surface.
There are so many cool studies that
I cannot cover here!

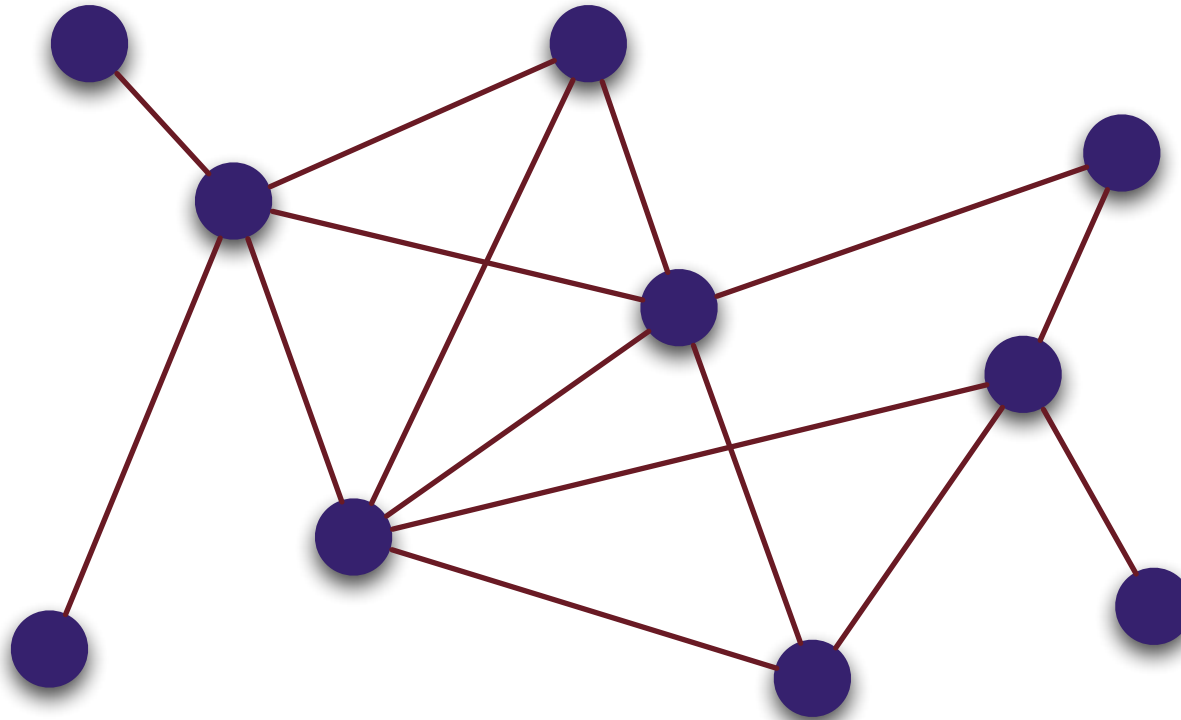
Visit:

<http://goo.gl/opEfTp>

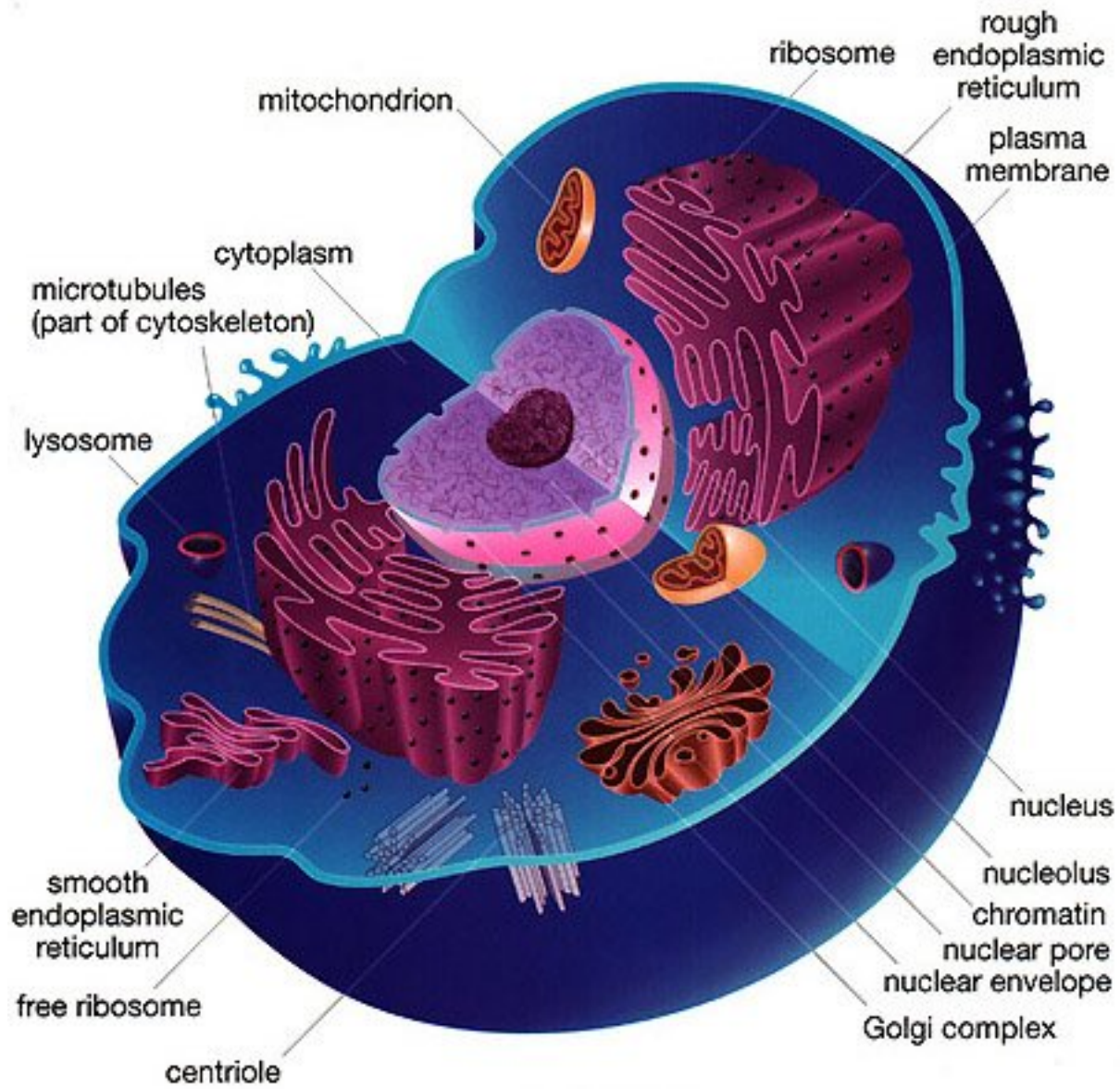
Network community analysis



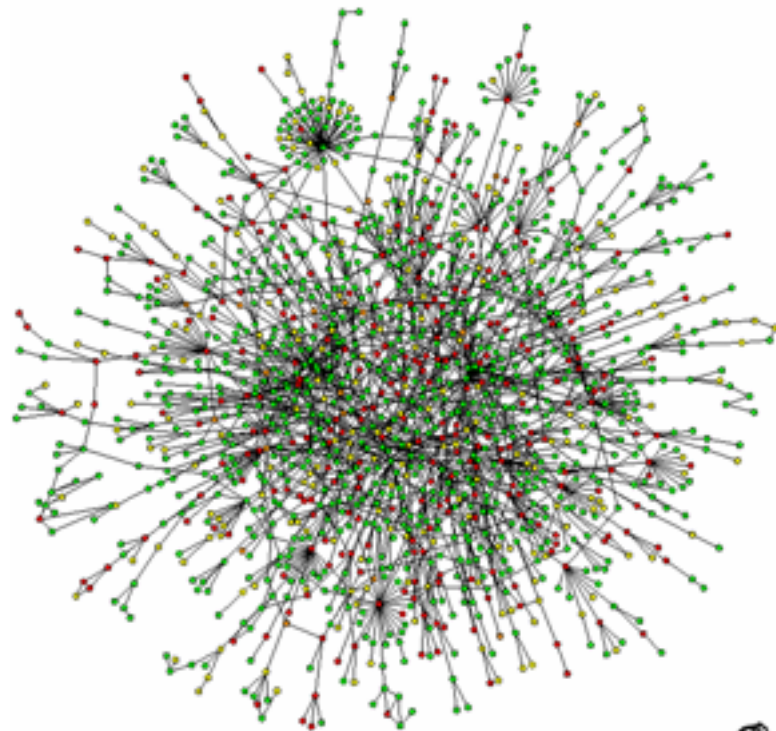
Nodes

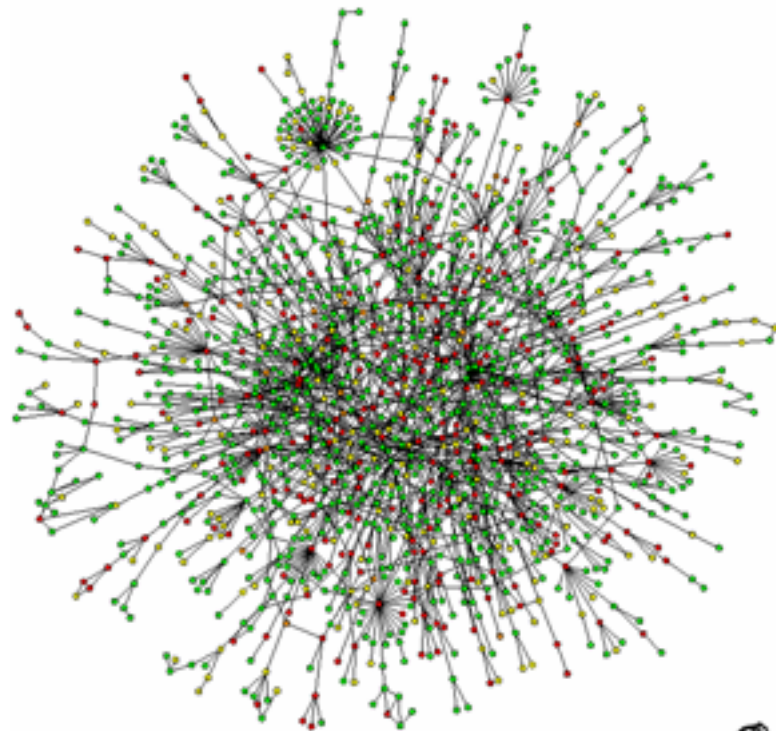
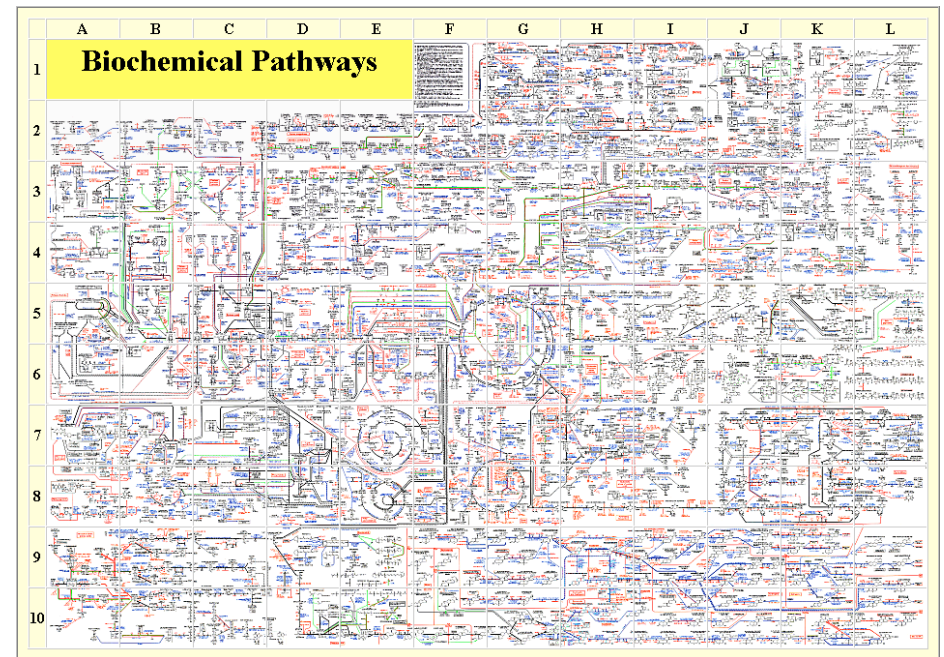


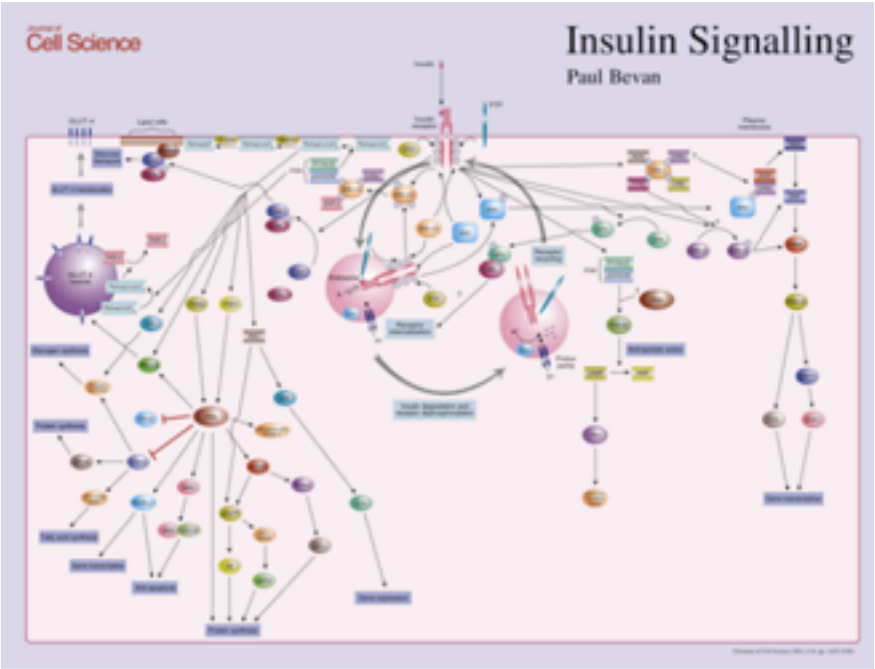
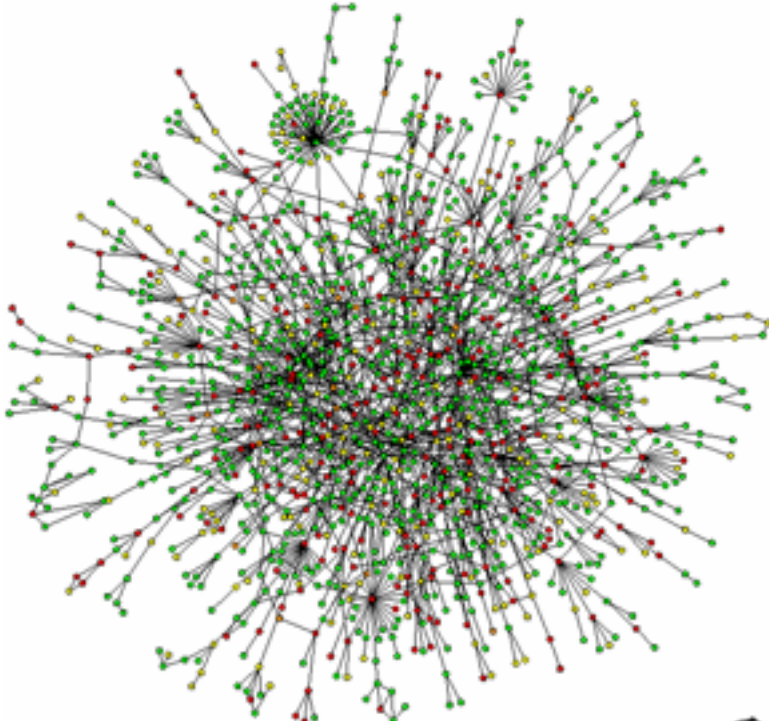
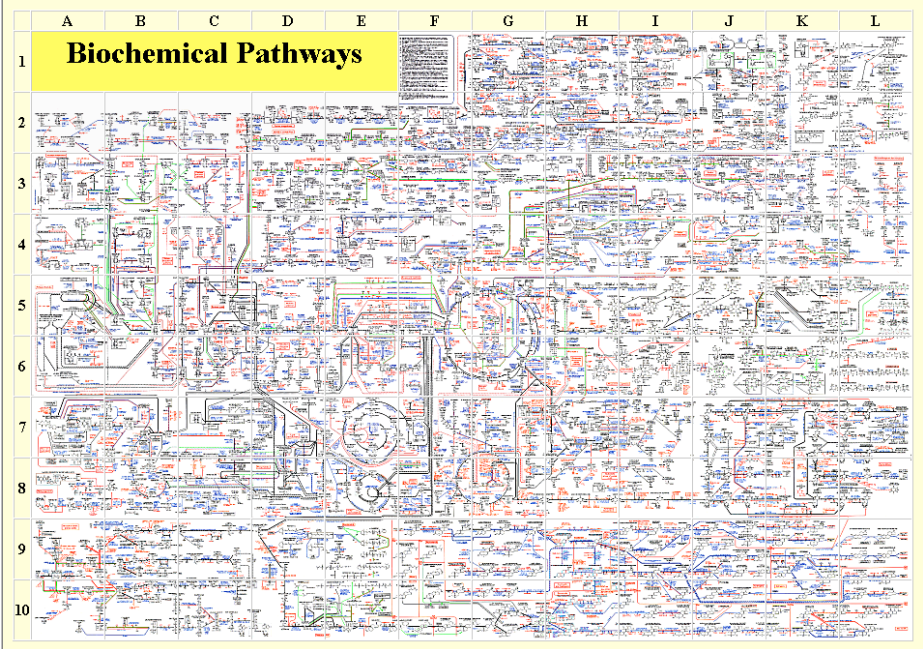
Links (edges) between nodes

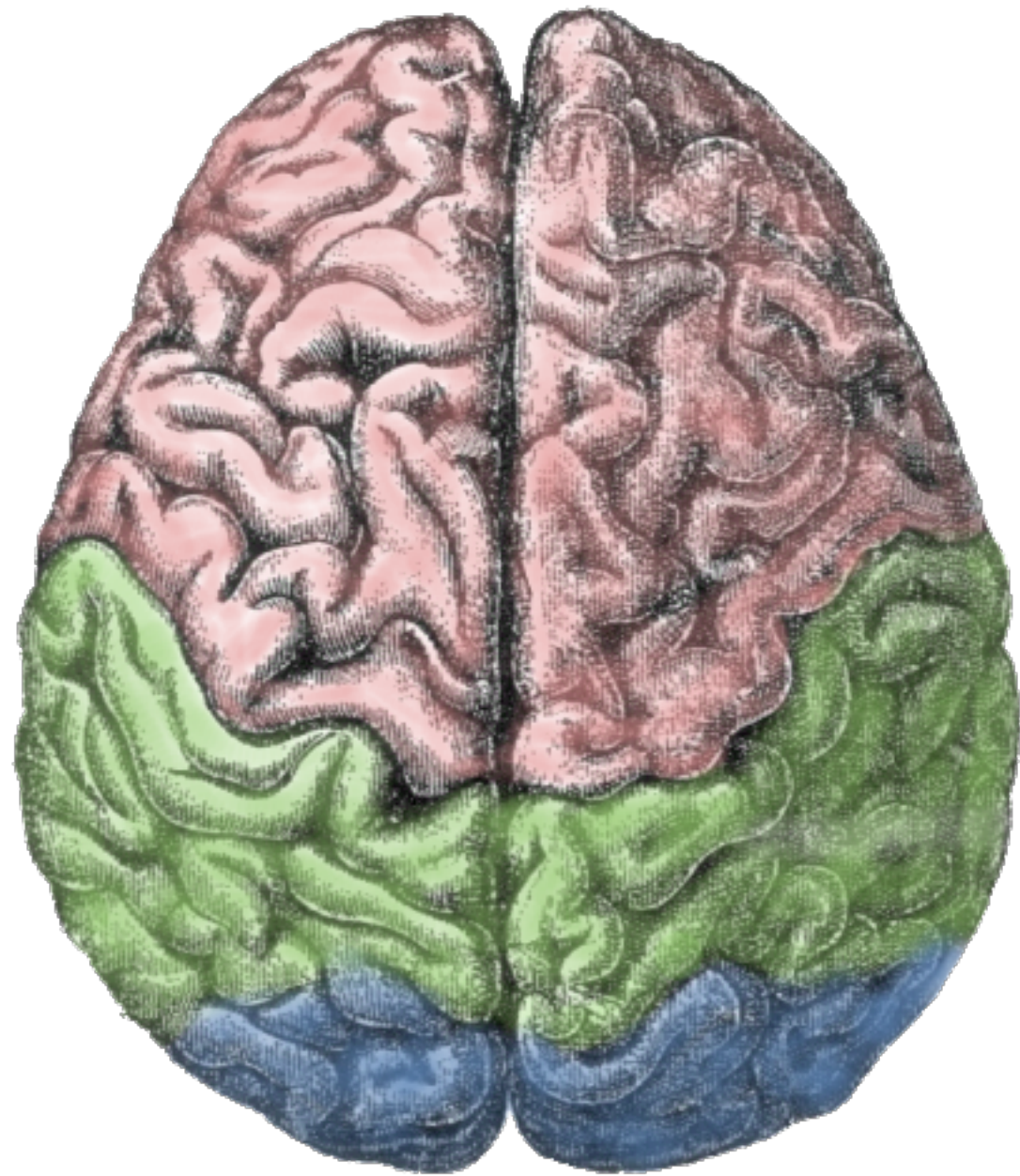
















facebook

December 2010



Networks = The maps of complex systems



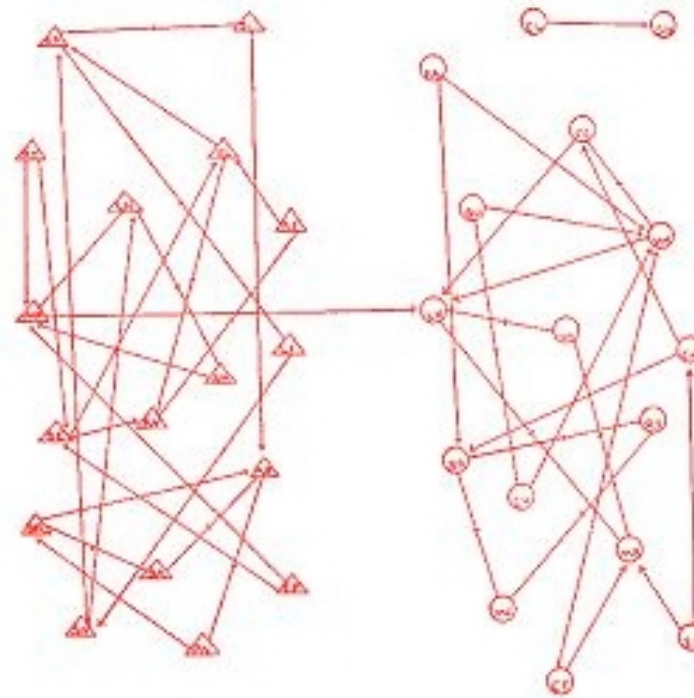
Network **community** analysis

What is a
network community?

EMOTIONS MAPPED BY NEW GEOGRAPHY

Charts Seek to Portray the
Psychological Currents of
Human Relationships.

New York Times
April 3, 1933



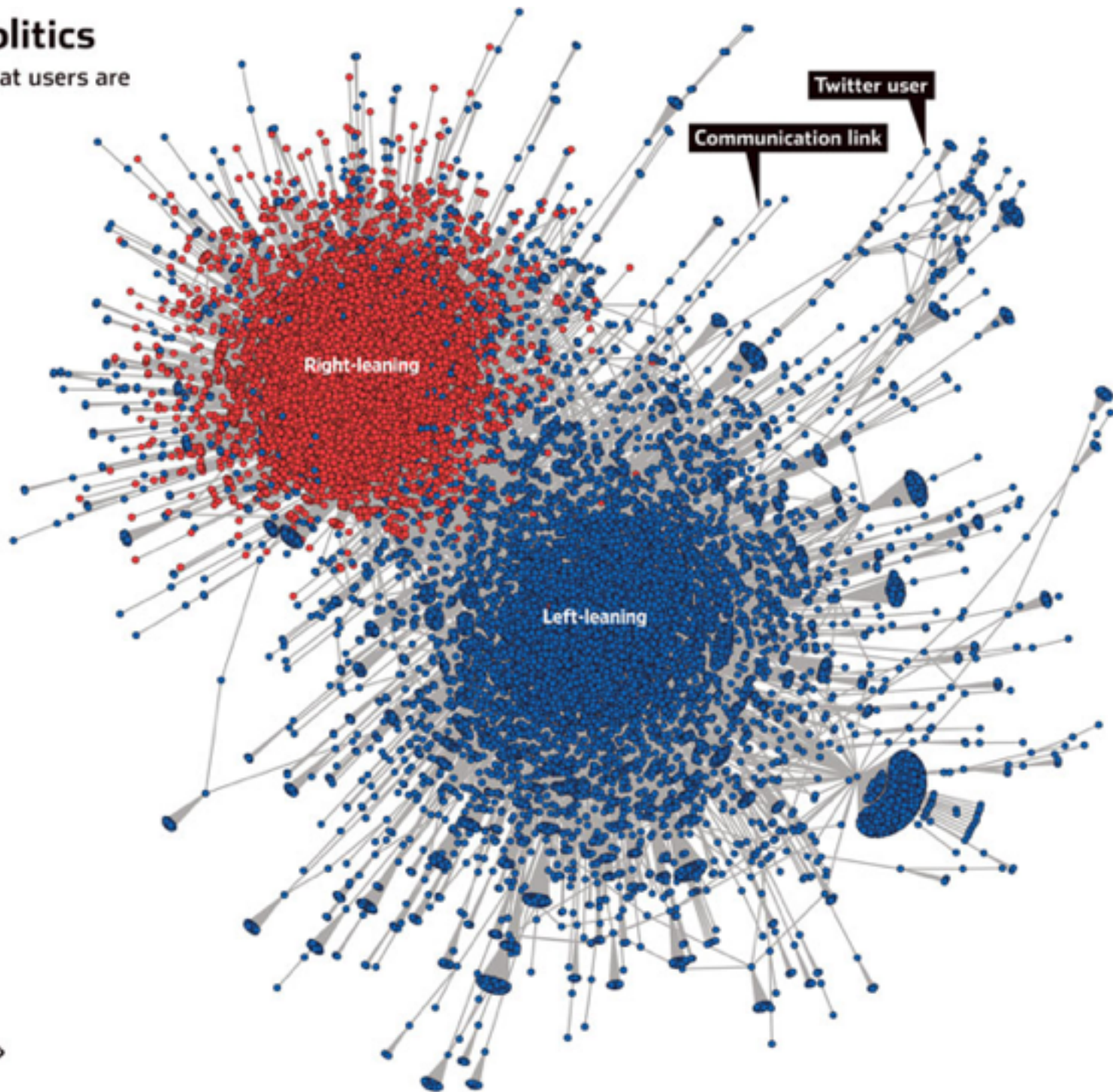
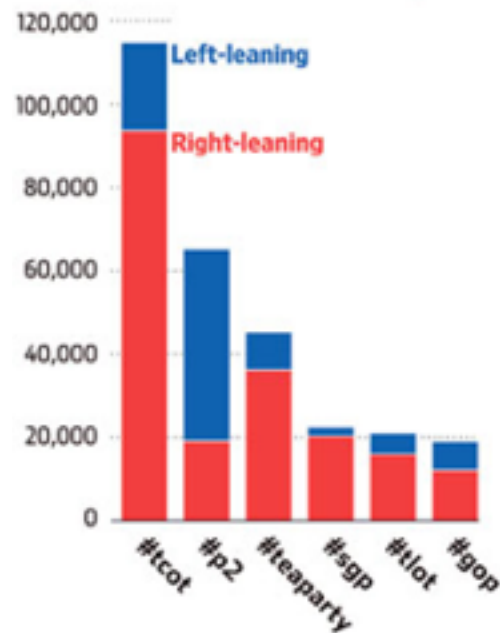
Moreno's "sociogram"

Twitter's Divided Politics

Political Twitter traffic reveals that users are polarized along party lines.*

Researchers at Indiana University analyzed 250,000 Twitter messages on political topics exchanged by 45,000 people during the 2010 mid-term congressional elections. This chart of 'retweets'—in which one user forwards another's message—shows that, though there were more left-leaning users, right-leaning users were more densely connected to one another. (Each dot is a Twitter user, and the lines show retweets.) Even so, as the chart illustrates, lines of communication do sometimes reach across the political divide.

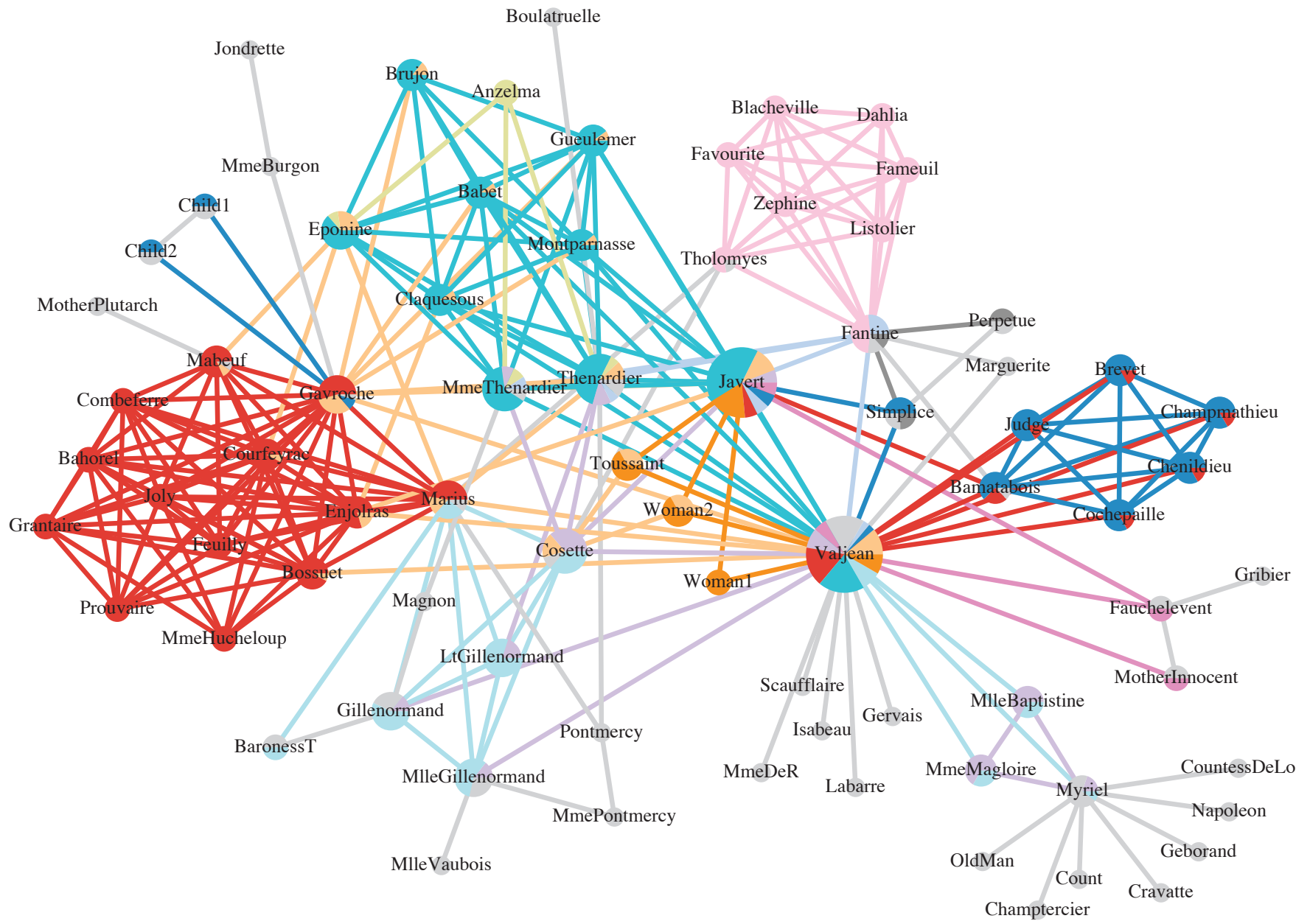
The most popular hashtags (short codes signaling the message's content), shown by number of tweets, researchers found that users on the left and right use each other's hashtags.



Hashtags: tcot, top conservatives on Twitter; p2, progressives 2.0; sgp, smart girl politics; tlol, top libertarians on Twitter.

*Data show 'retweets' of other users' messages. Political leaning designations are based on algorithmically-determined communities of users which correlate with political affiliation.

Source: Center for Complex Networks and Systems Research, Indiana University



Cohesiveness

Separation

Group cohesiveness

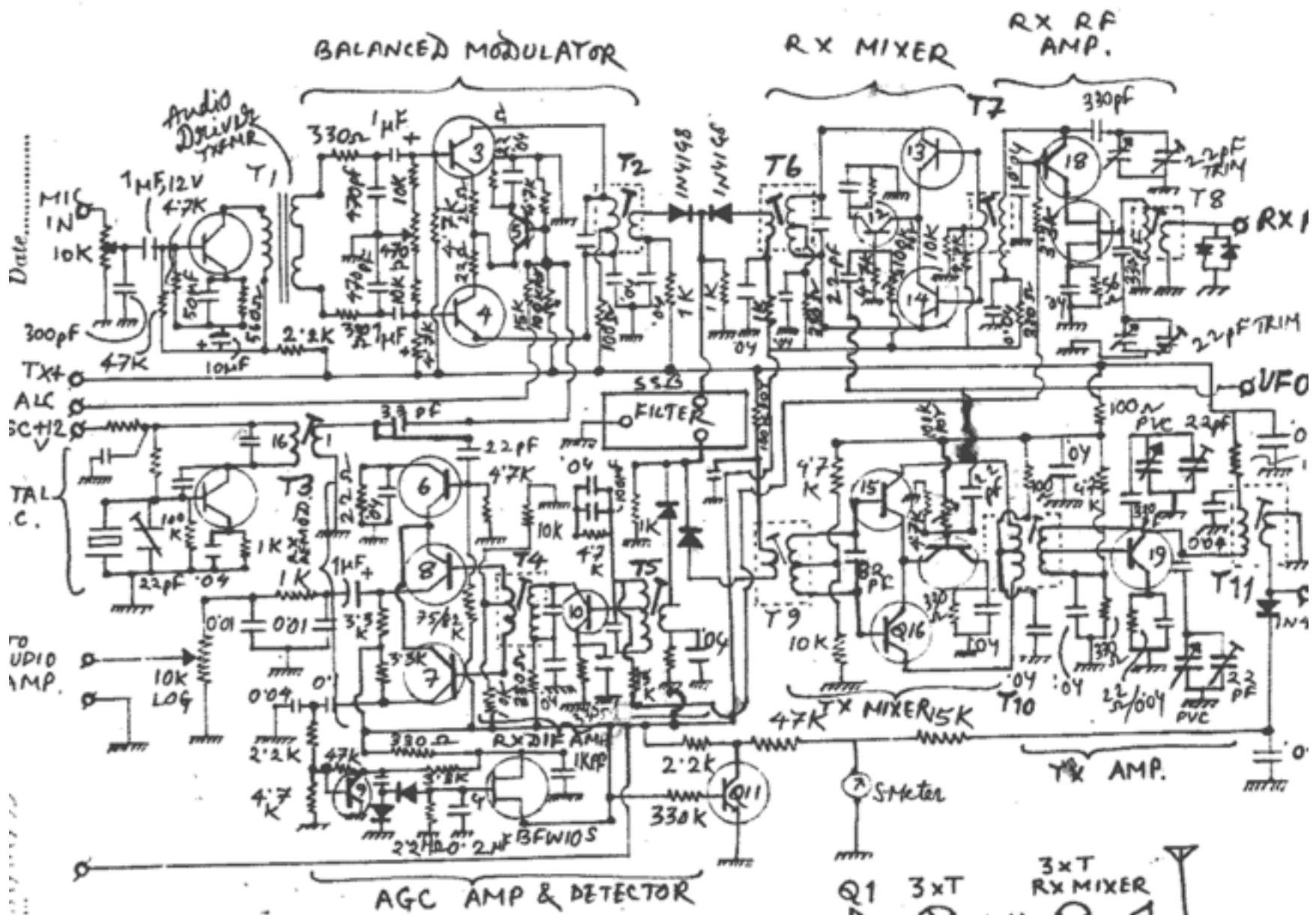
(Moreno & Jennigs 1938, Festinger 1950, Gross & Martin 1952)

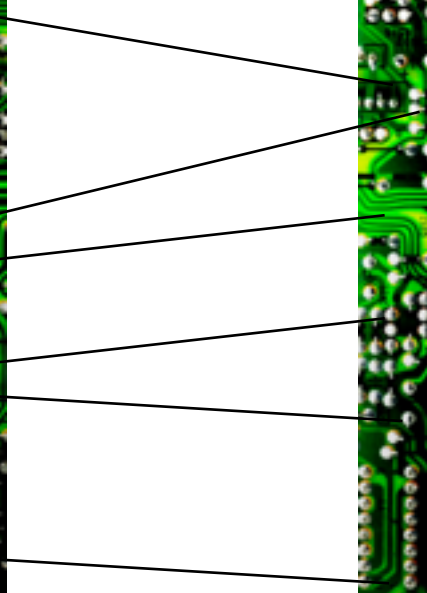
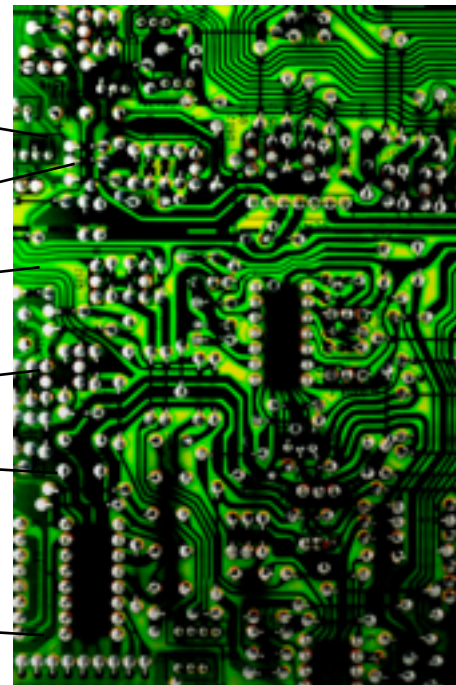
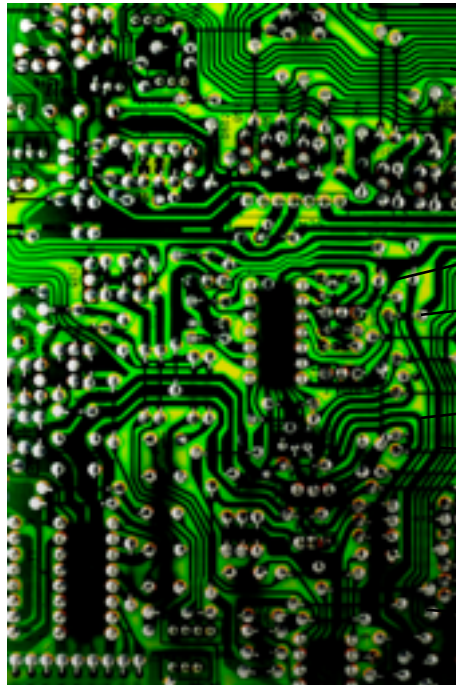
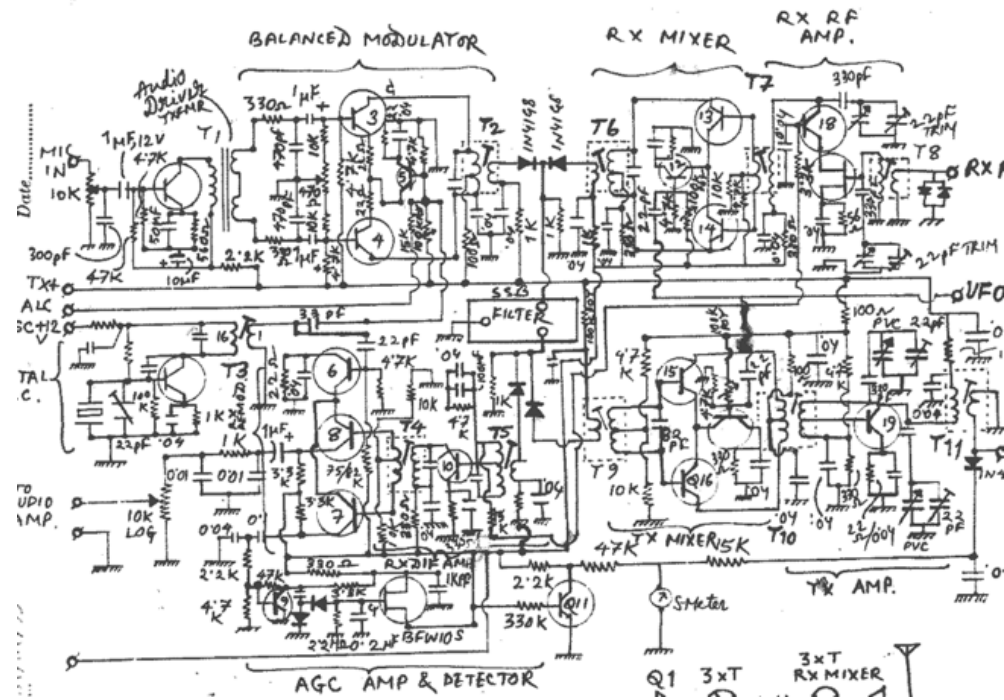
Graph partitioning

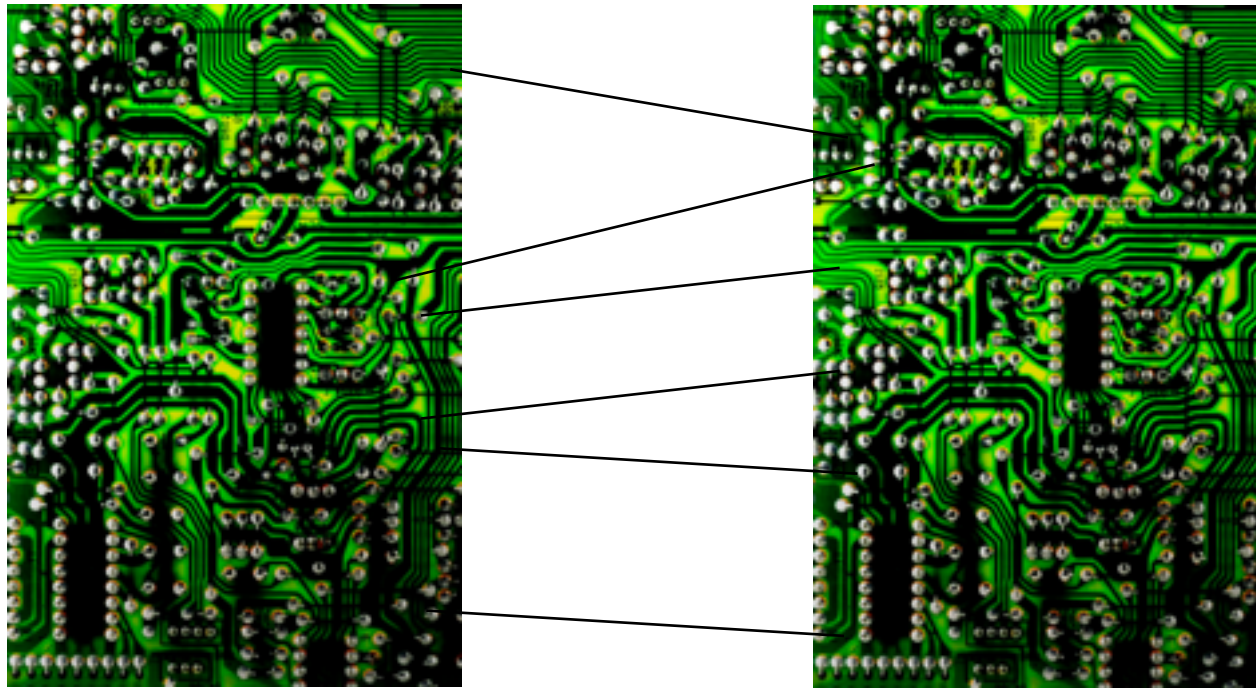
(Kernighan & Lin 1970)

Why do we care?

Original motivation:
Computation







How to minimize the
number of wires?



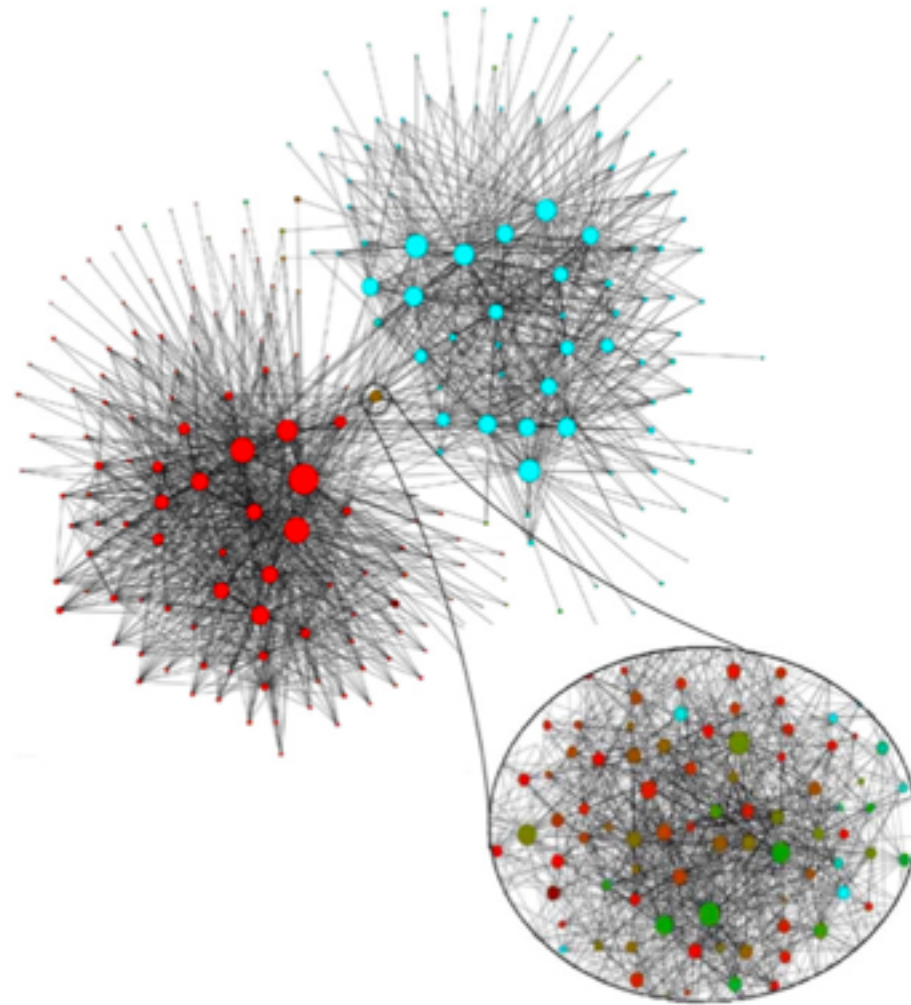
How to minimize the communication between computers?

Circuits, Communication
between softwares ~
Networks

Functional modules ~
Communities

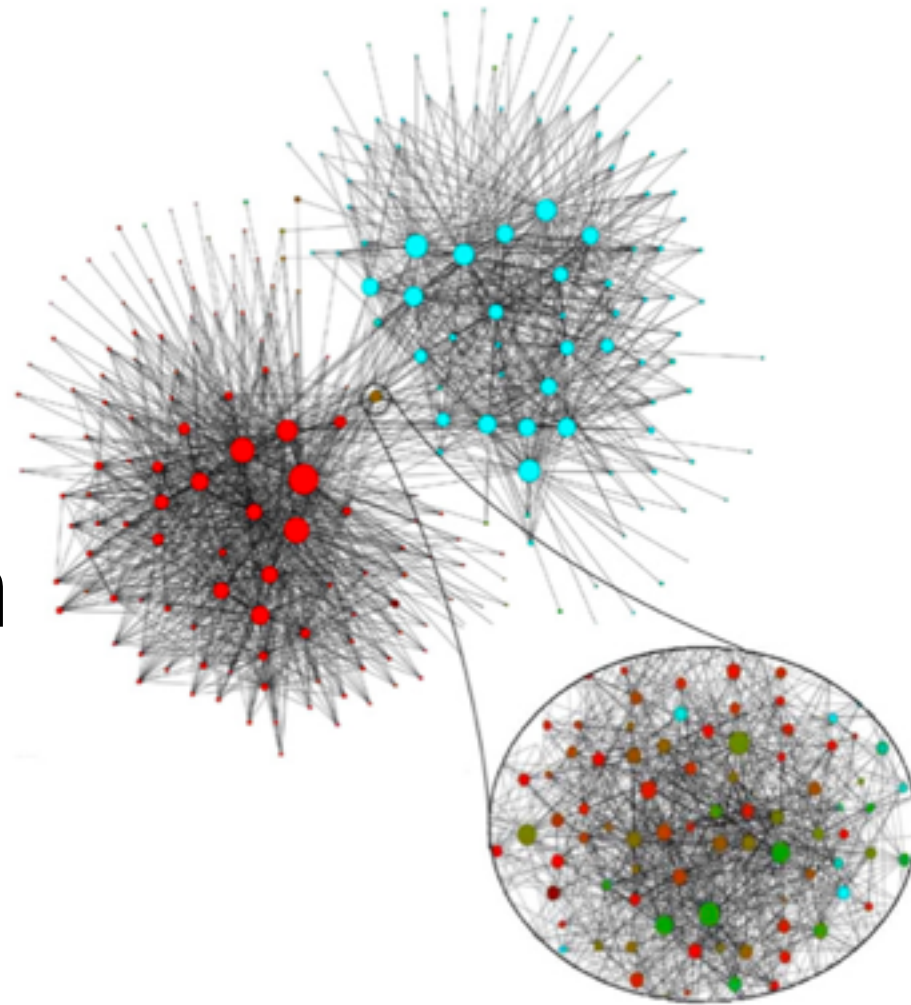
Correspondence to
**functional, structural
units**

Belgian communication network

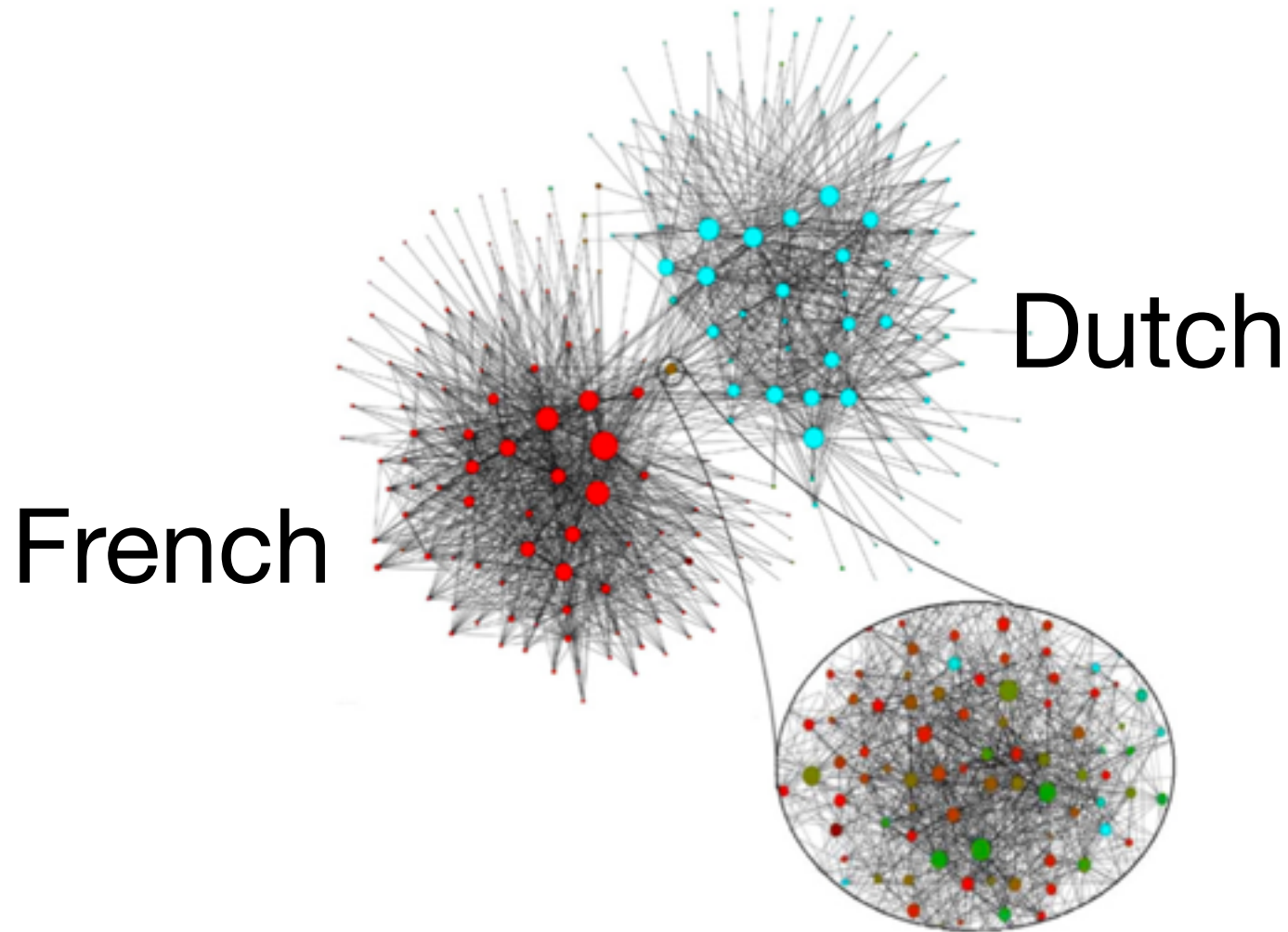


Belgian communication network

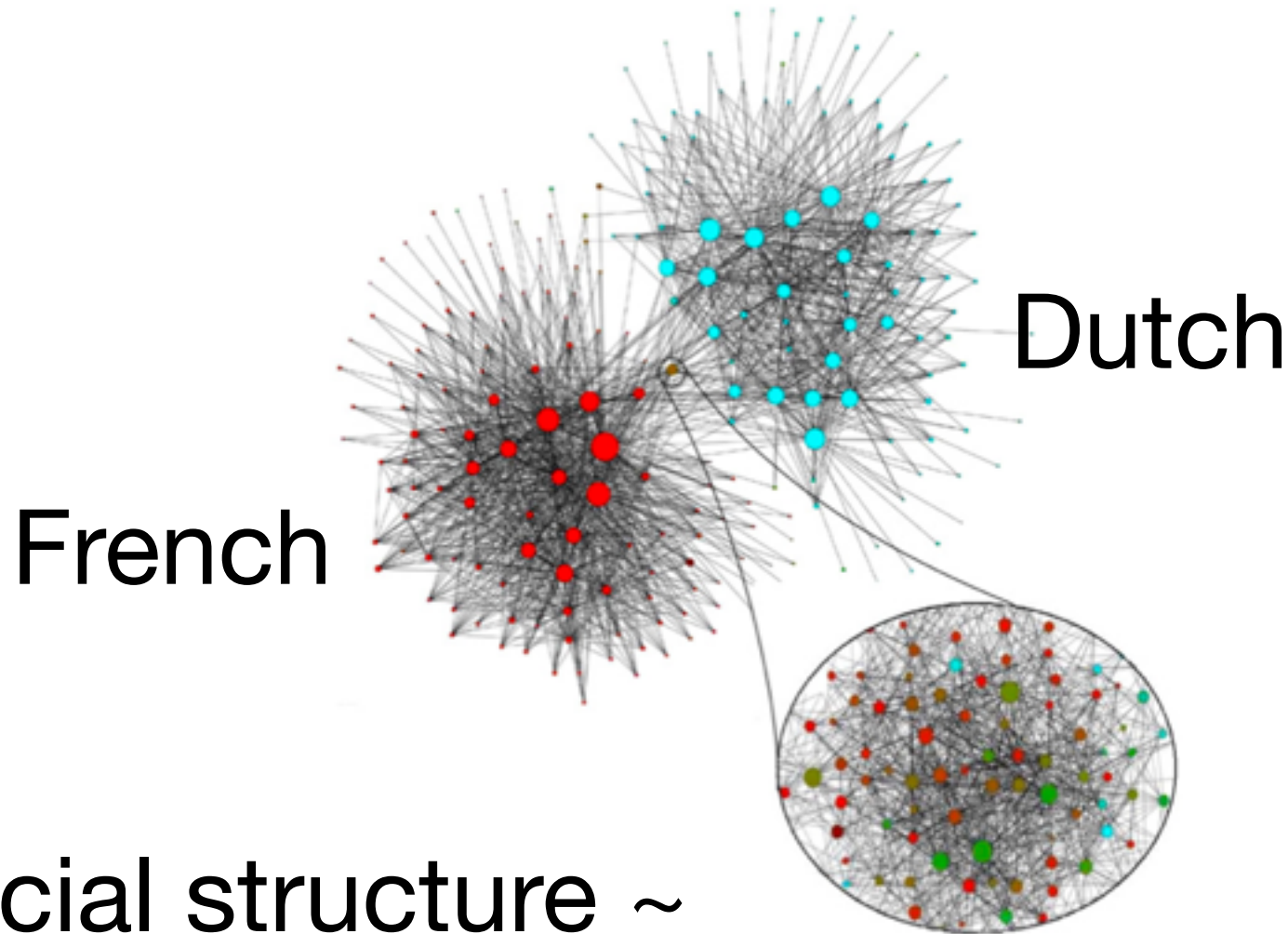
French



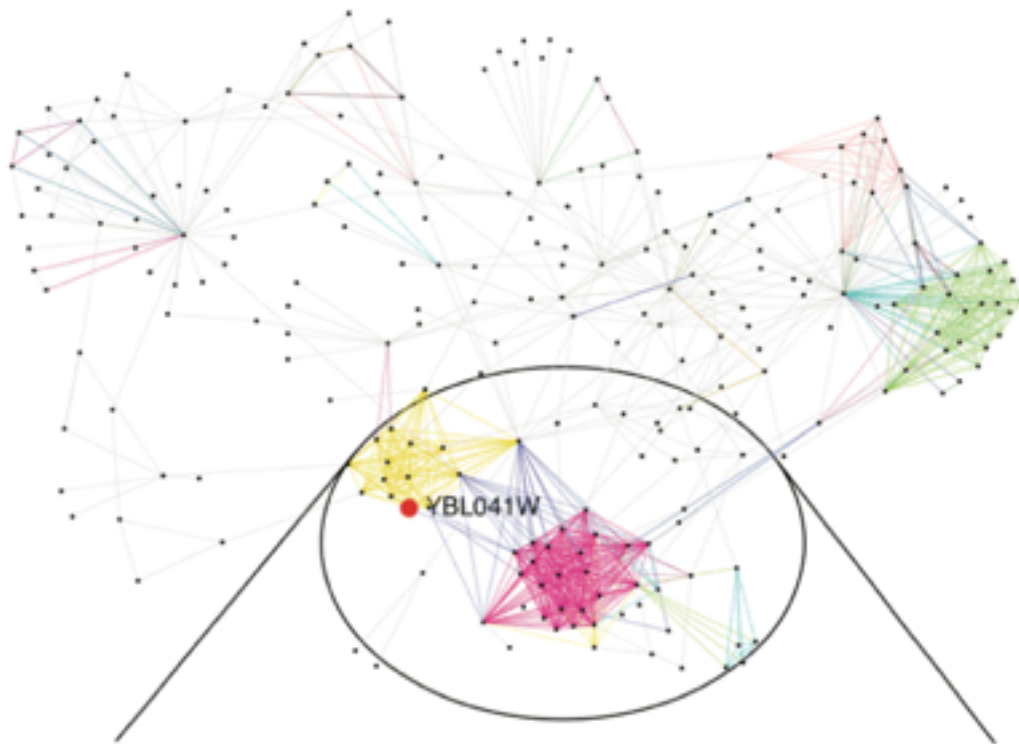
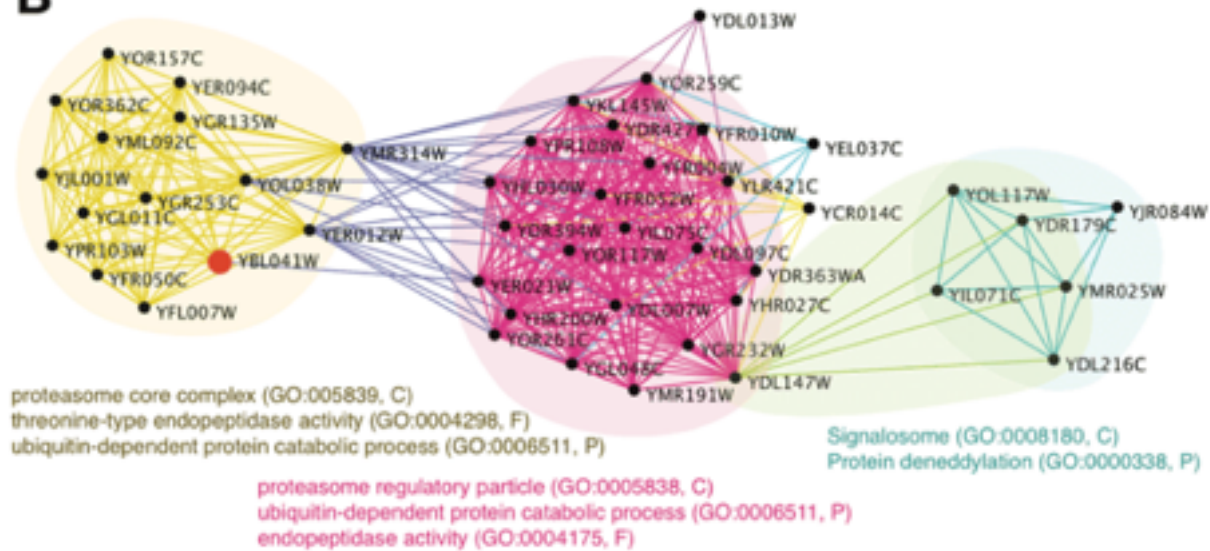
Belgian communication network

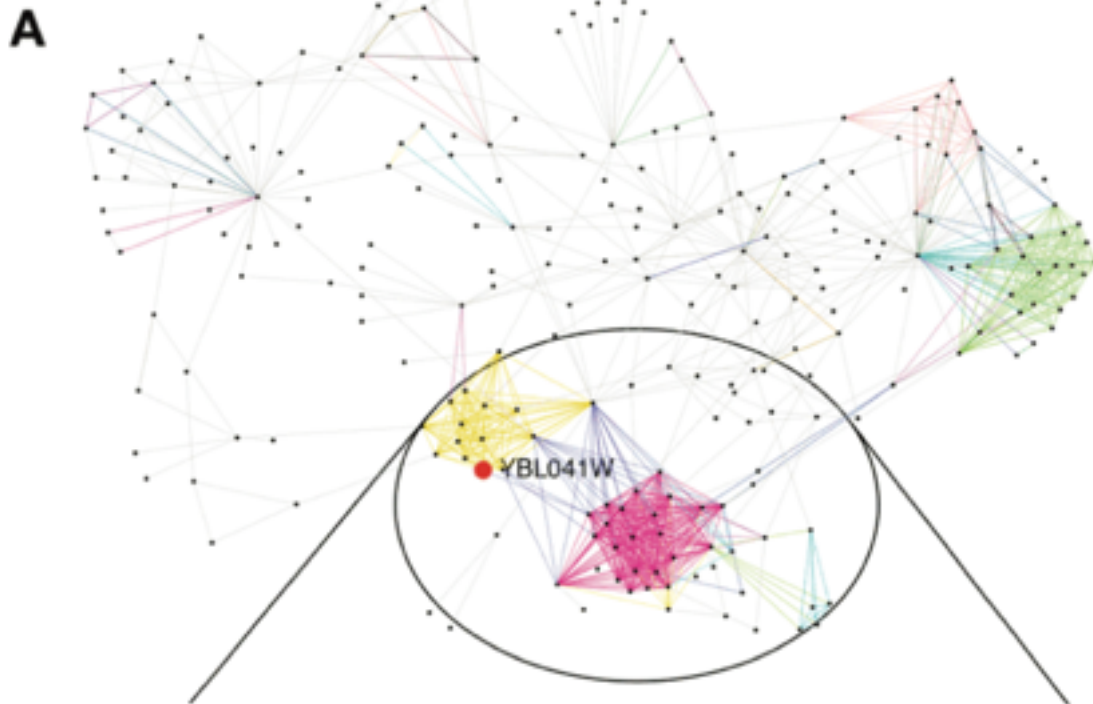


Belgian communication network



Social structure ~
Communities

A**B**



Protein complexes
 ~
communities

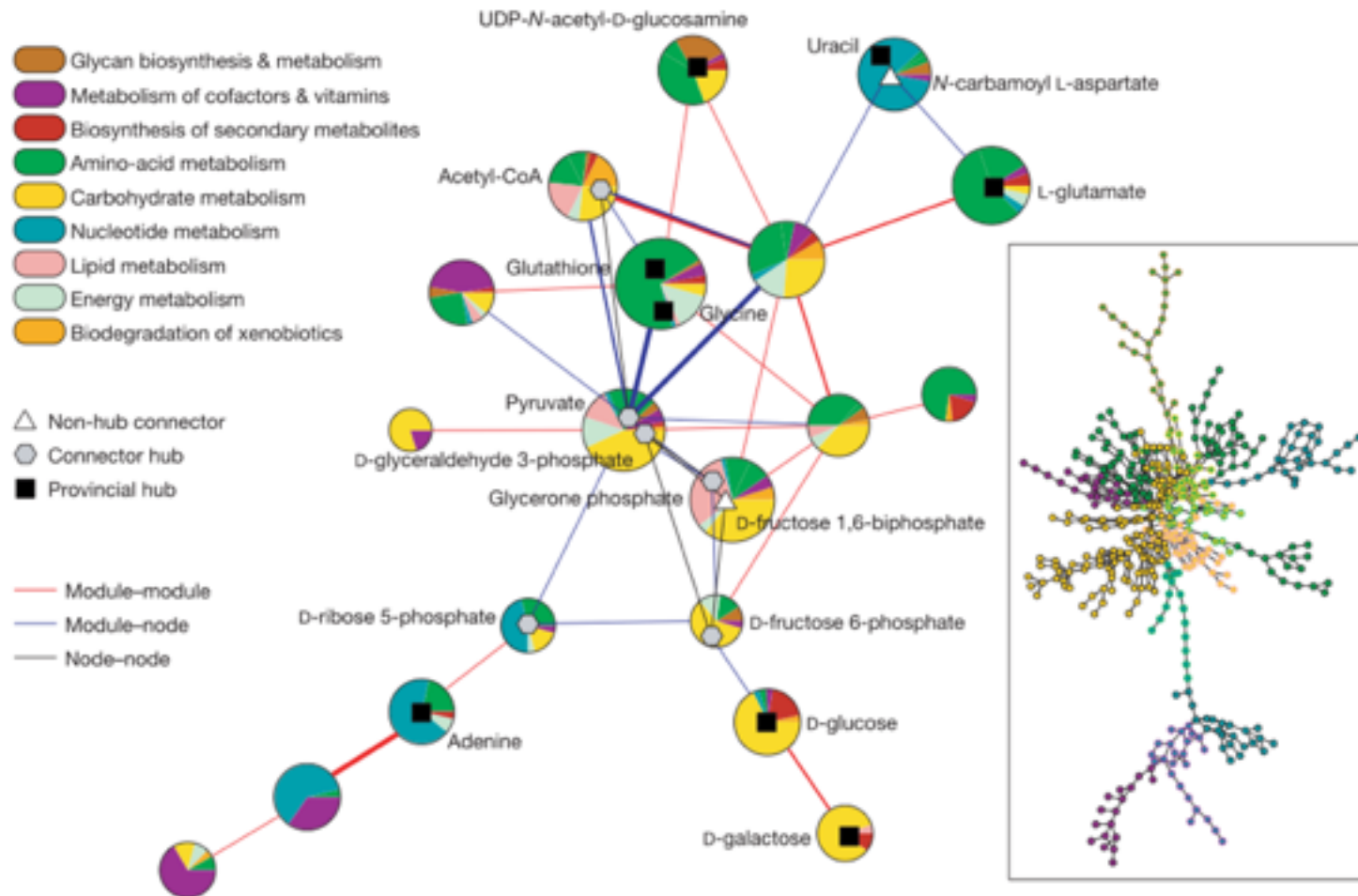


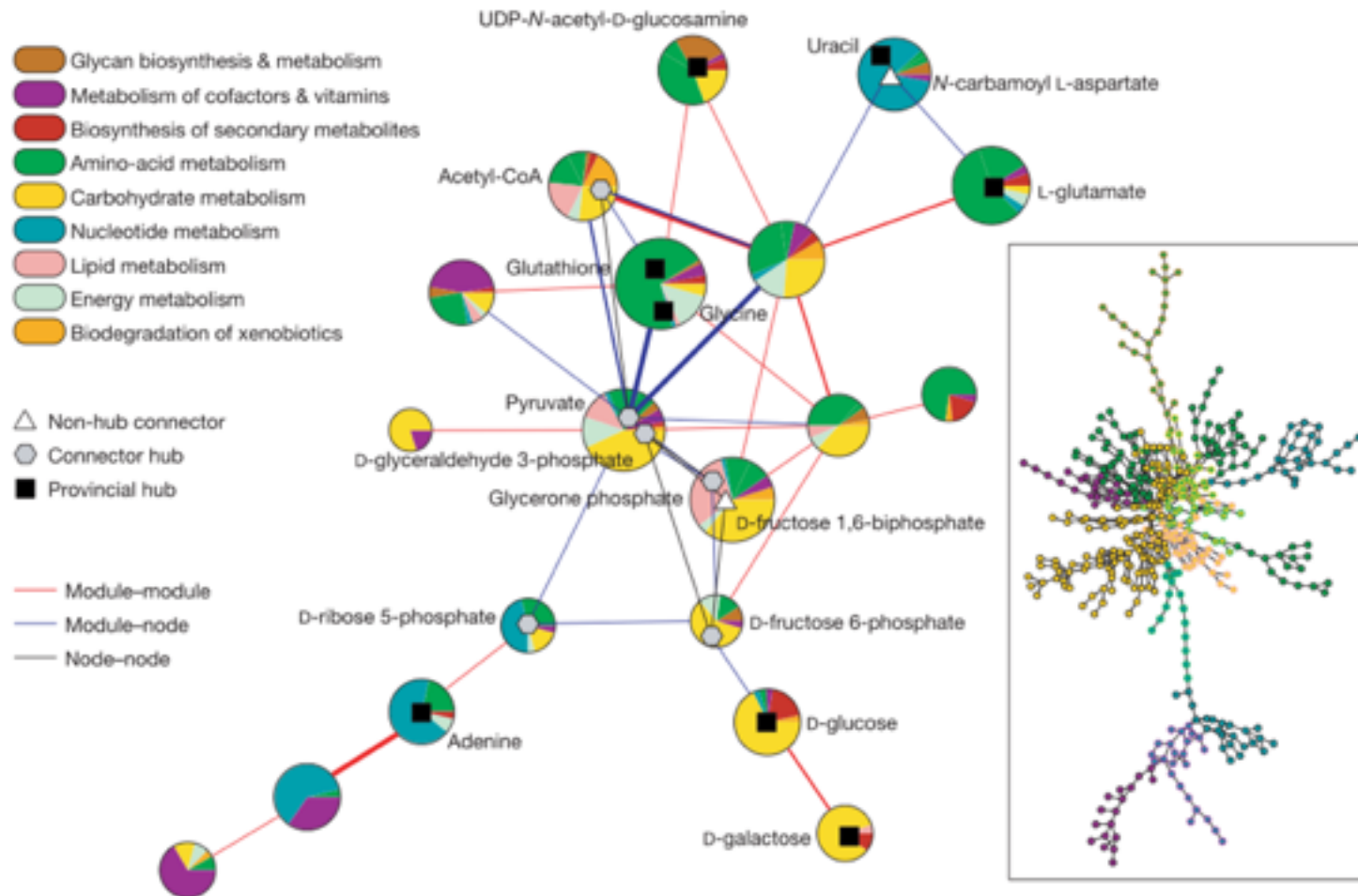
proteasome core complex (GO:005839, C)
 threonine-type endopeptidase activity (GO:0004298, F)
 ubiquitin-dependent protein catabolic process (GO:0006511, P)

Signalosome (GO:0008180, C)
 Protein deneeddylation (GO:0000338, P)

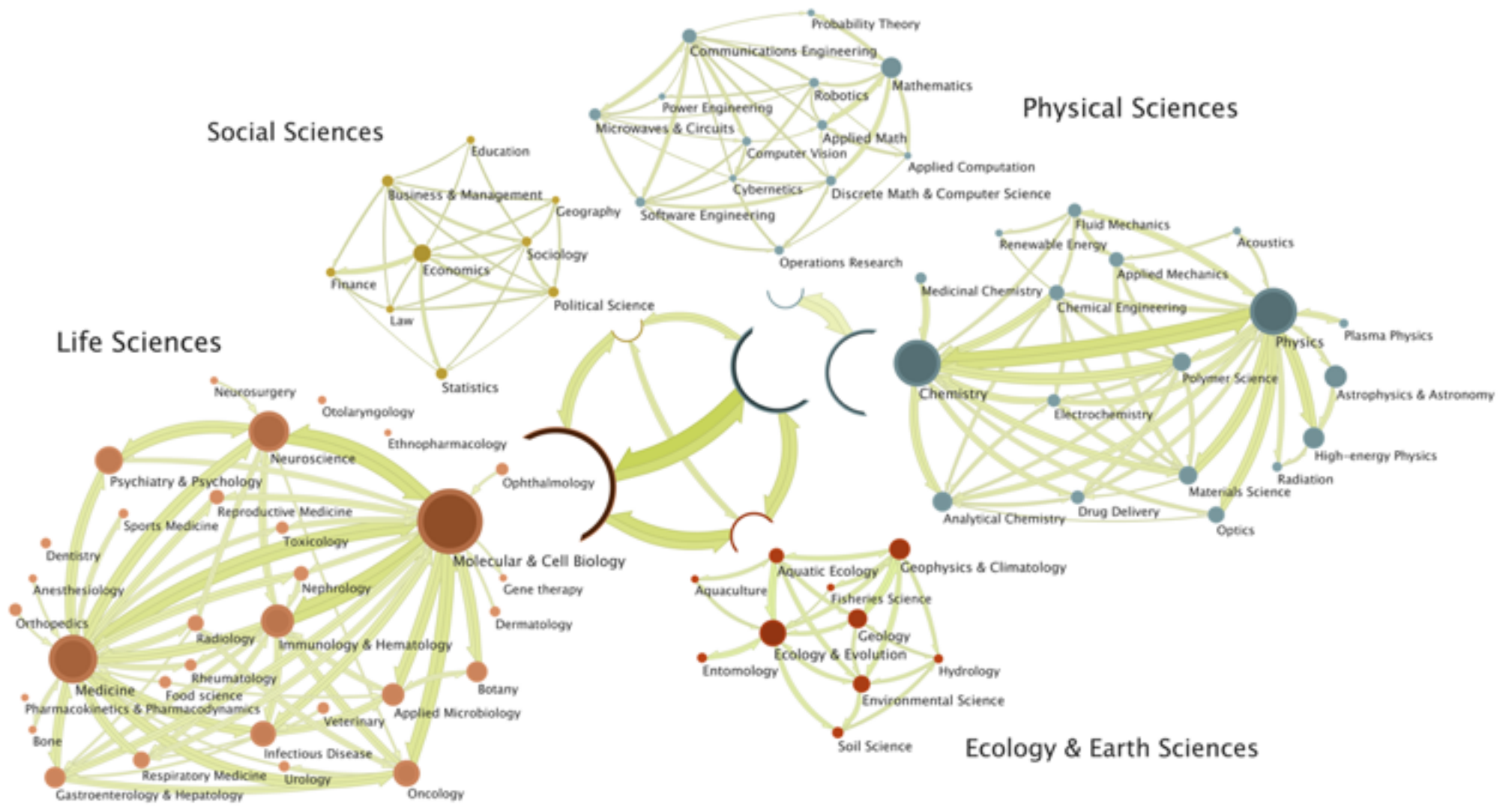
proteasome regulatory particle (GO:0005838, C)
 ubiquitin-dependent protein catabolic process (GO:0006511, P)
 endopeptidase activity (GO:0004175, F)



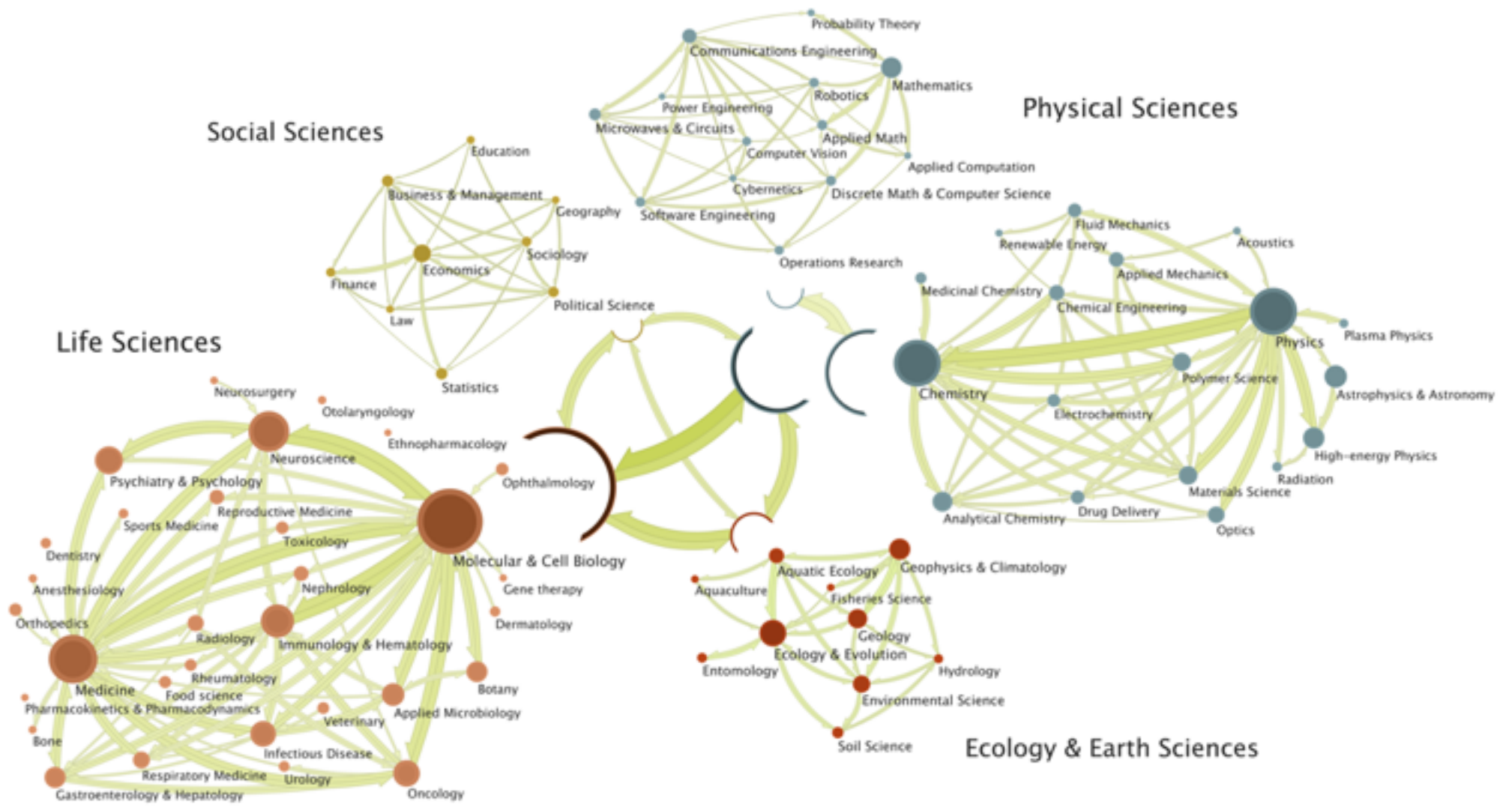




Metabolic pathways ~ **communities**



M. Rosvall, C. T. Bergstrom, PLoS One (2011)



Disciplines ~ **communities**

Social Networks

Social circles,
communities

Biological networks

Protein complexes,
functional modules

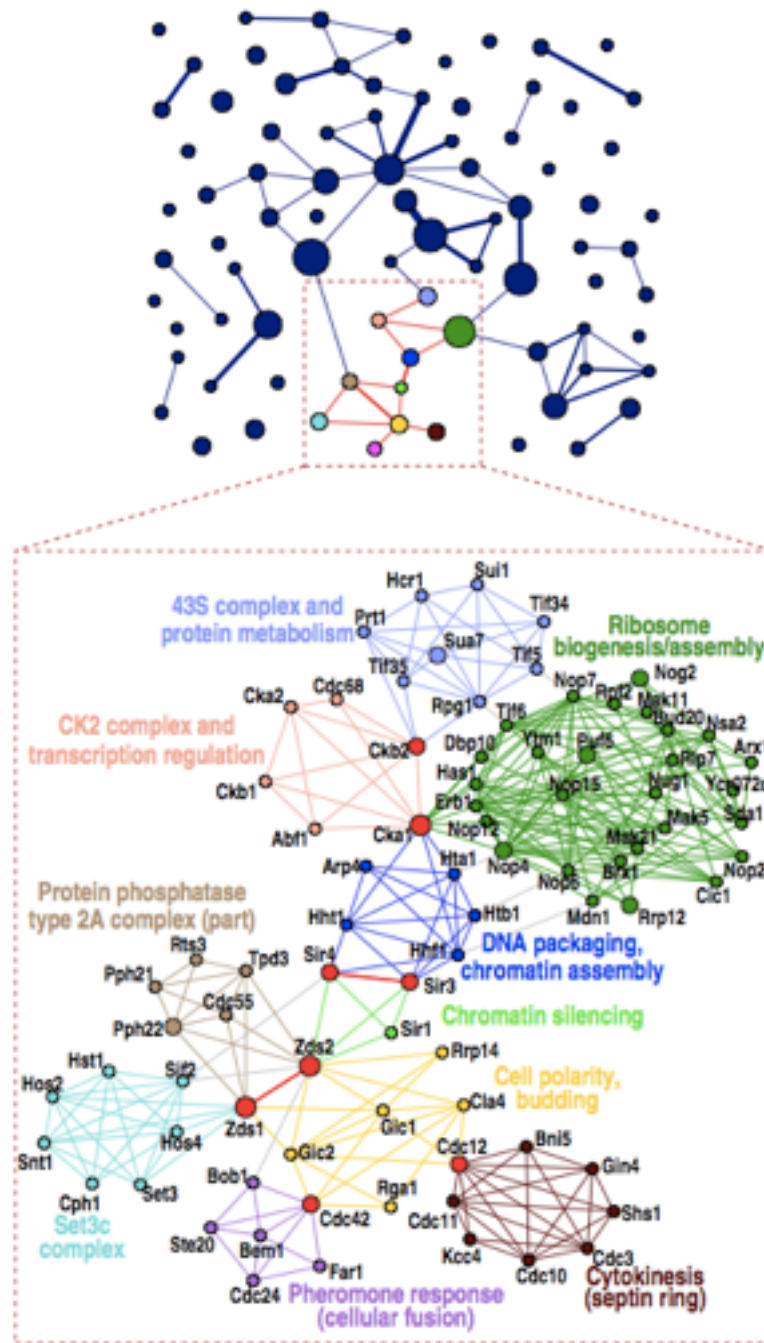
Citation networks

Disciplines,
scientific communities

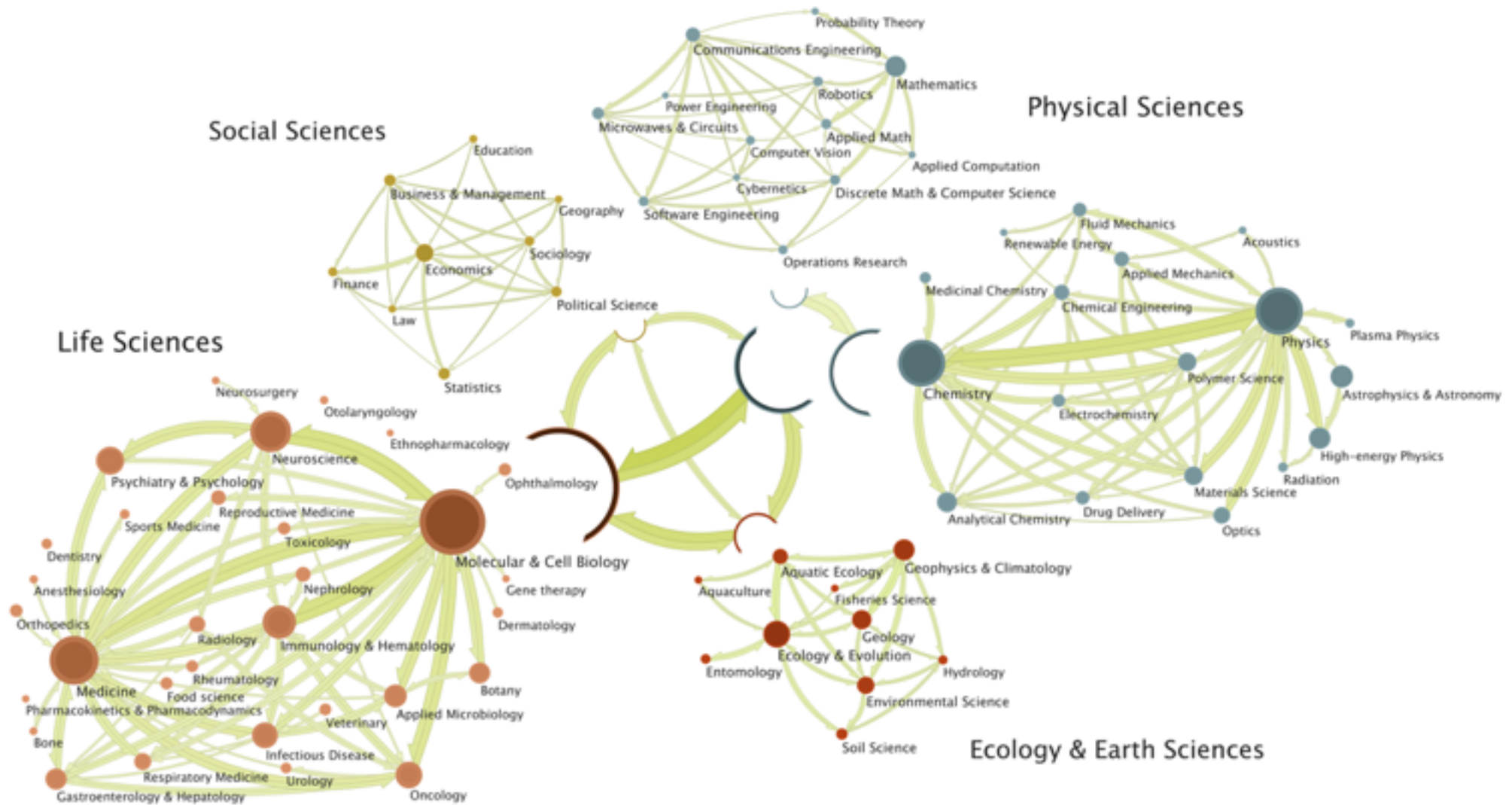
...

...

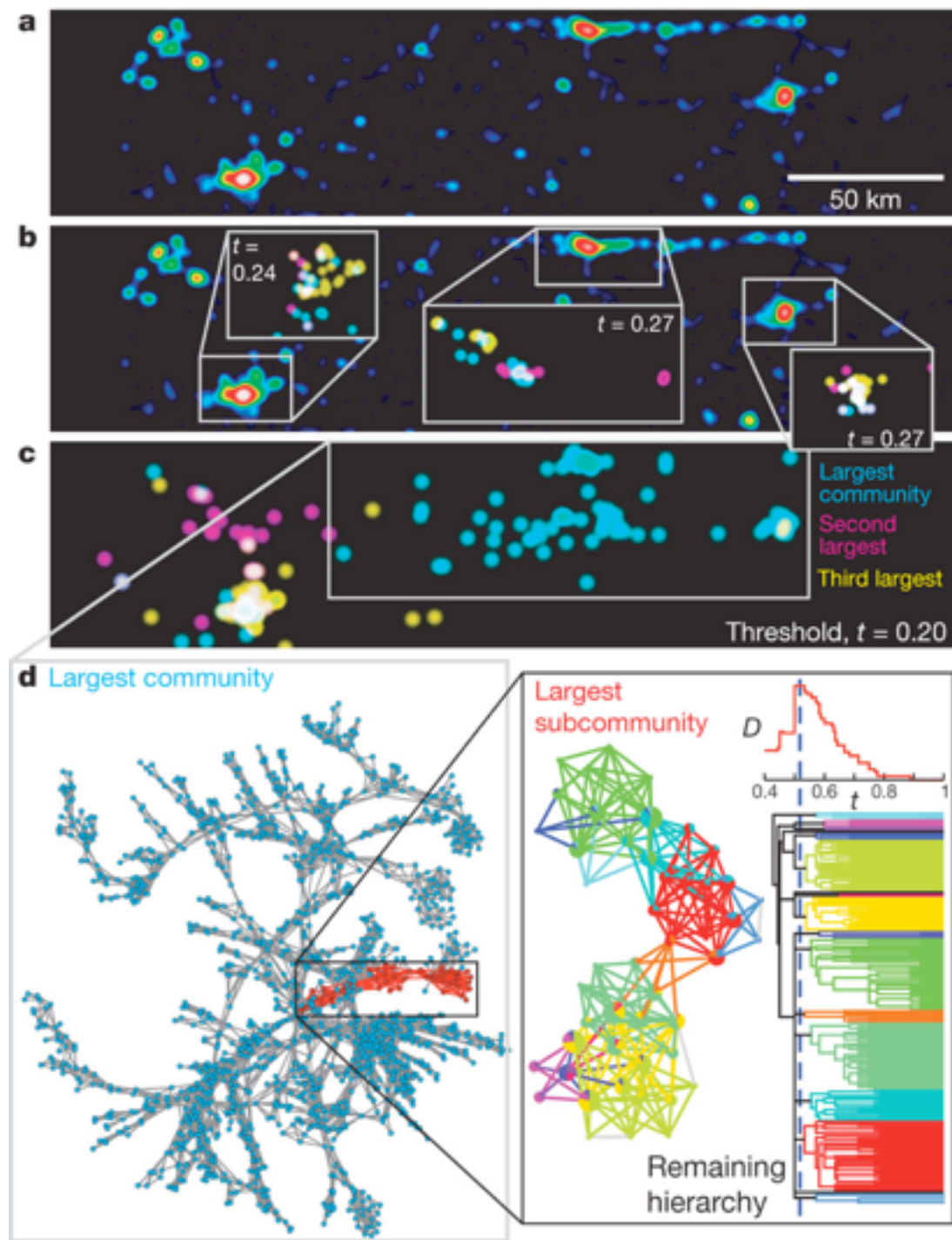
Finding communities:
A nice way to
overview
the whole system



G. Palla, I. Derenyi, I. Farkas, T. Vicsek, *Nature* (2005).



M. Rosvall, C. T. Bergstrom, *PLoS One* (2011)



Network community **analysis**

How to
define
communities?

Cohesiveness, Separation

or both

A nice review:

J. Yang and J. Leskovec, Defining and Evaluating
Network Communities based on Ground-truth,
ICDM 2012

“Cohesiveness”

Clique percolation
Link communities

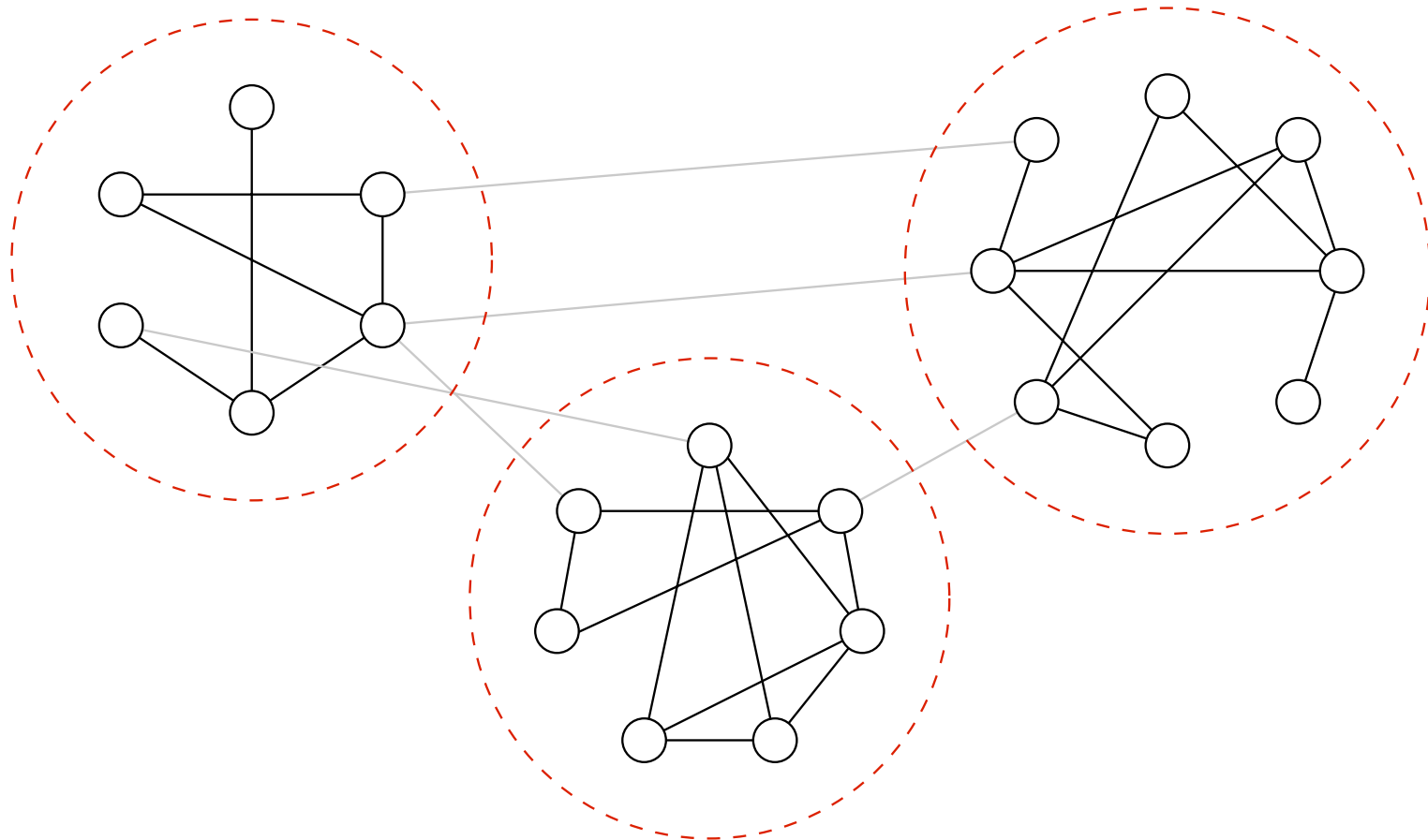
“Separation”

Girvan-Newman algorithm

Graph cuts

Spectral clustering

Cohesiveness + Separation



Modularity

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

of links within
- # of expected links

How to detect
communities?

We should be able to

1. evaluate a community structure
2. explore possible structures effectively

**Wait, can we just
check every possible
configurations?**

**Bell Number: # of
partitions of a set of size n .**

**Bell Number: # of
partitions of a set of size n.**

$$B_3 = 5$$

**Bell Number: # of
partitions of a set of size n.**

$$B_3 = 5$$

$$B_{100} = ?$$

16187060274460683058556806
28161135741330684513088812
39989840947008912873079240
70443511081340194490281914
80663320741161870602744606
83058556806281611357413306
84513088812399898409470089
12873079240704435110813401
9449028191480663320741

Impossible to
enumerate

Fundamental problem of
community detection

1. evaluate a community structure

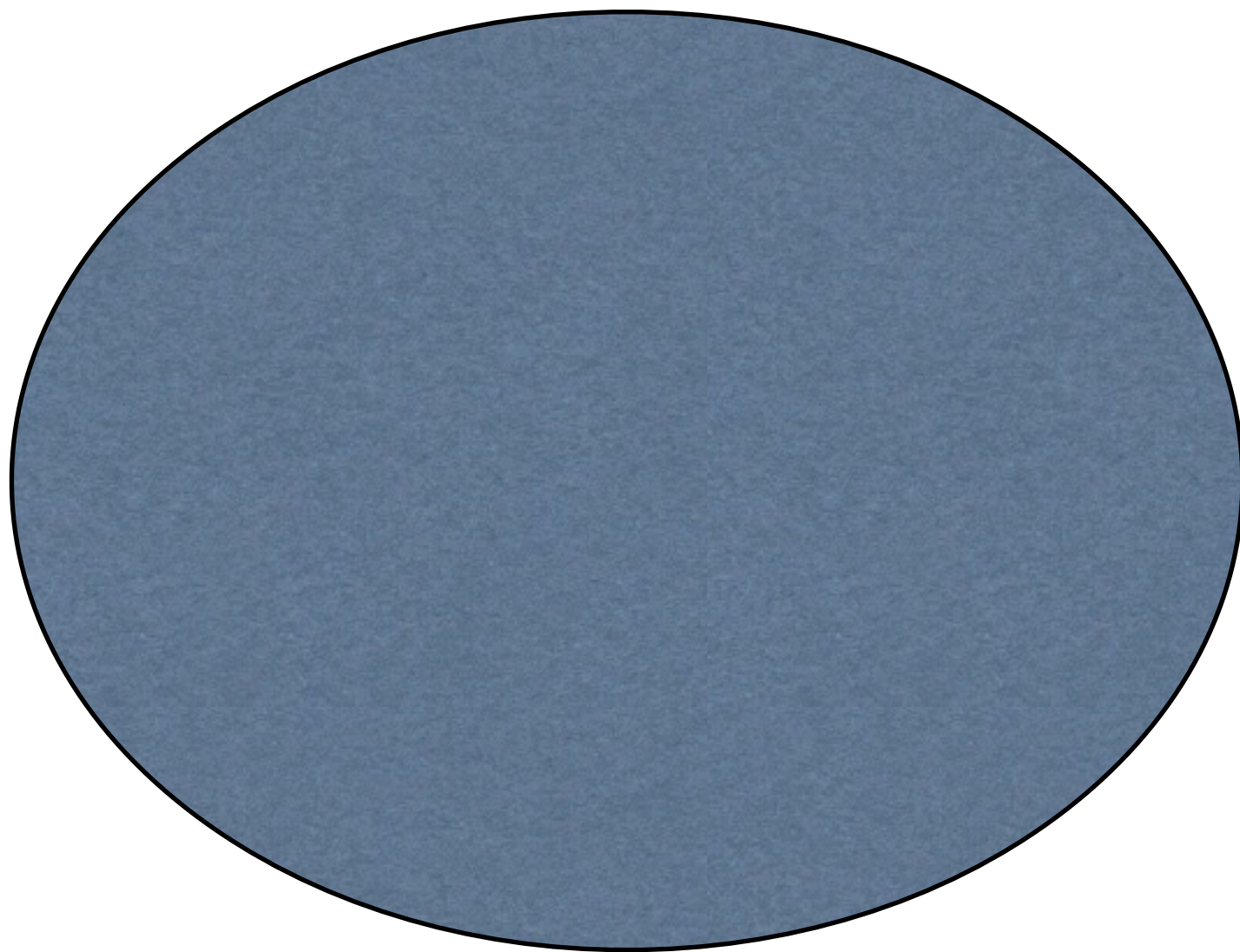
- Modularity, cliques, map equation, partition density, ...

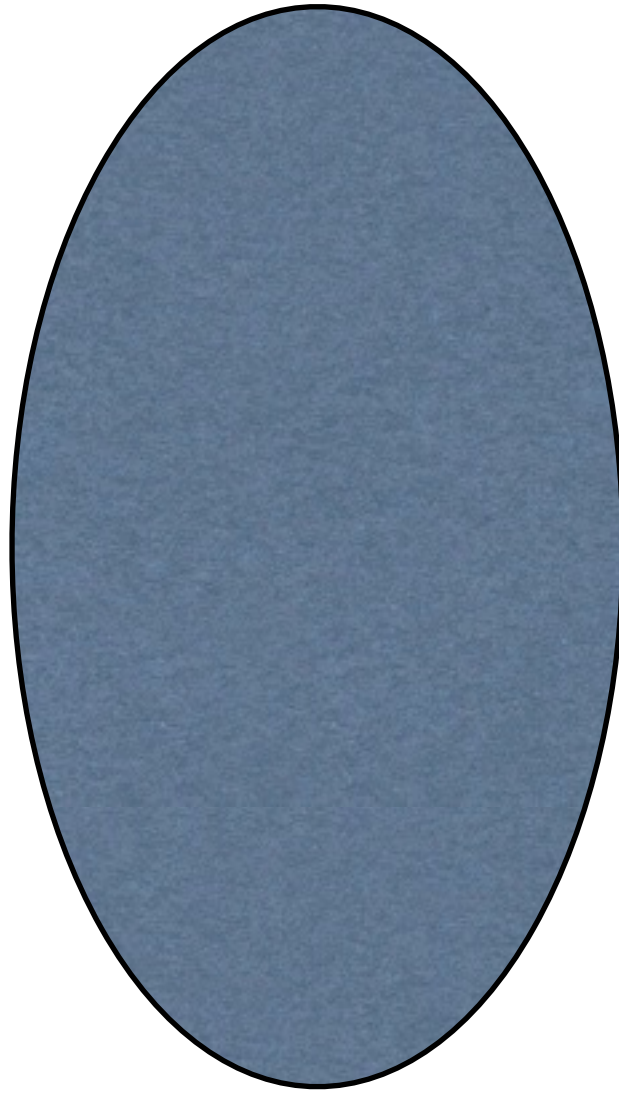
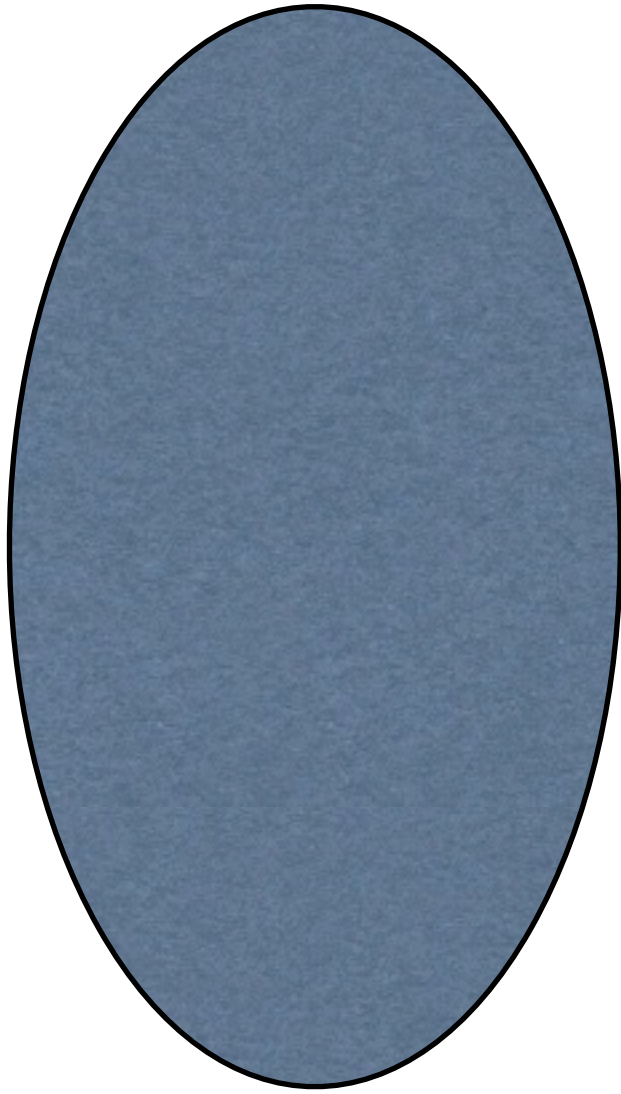
2. explore possible structures effectively

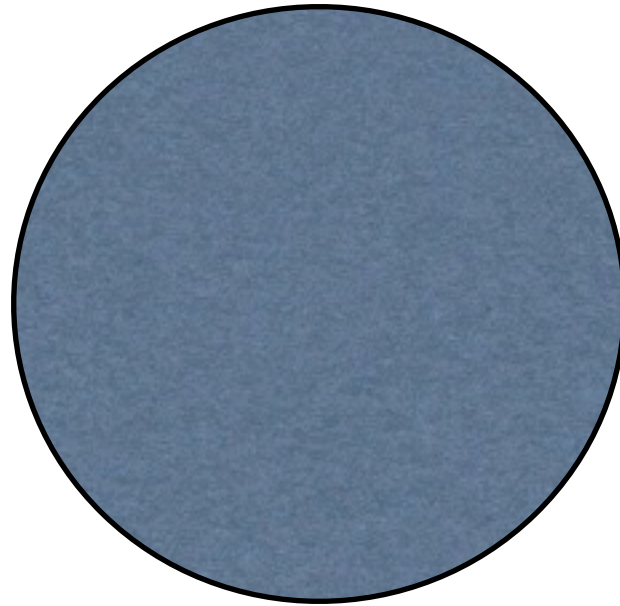
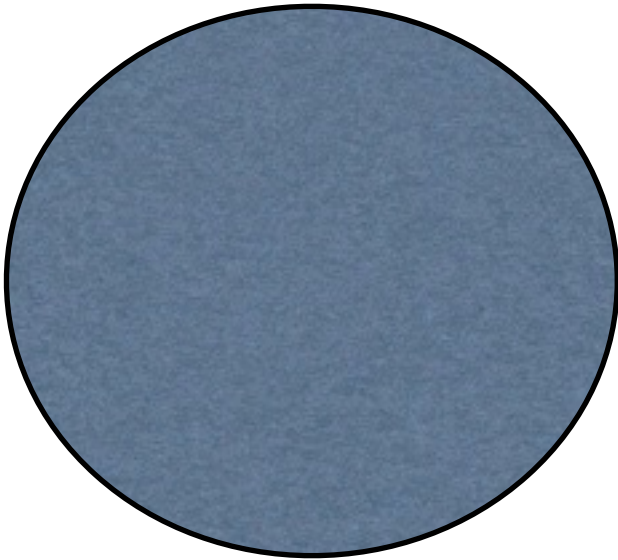
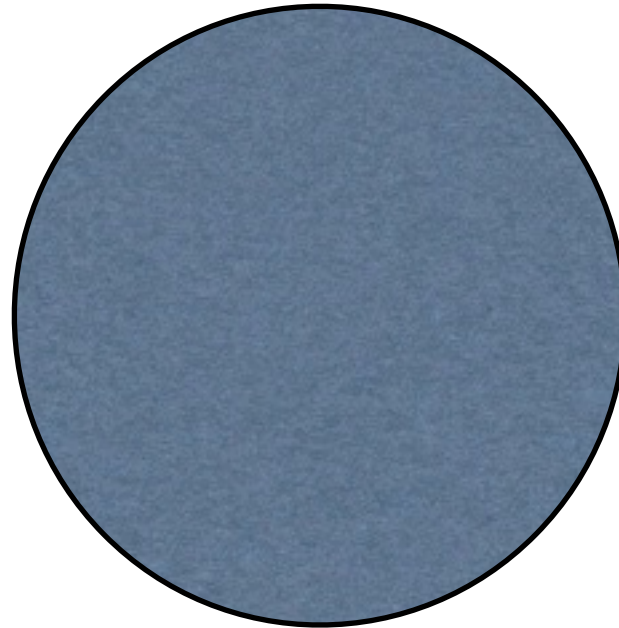
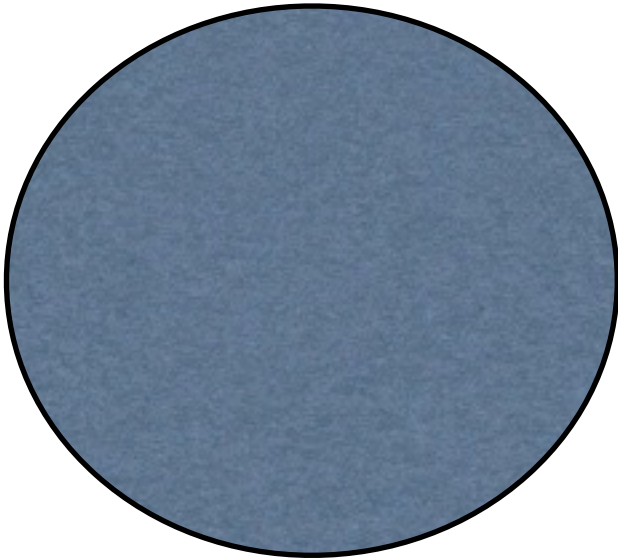
- Many heuristics, Divisive & agglomerative clustering, Monte-carlo, ...

Modularity-based methods

Divisive vs.
Agglomerative



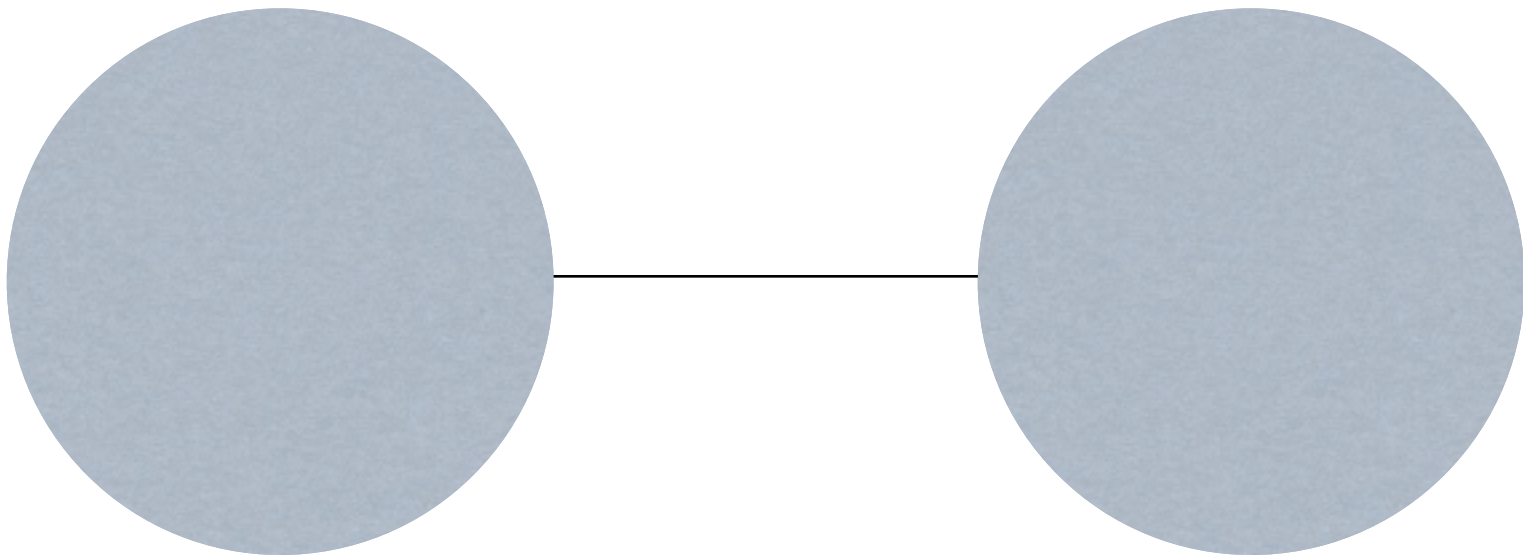


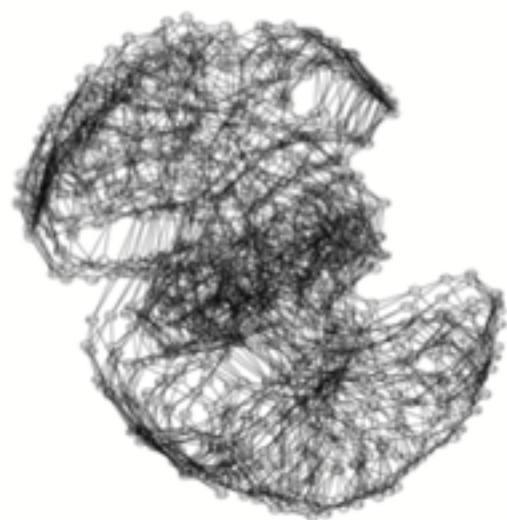


Girvan-Newman algorithm

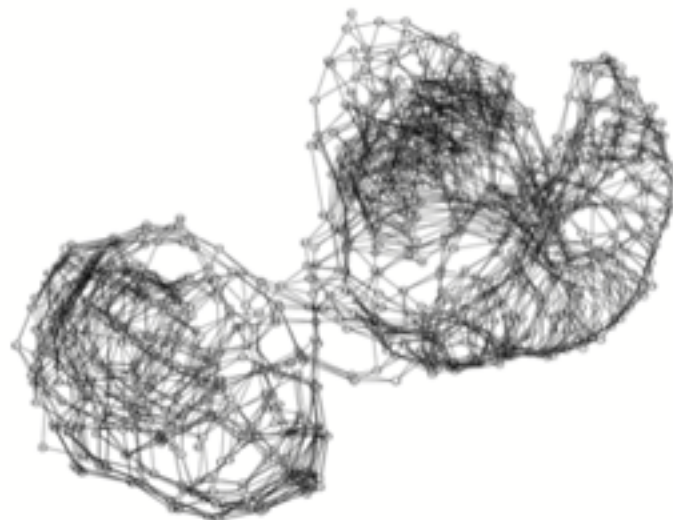
M. Newman, M. Girvan, PNAS (2002)

Idea

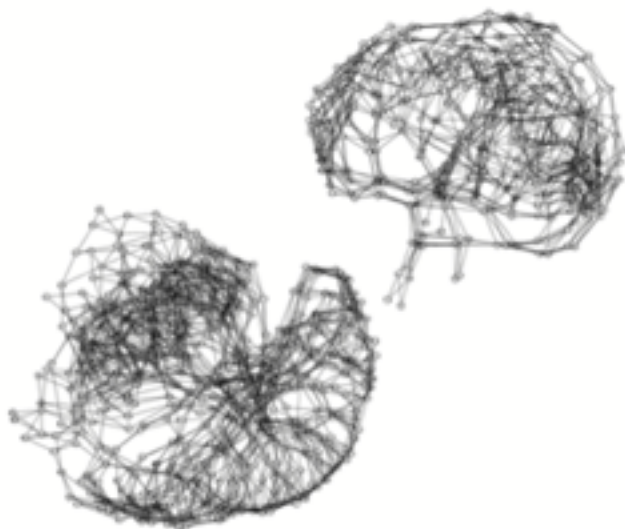




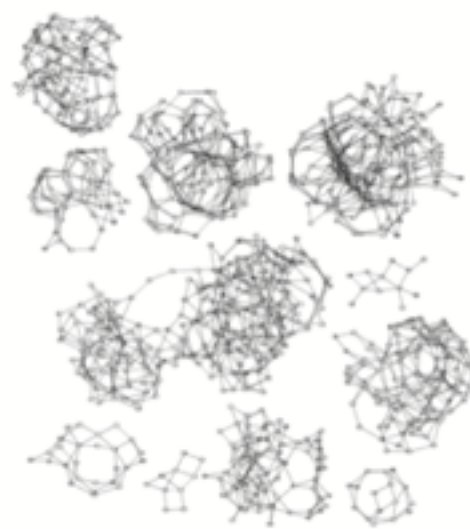
0 cuts



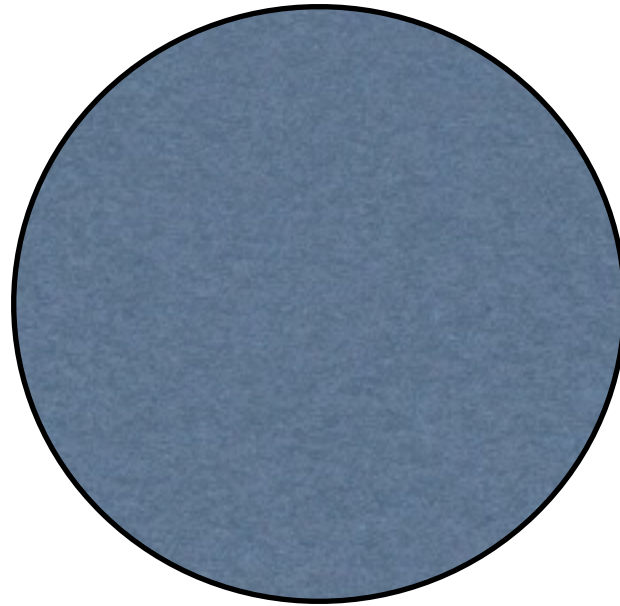
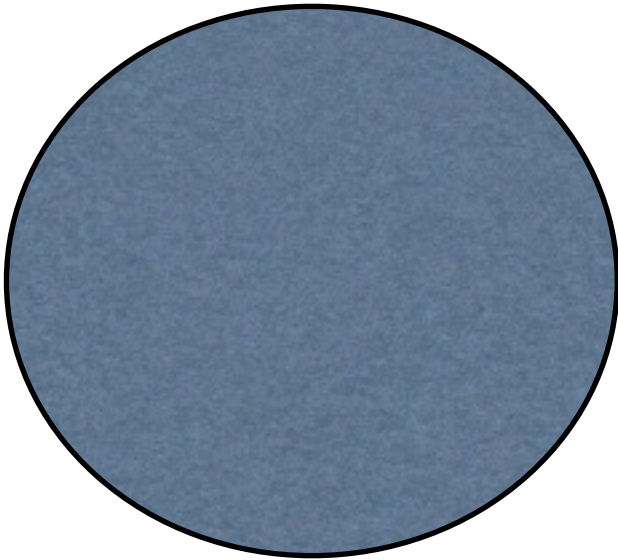
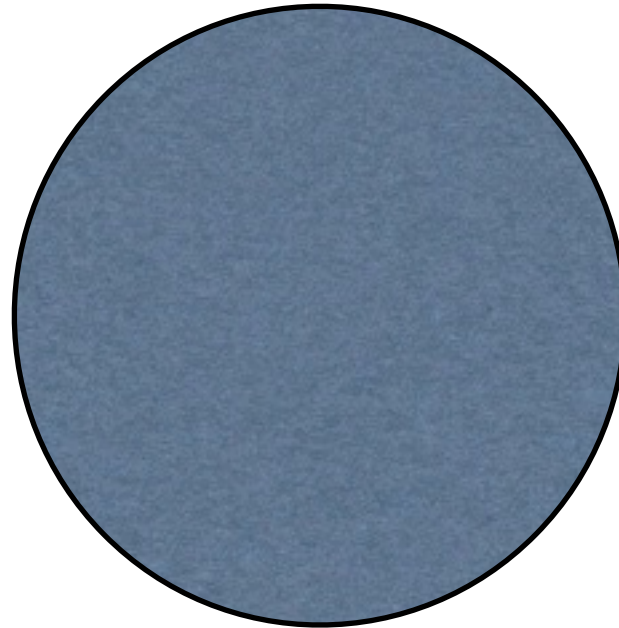
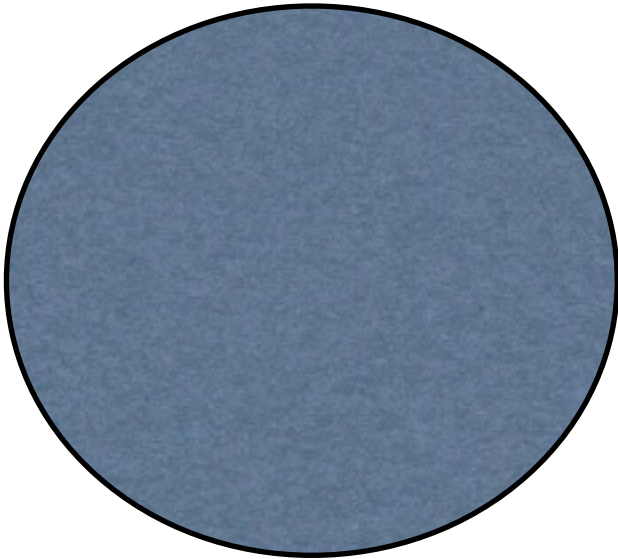
100 cuts

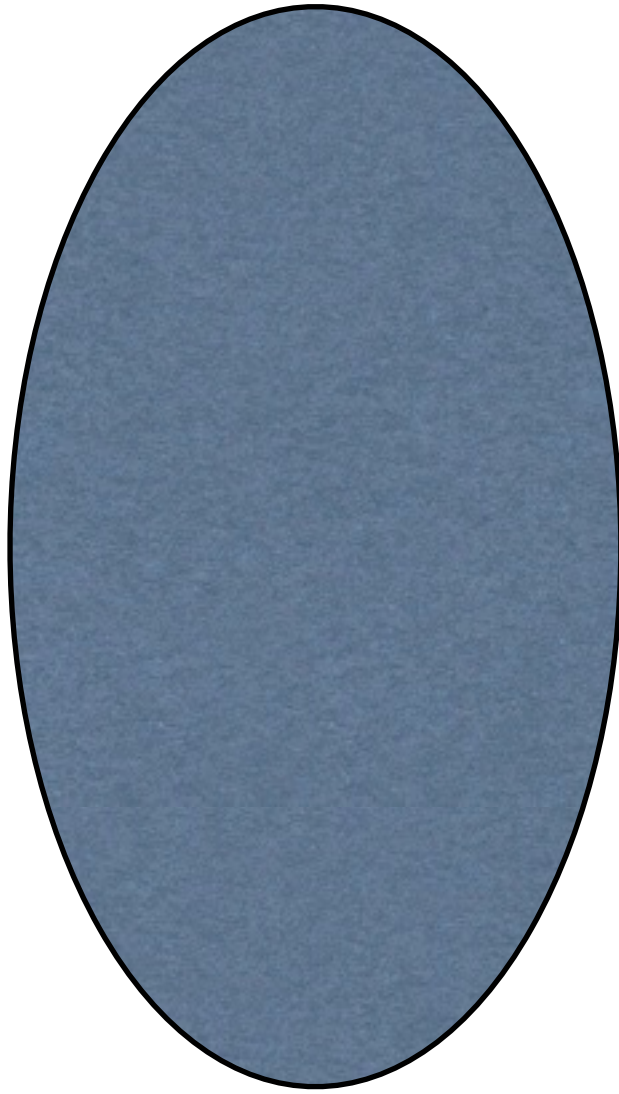
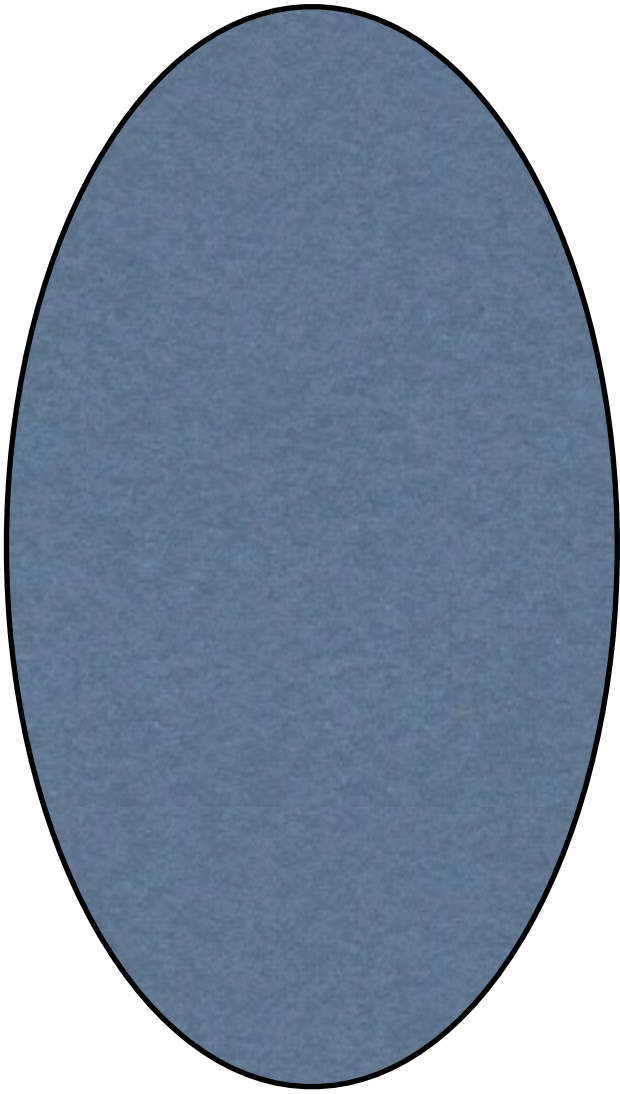


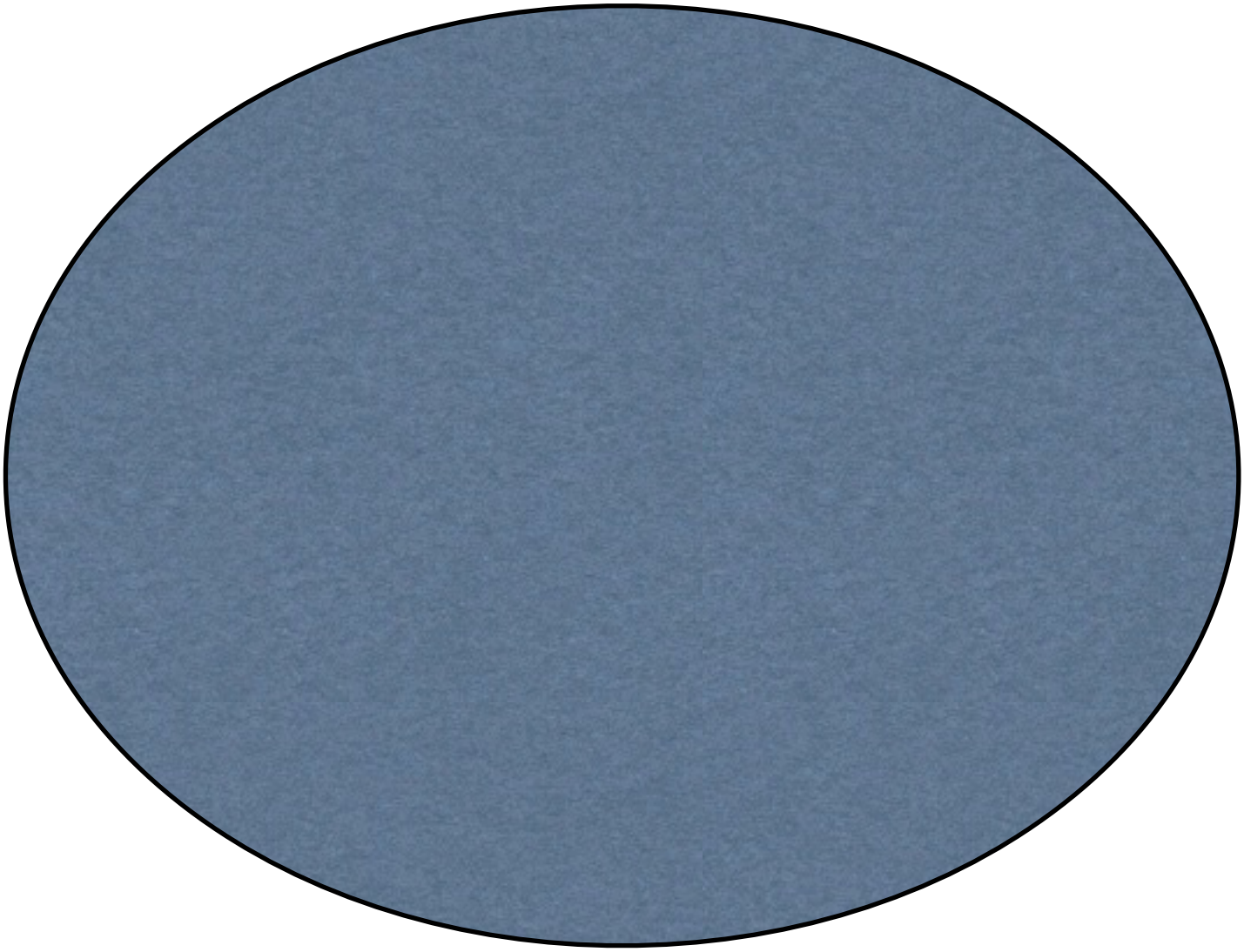
120 cuts



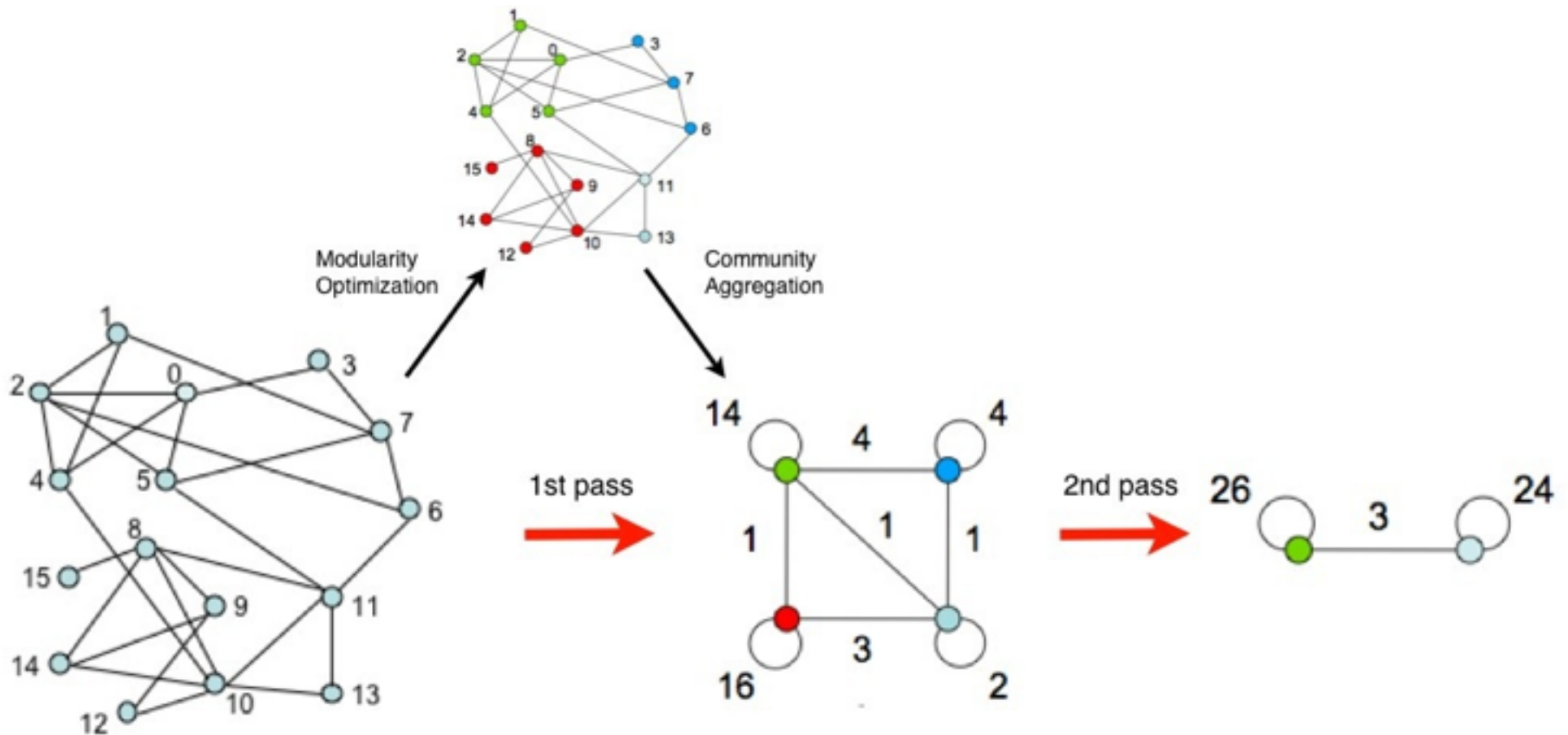
500 cuts







Louvain method



Various optimization techniques

- A. Clauset, M. Newman, C. Moore: Greedy optimization
- R. Guimera, L. A. N. Amaral: Extremal optimization
- V. D. Blondel, J.-L. Guillaume, R. Lambiotte, E. Lefebvre: Hierarchical aggregation
- **Any** optimization technique can be used.

“Cliques”

What is a 'perfect
community'?

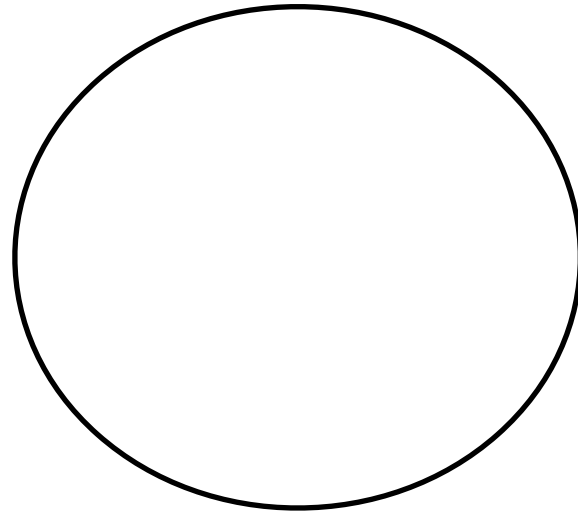
A clique!

Then, how about
finding quasi-cliques?

Clique Percolation Method

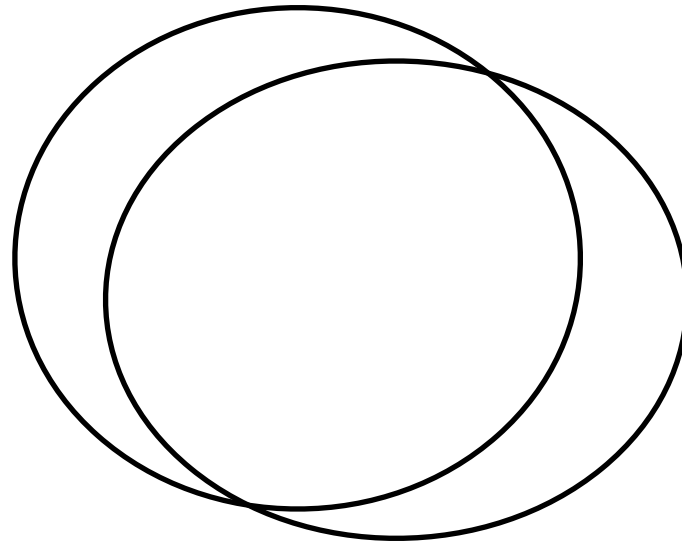
- “Rolling” a clique to find a quasi-clique.
- Quasi-cliques are communities.

Clique Percolation Method



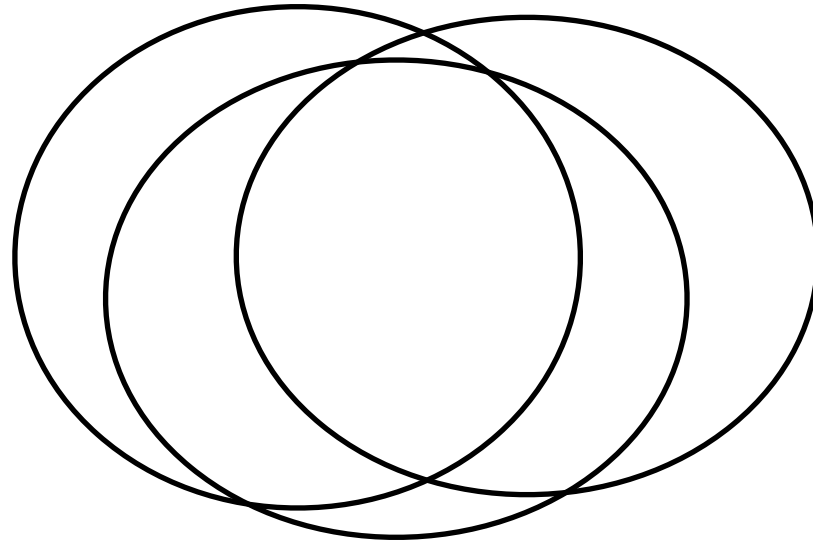
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Clique Percolation Method



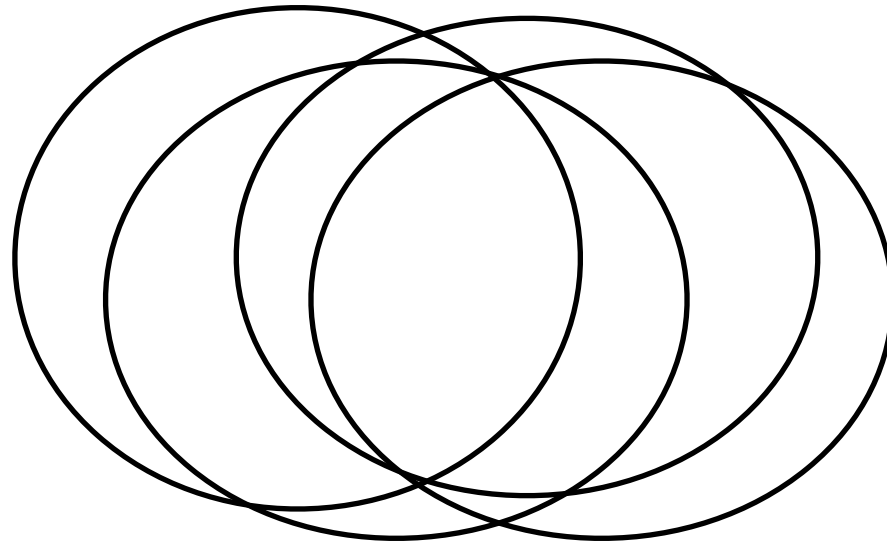
- “Rolling” a clique to find a quasi-clique.
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Clique Percolation Method

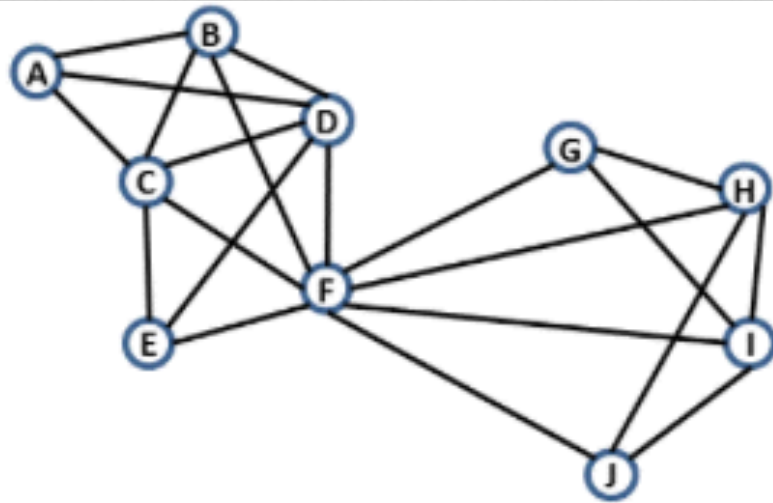


- “Rolling” a clique to find a quasi-clique.
- Quasi-cliques are communities.

Clique Percolation Method

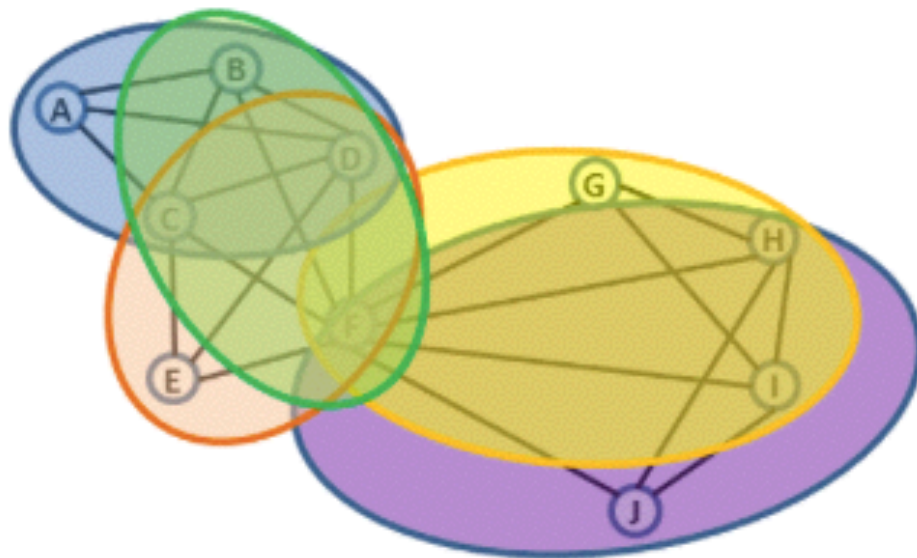


- “Rolling” a clique to find a quasi-clique.
- Quasi-cliques are communities.



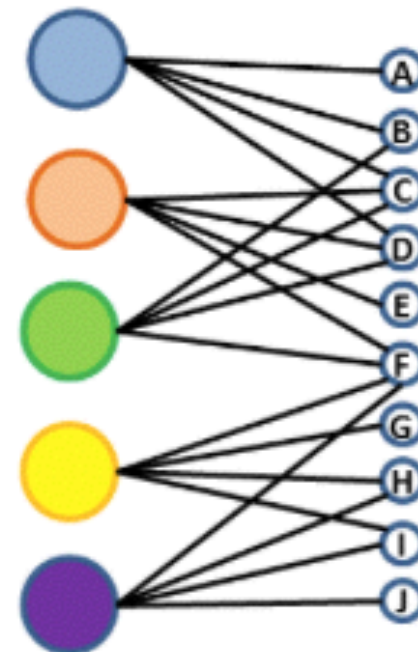
Original Graph

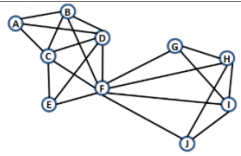
Step 1: Find all K-Cliques ($K = 4$)



Bi-partite graph

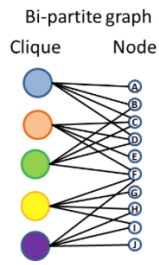
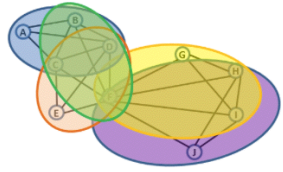
Clique Node



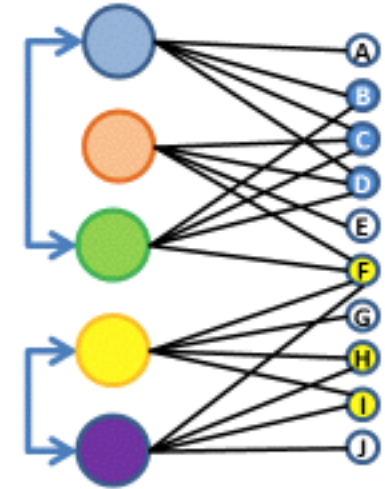
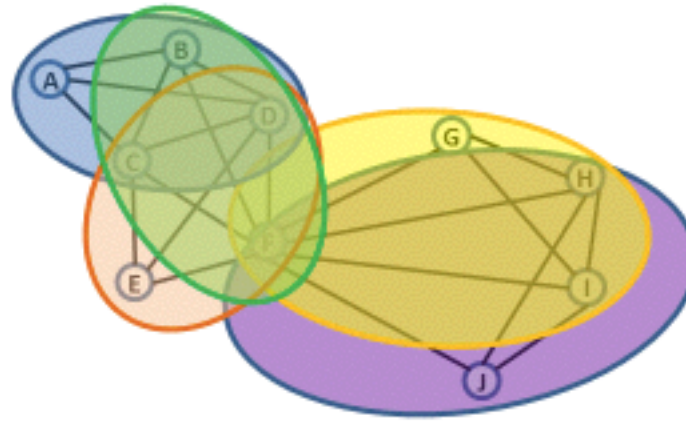


Original Graph

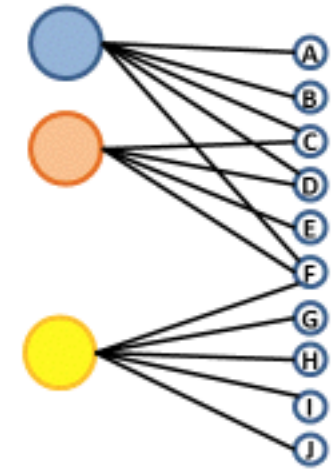
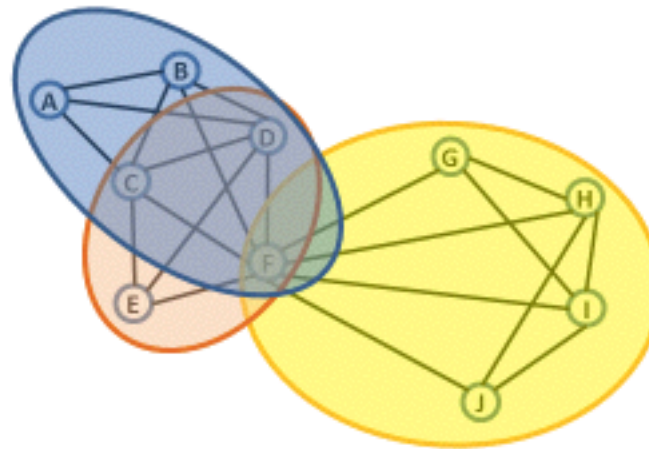
Step 1: Find all K-Cliques (K = 4)



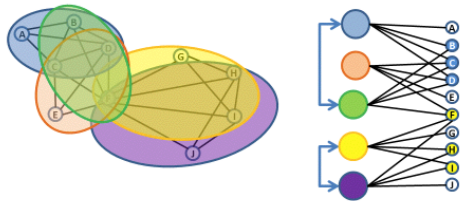
Step 2: Combine adjacent cliques (with K-1 = 3 shared nodes)



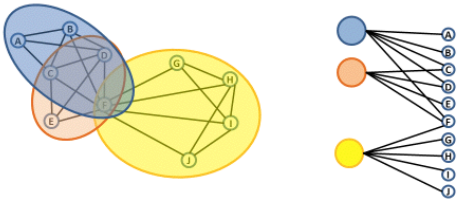
After merging adjacent cliques



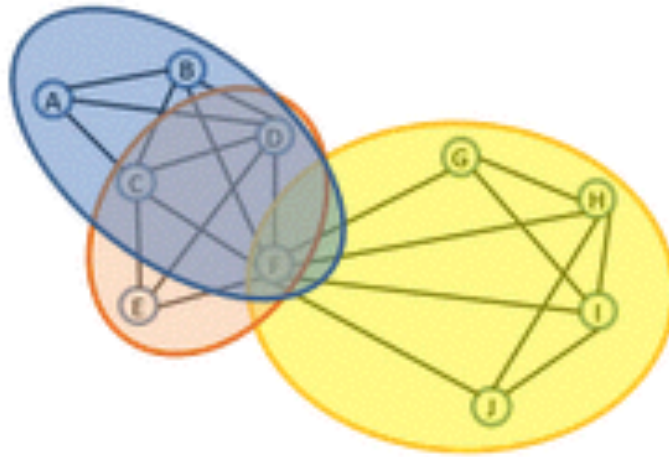
Step 2: Combine adjacent cliques (with $K-1 = 3$ shared nodes)



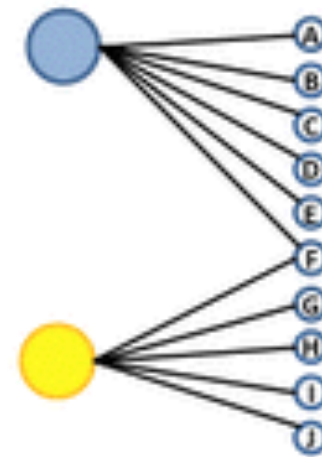
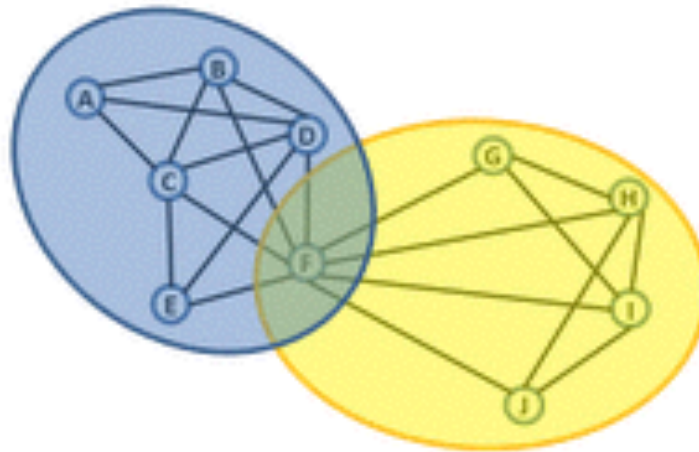
After merging adjacent cliques

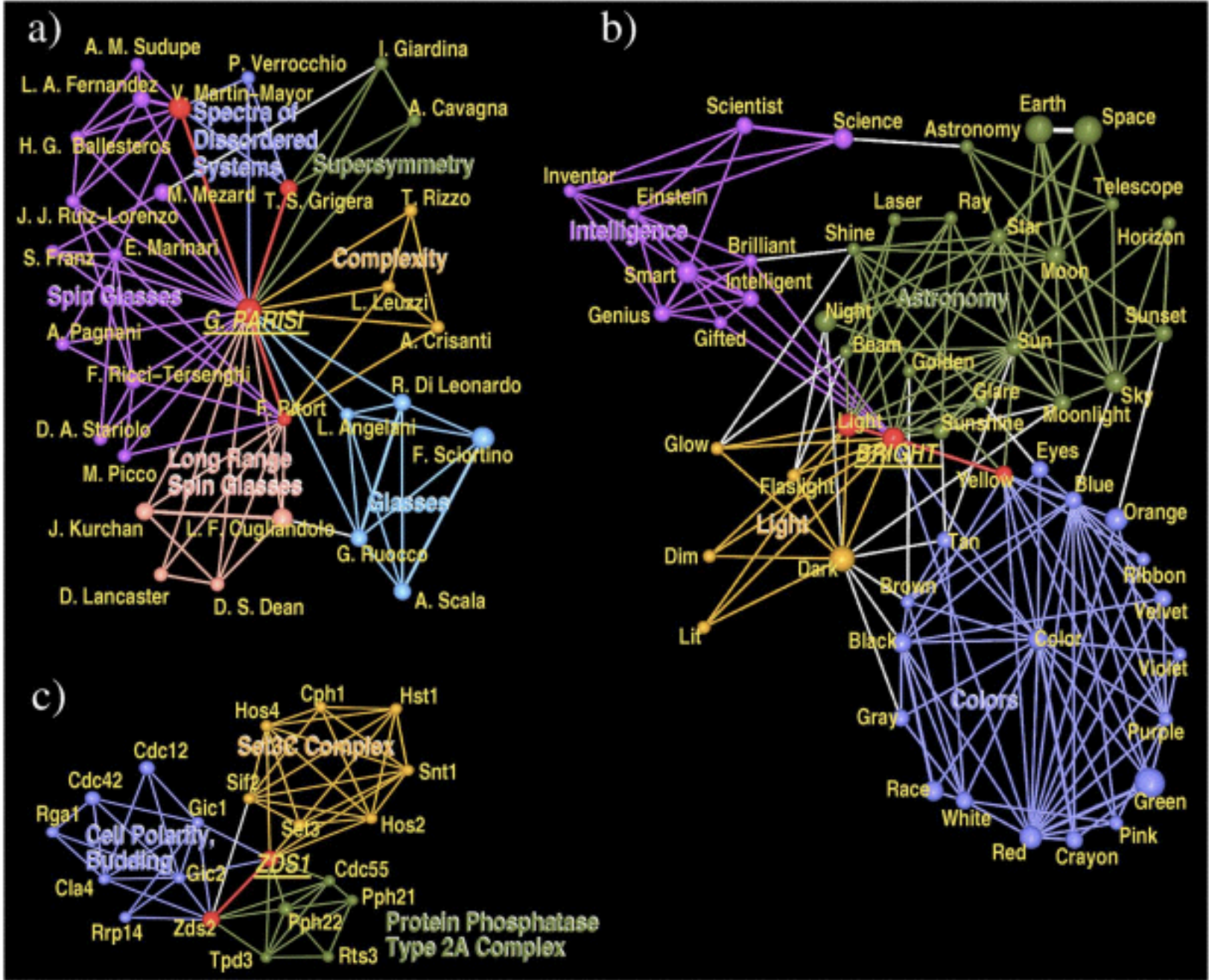


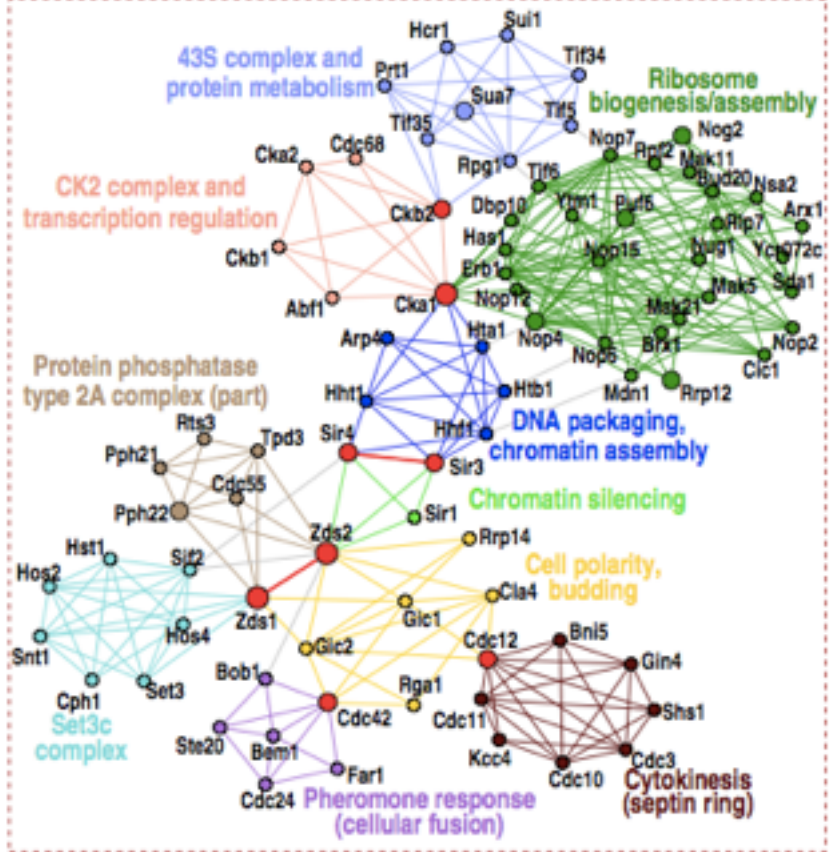
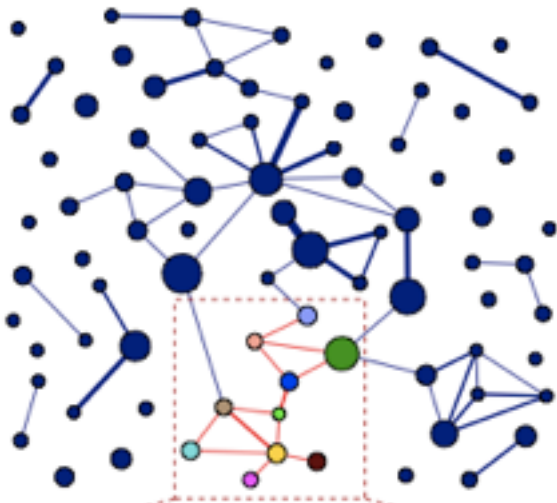
Step 3: Combine adjacent cliques (with $K-1 = 3$ shared nodes)



After merging adjacent cliques, there are 2 overlapping communities





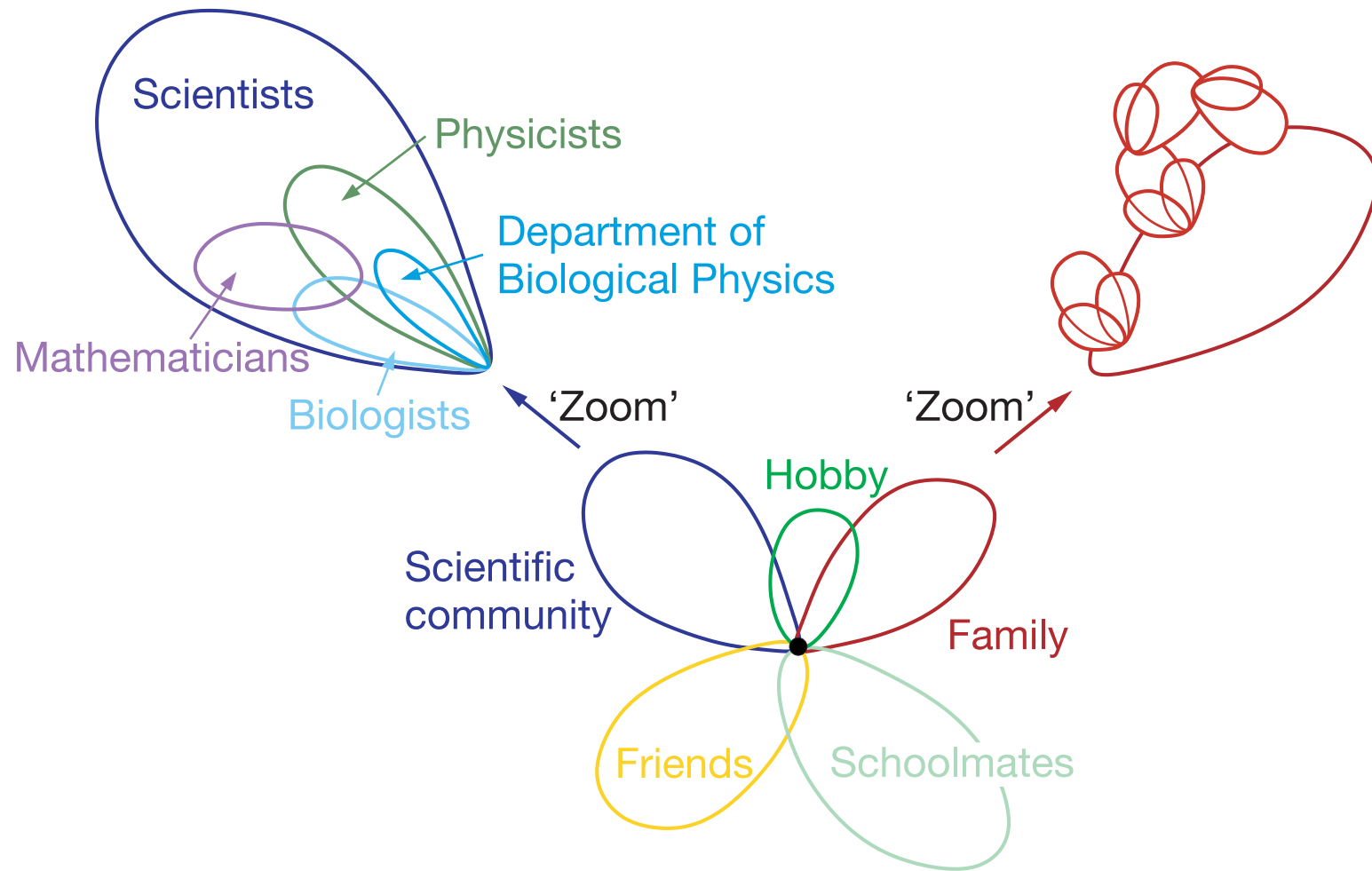


“Information”

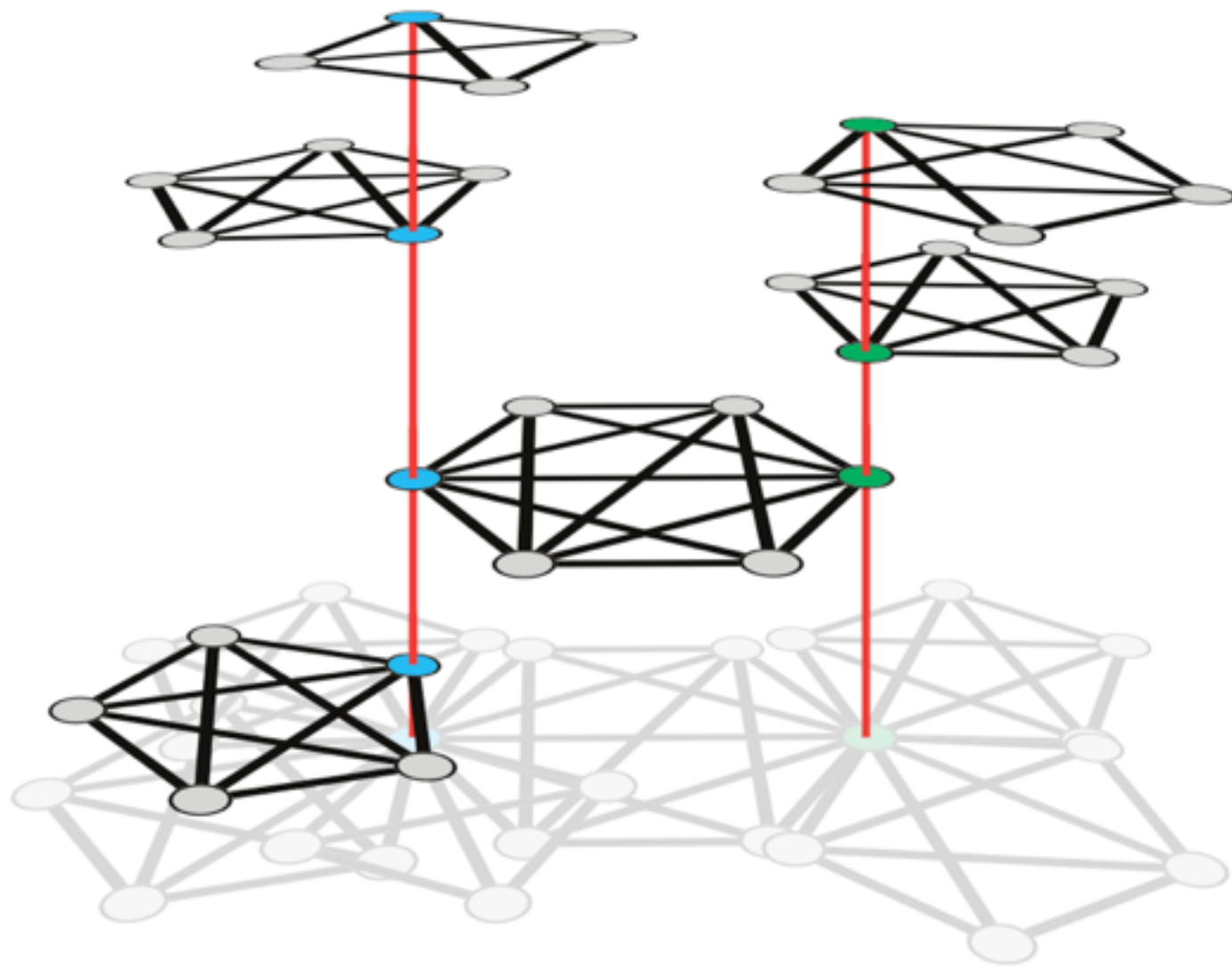
If there is a random walker on the network, it will be trapped inside each community.

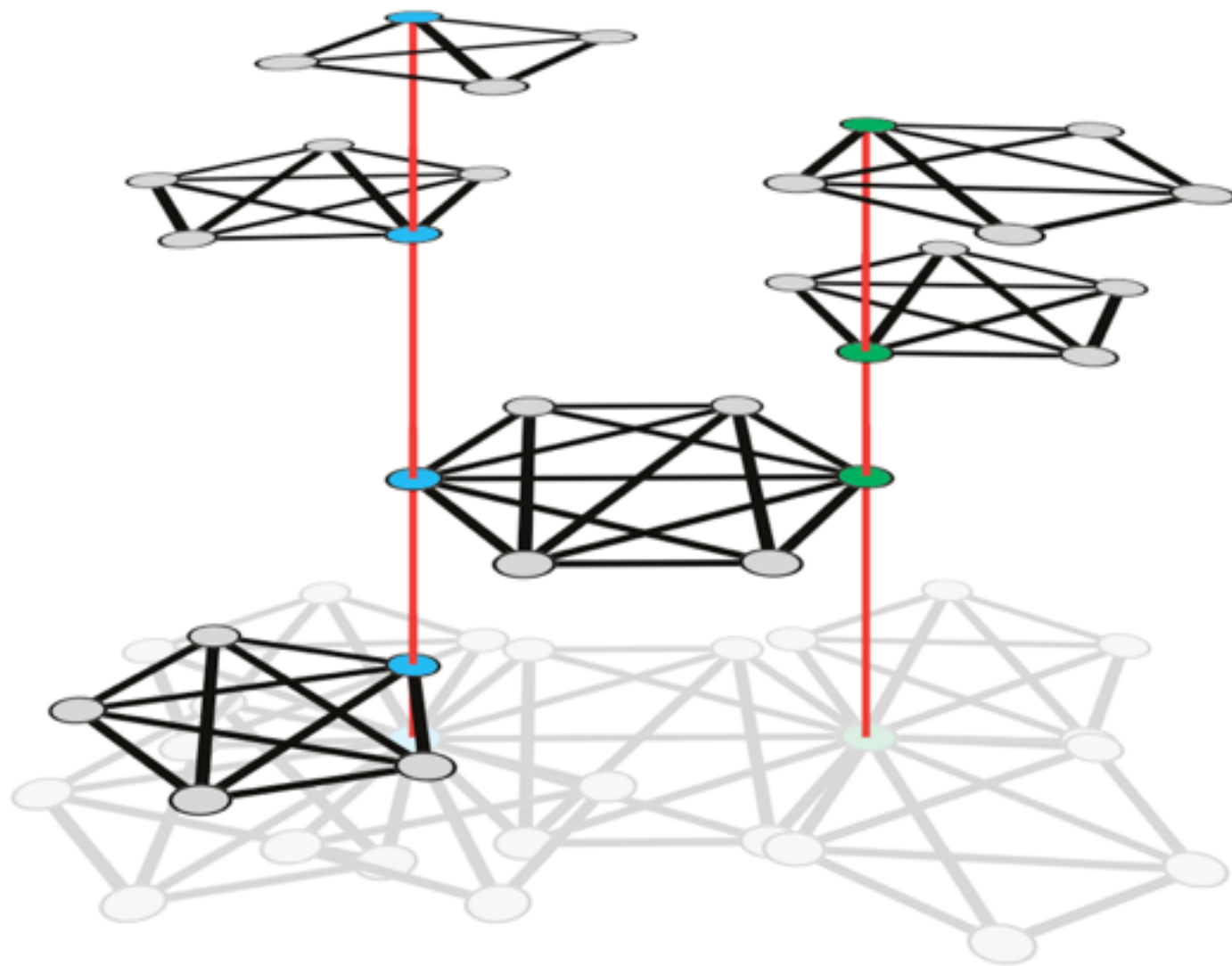
Demo

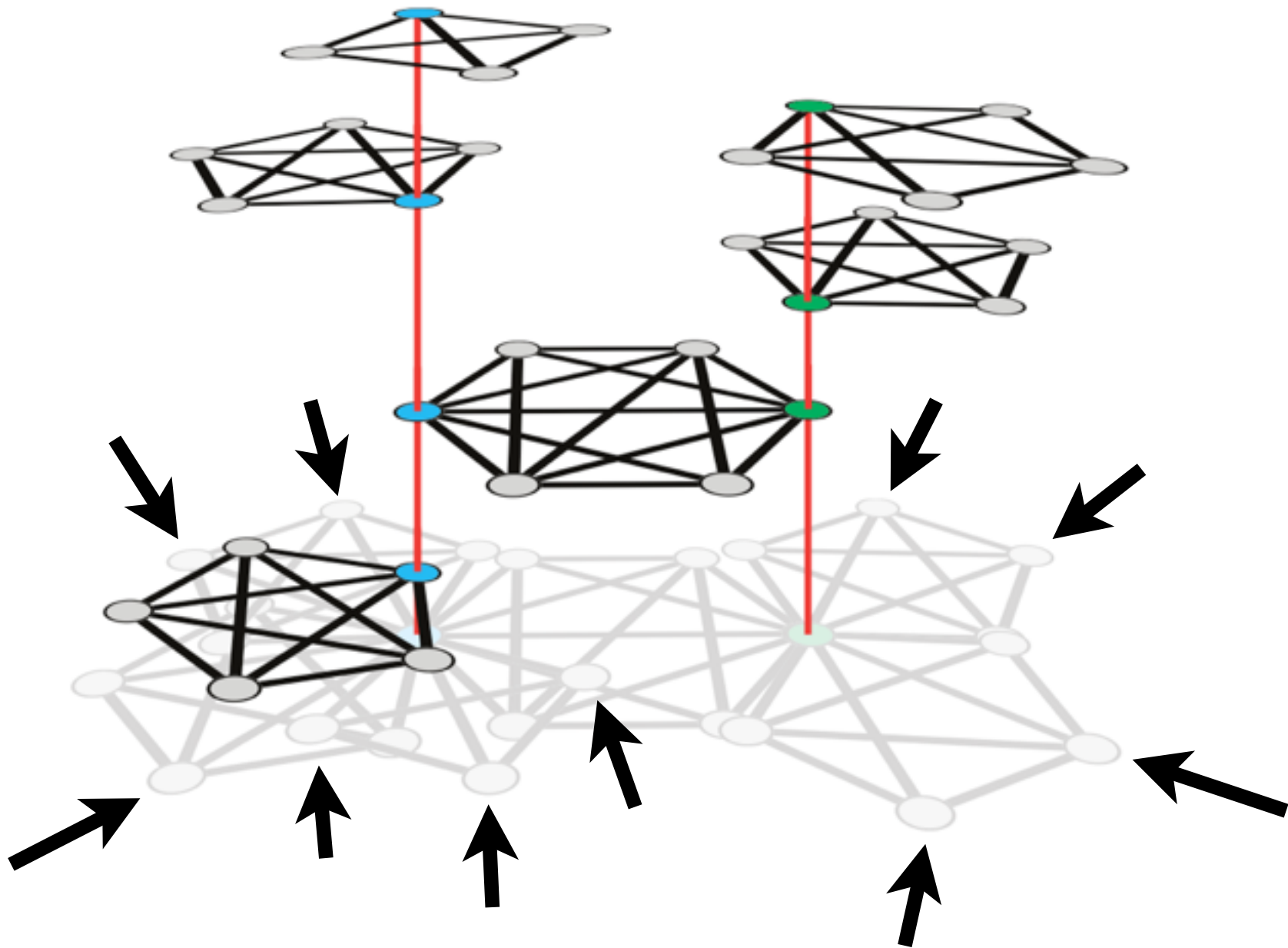
“Overlap”

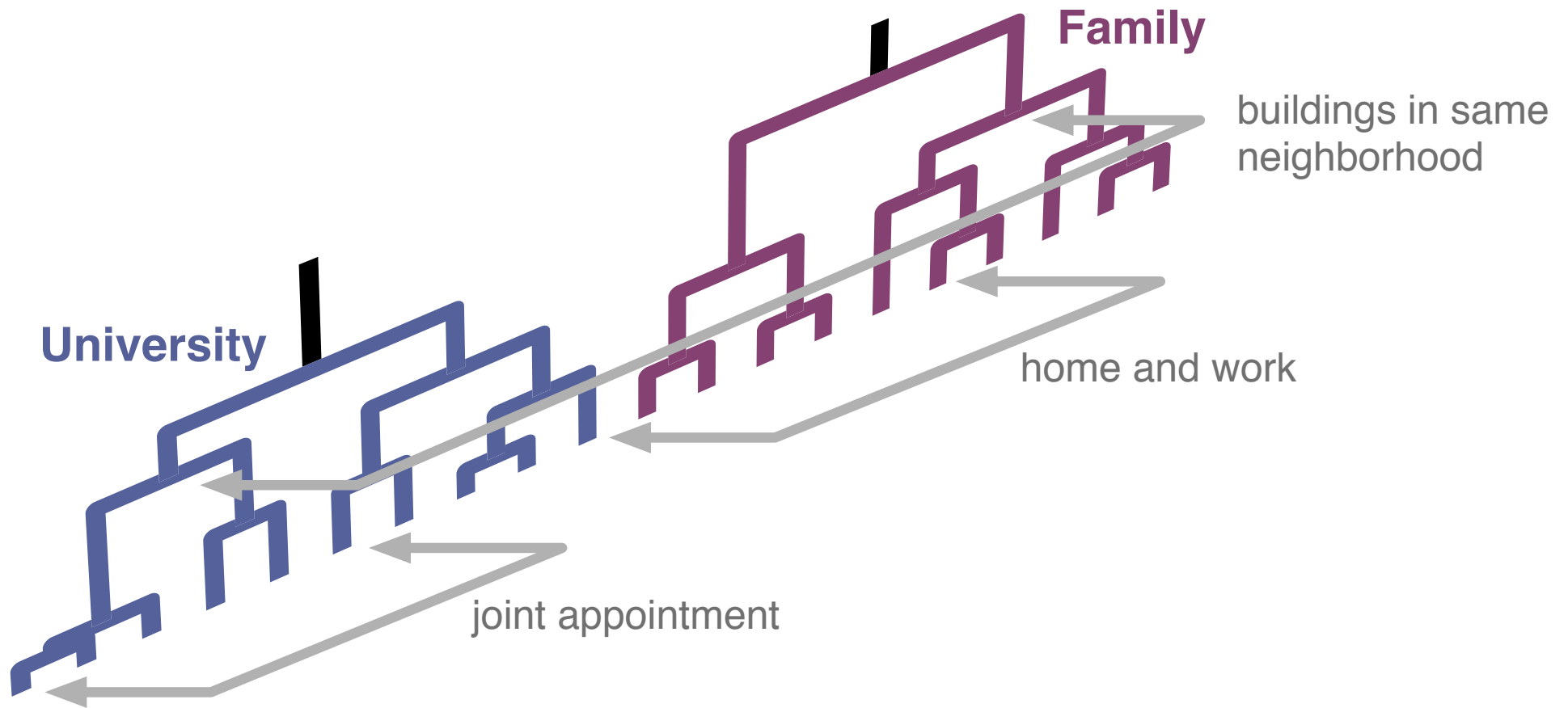


G. Palla, I. Derényi, I. Farkas & T. Vicsek, *Nature* (2005)



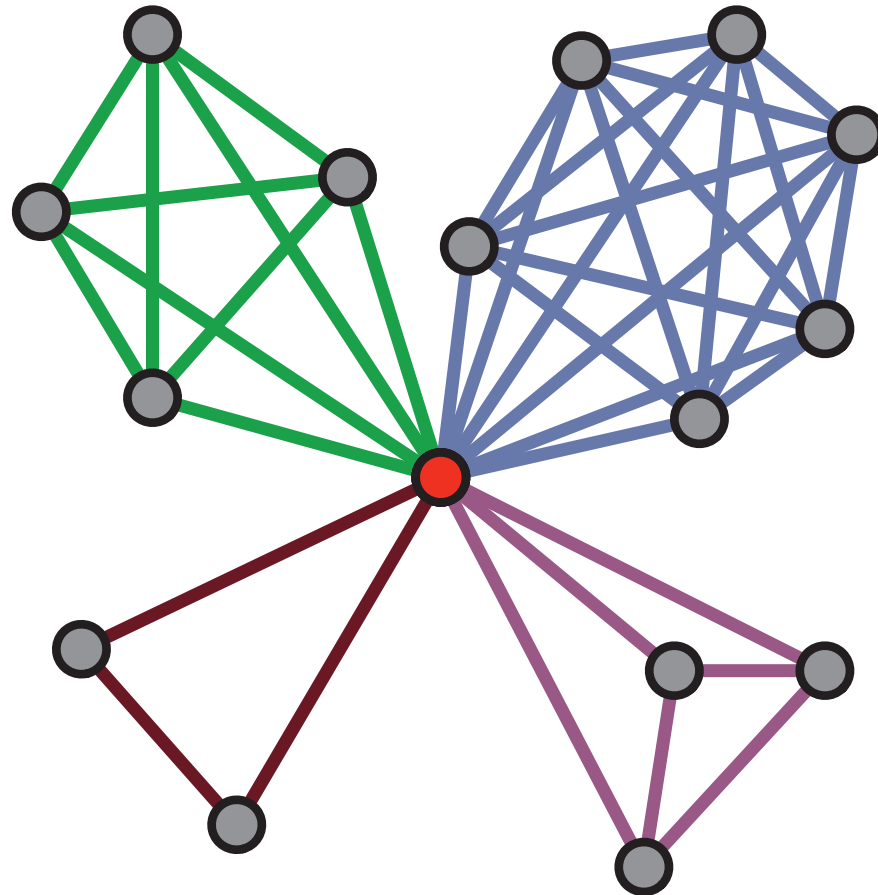




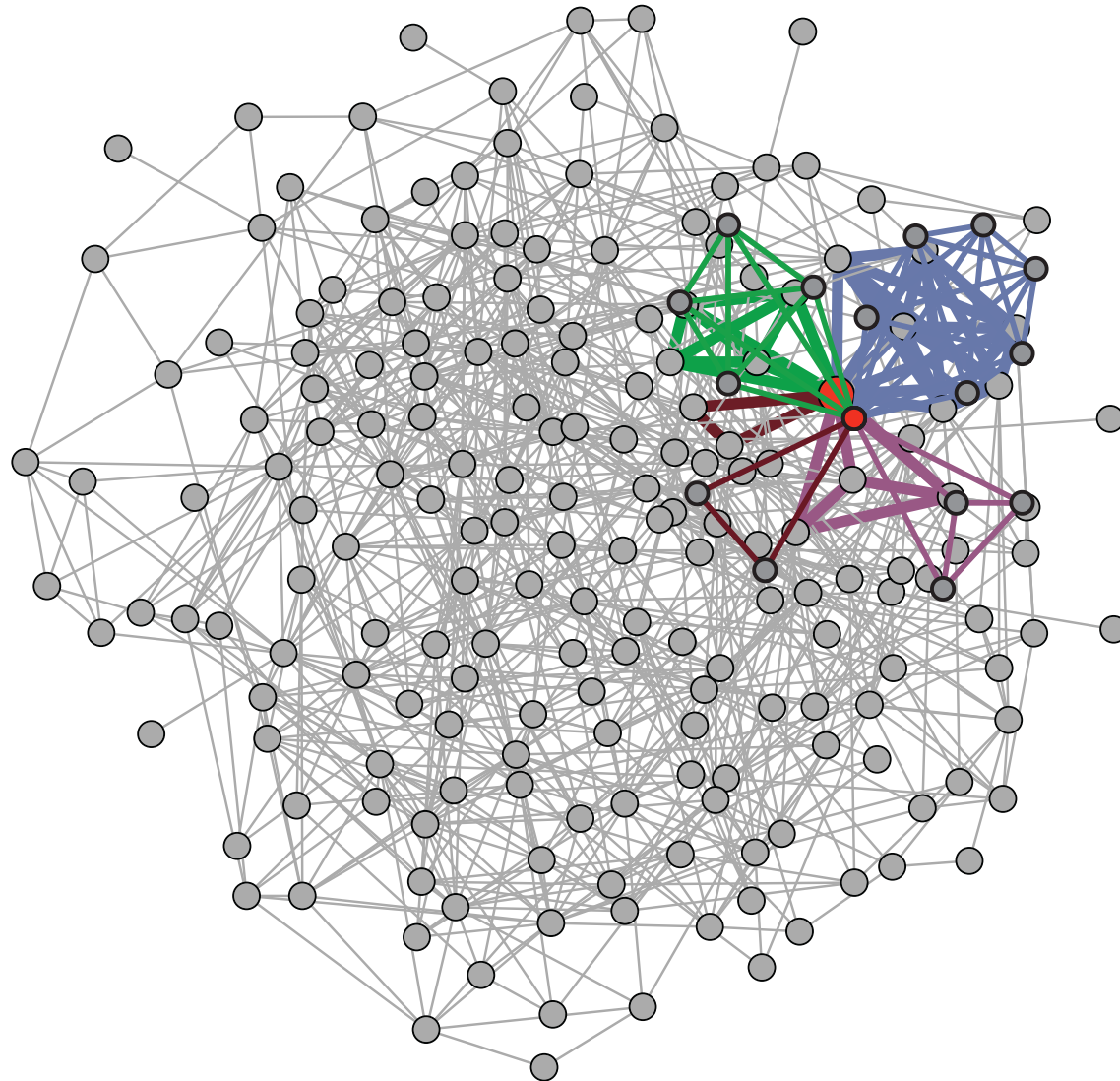


It is **impossible** to obtain a single dendrogram.

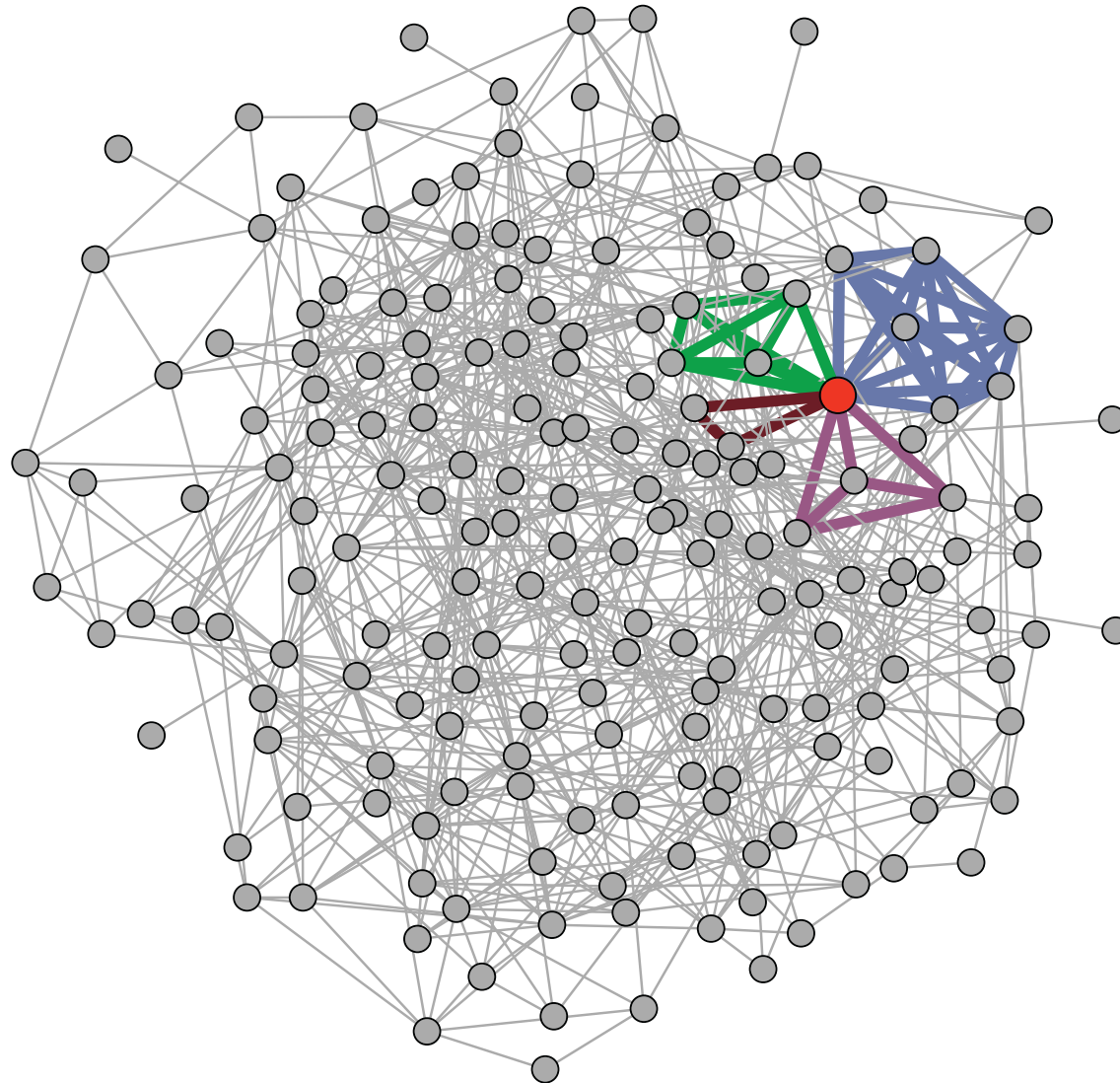
Simple local structure

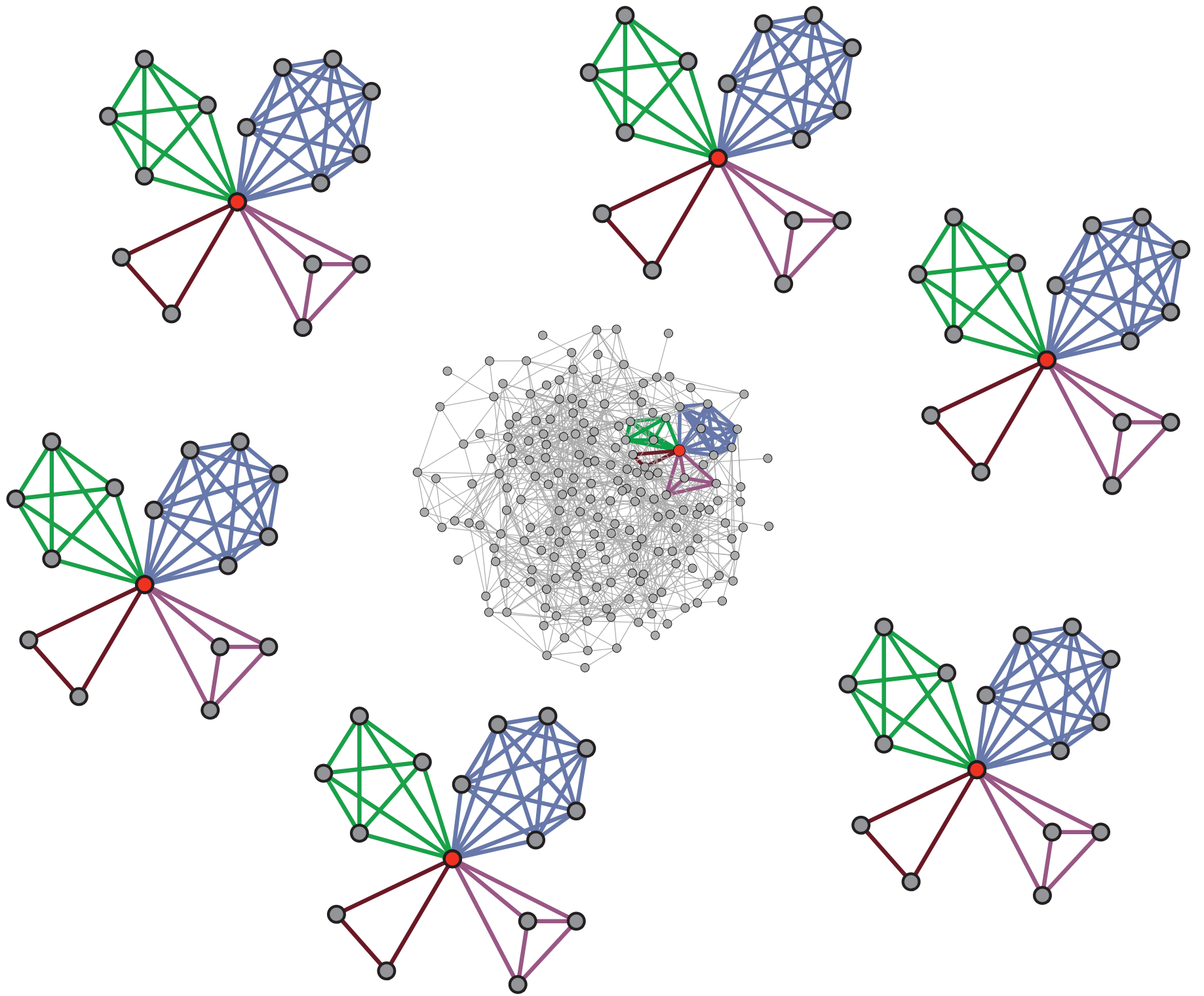


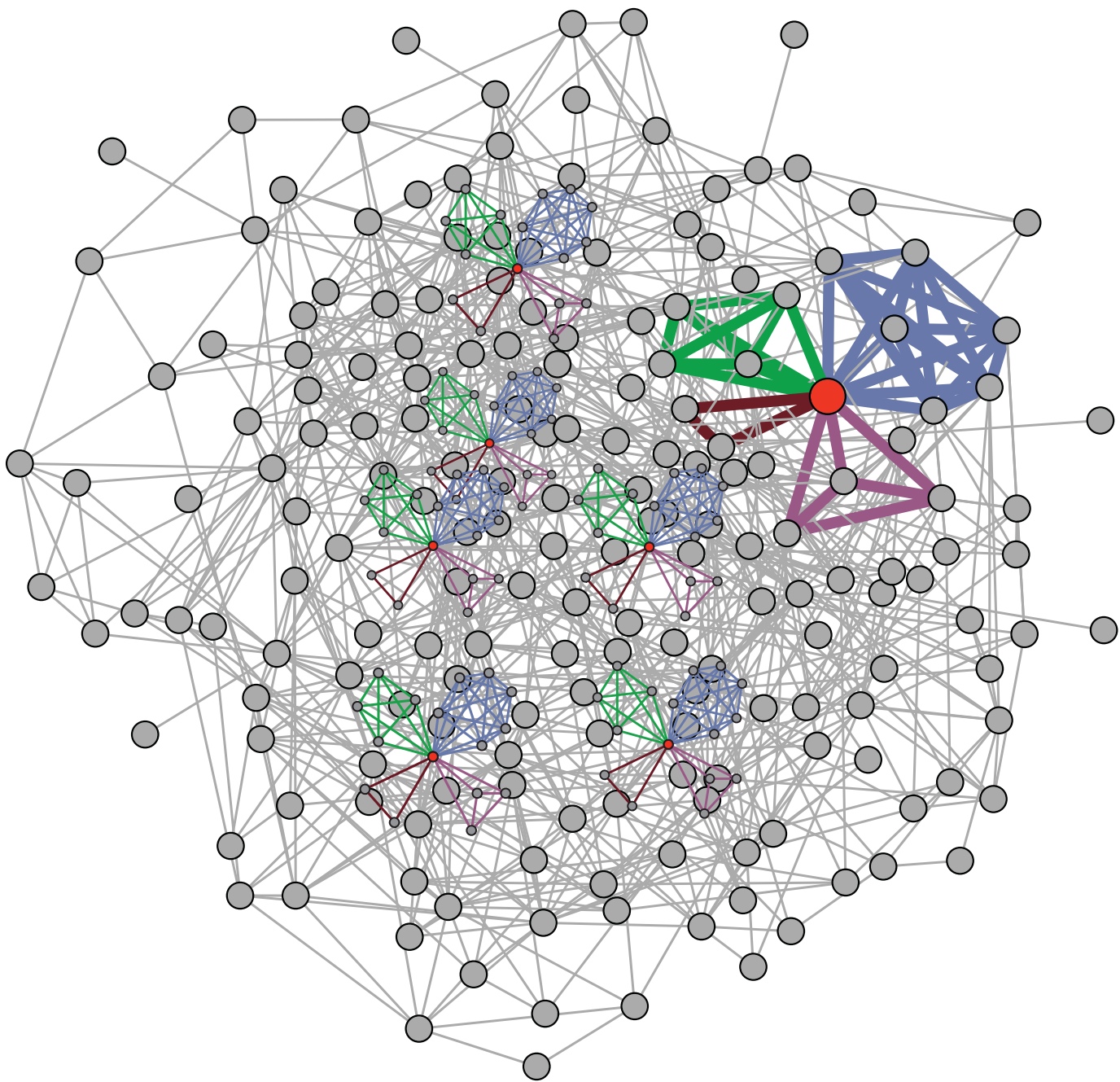
Complex global structure

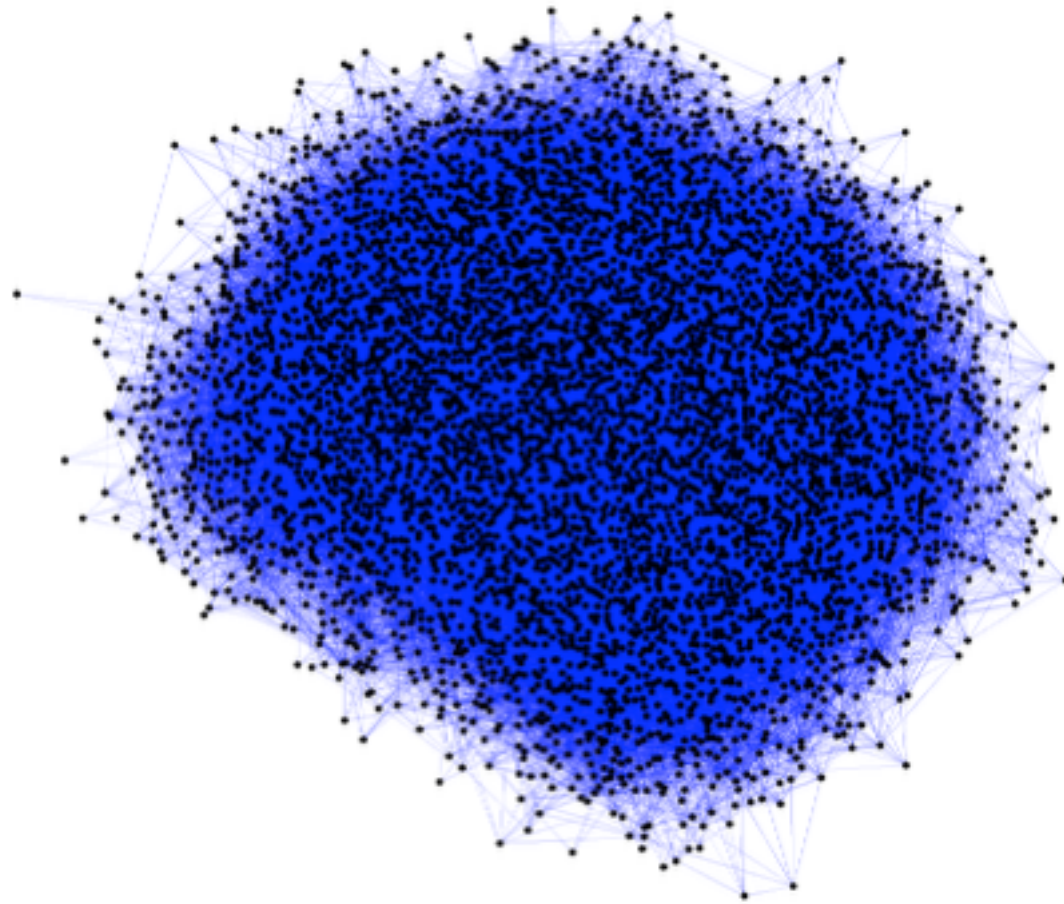


Complex global structure



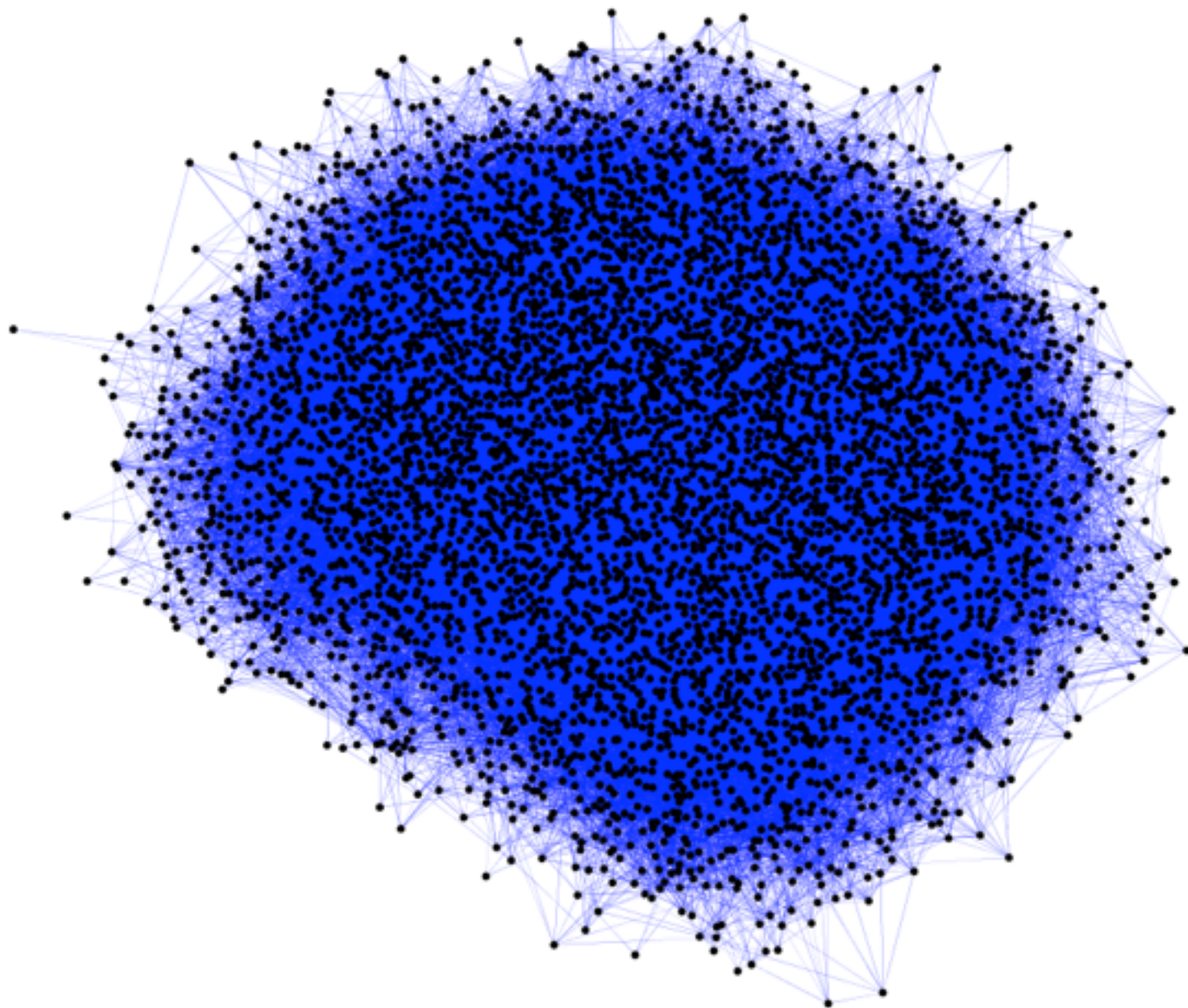




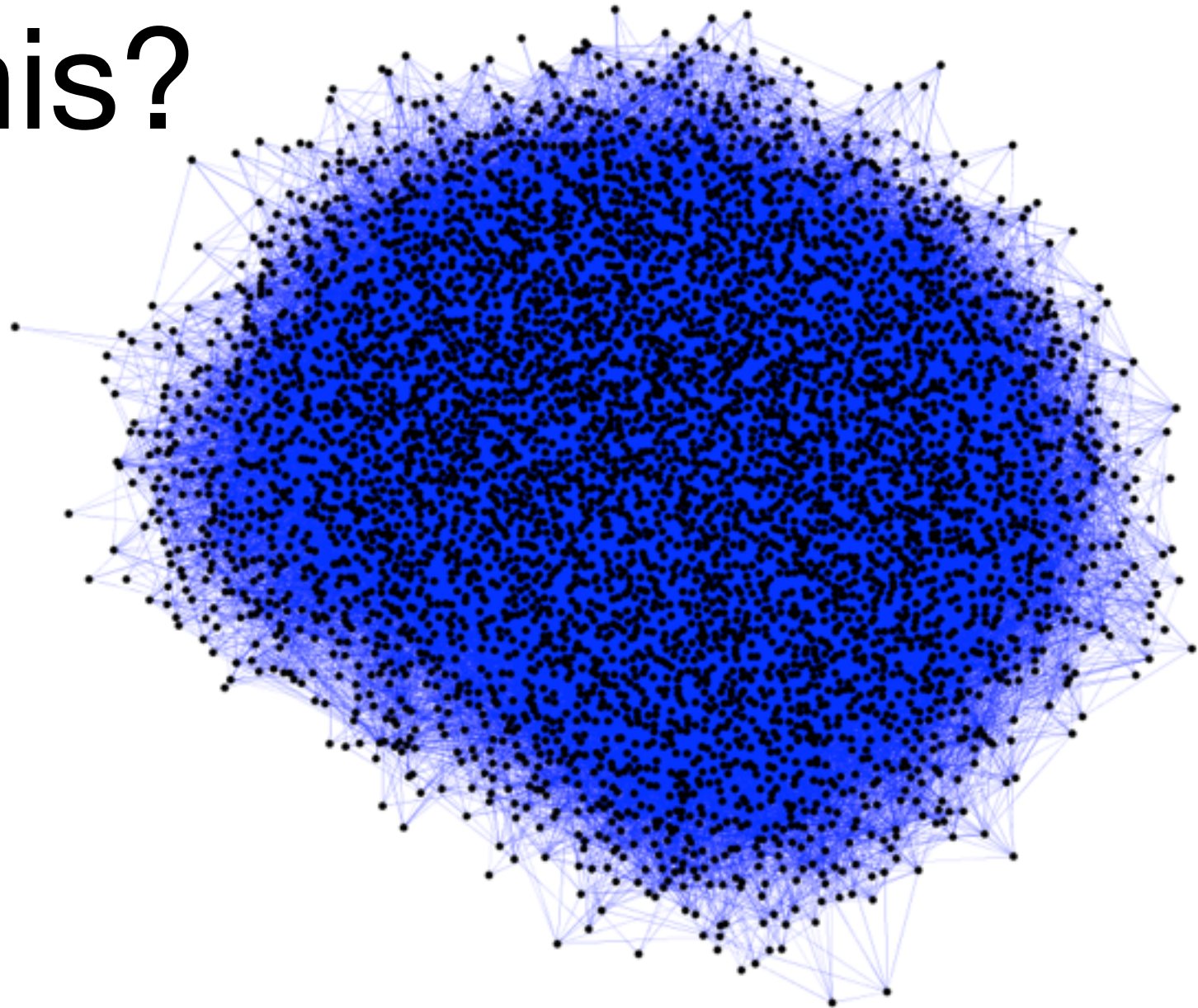


This is a modular network.

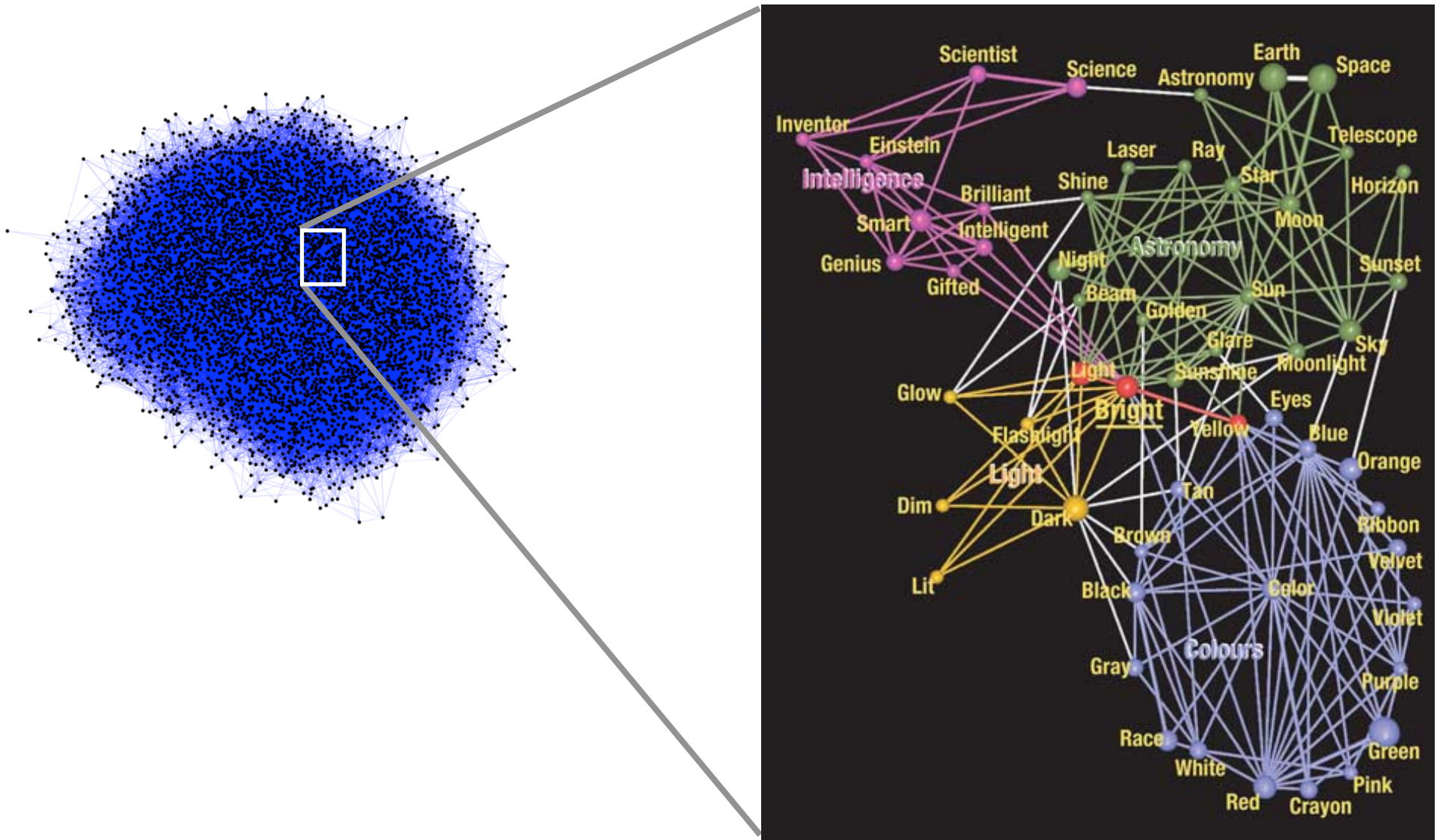
What is this?



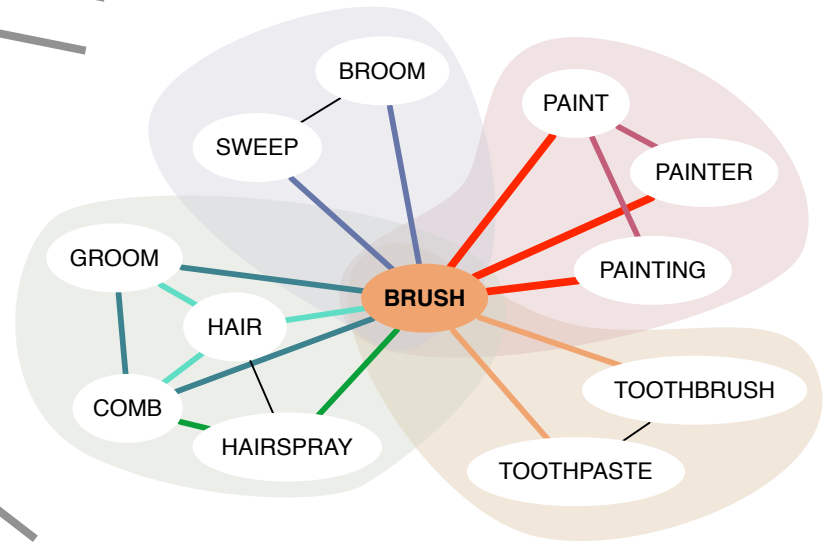
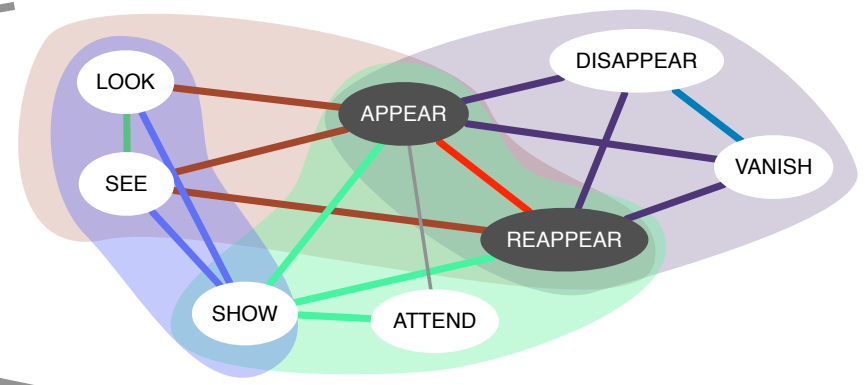
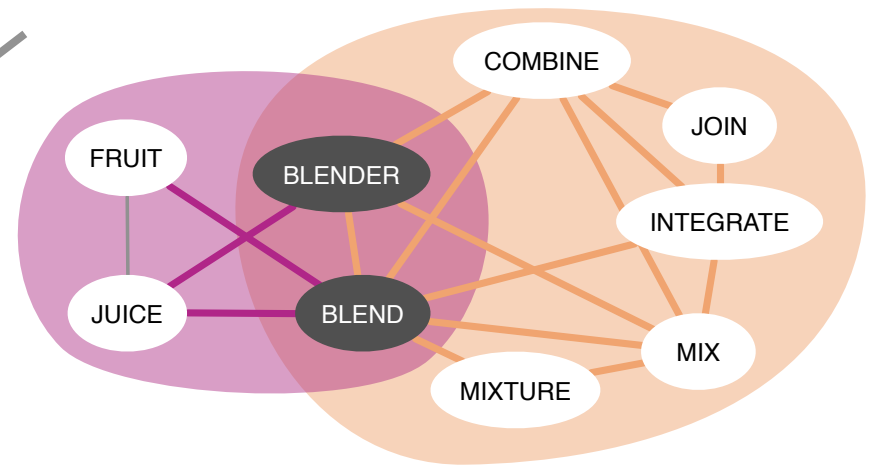
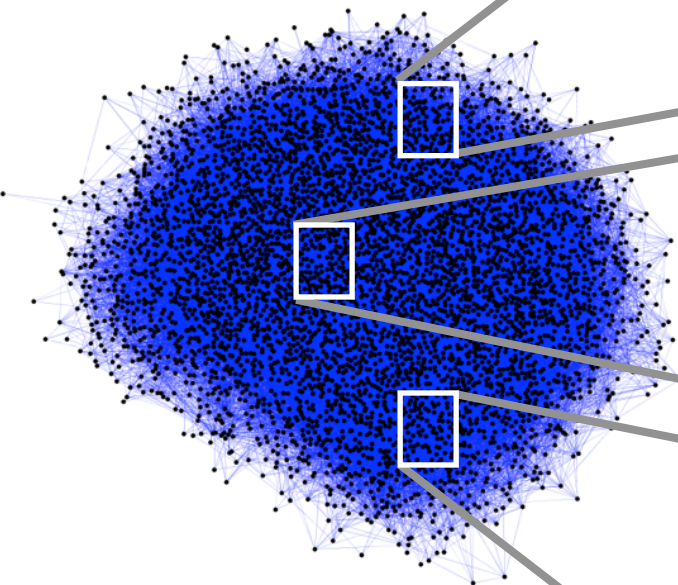
What the xxxx
is this?



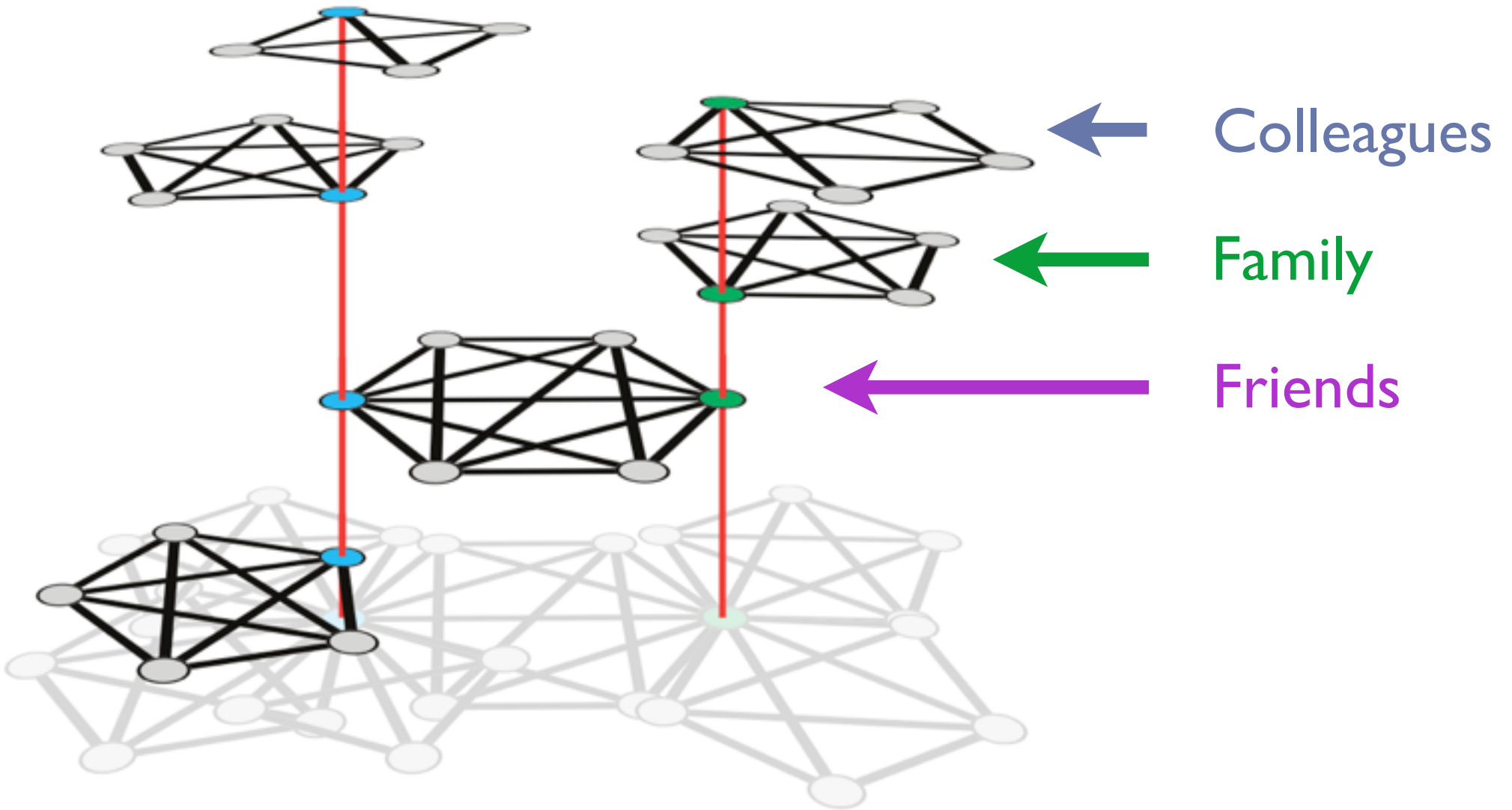
Word association network: Network of “commonly associated English words”

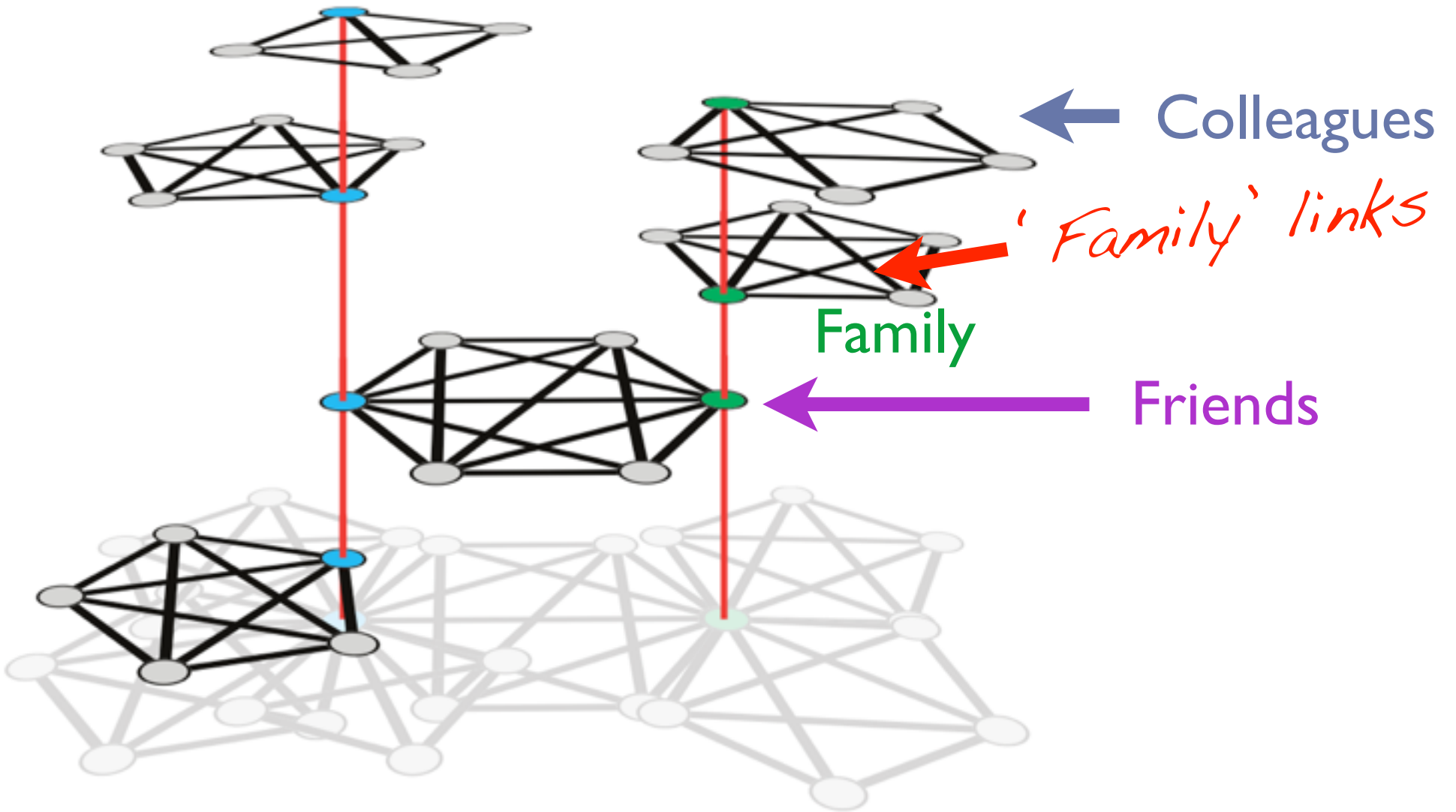


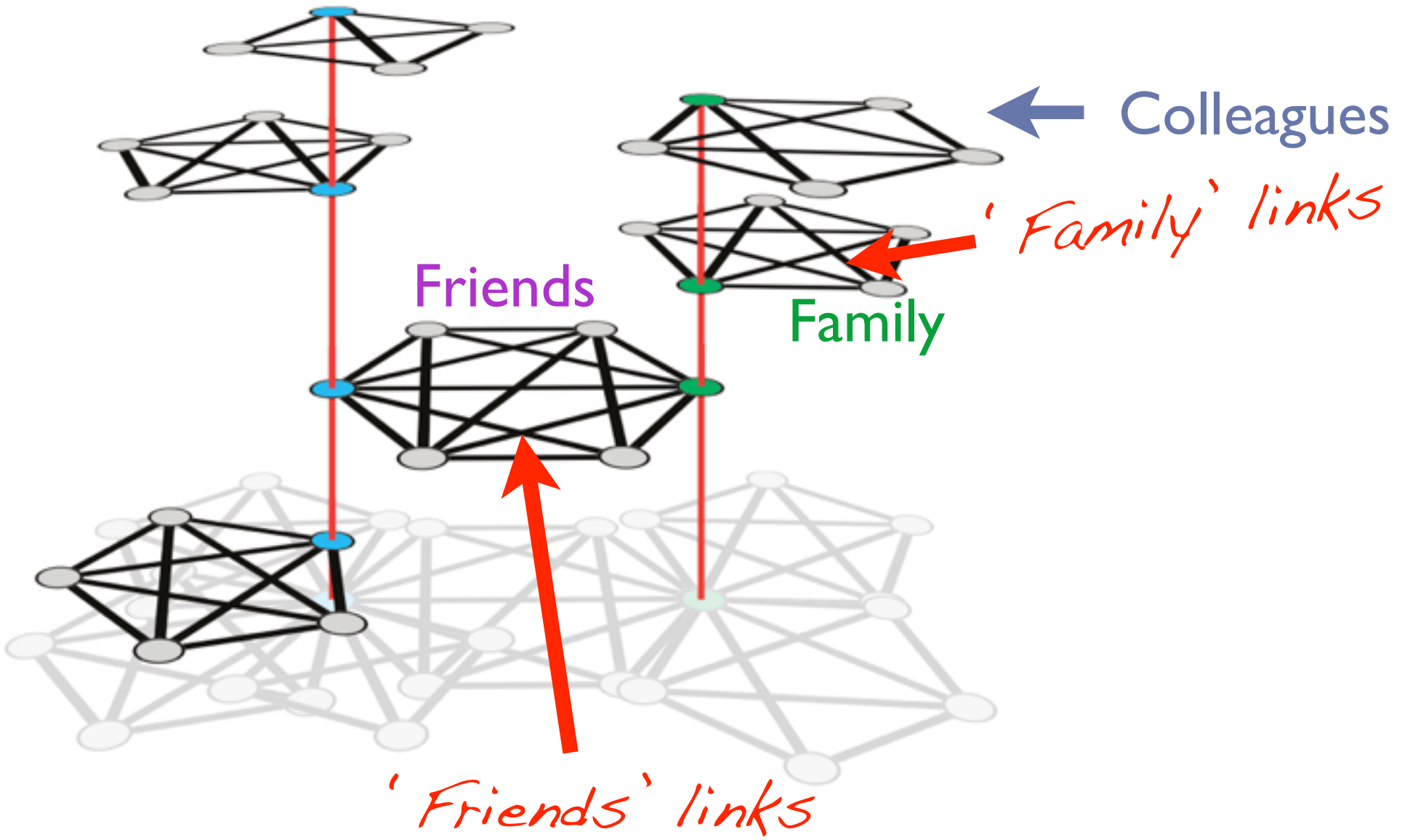
G. Palla, I. Derényi, I. Farkas & T. Vicsek, *Nature*, 2005



Link communities







'Nerds & geeks' links

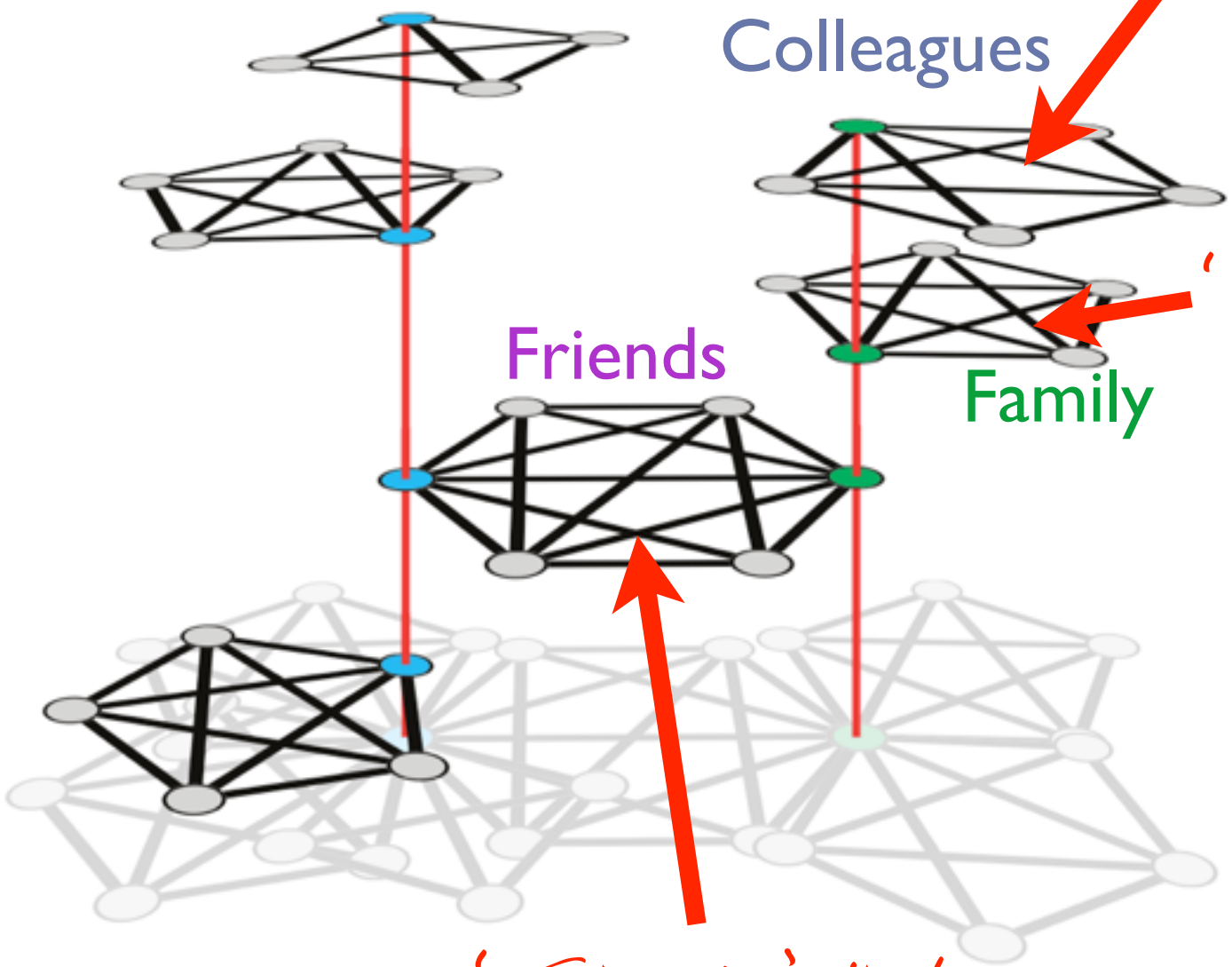
Colleagues

'Family' links

Friends

Family

'Friends' links



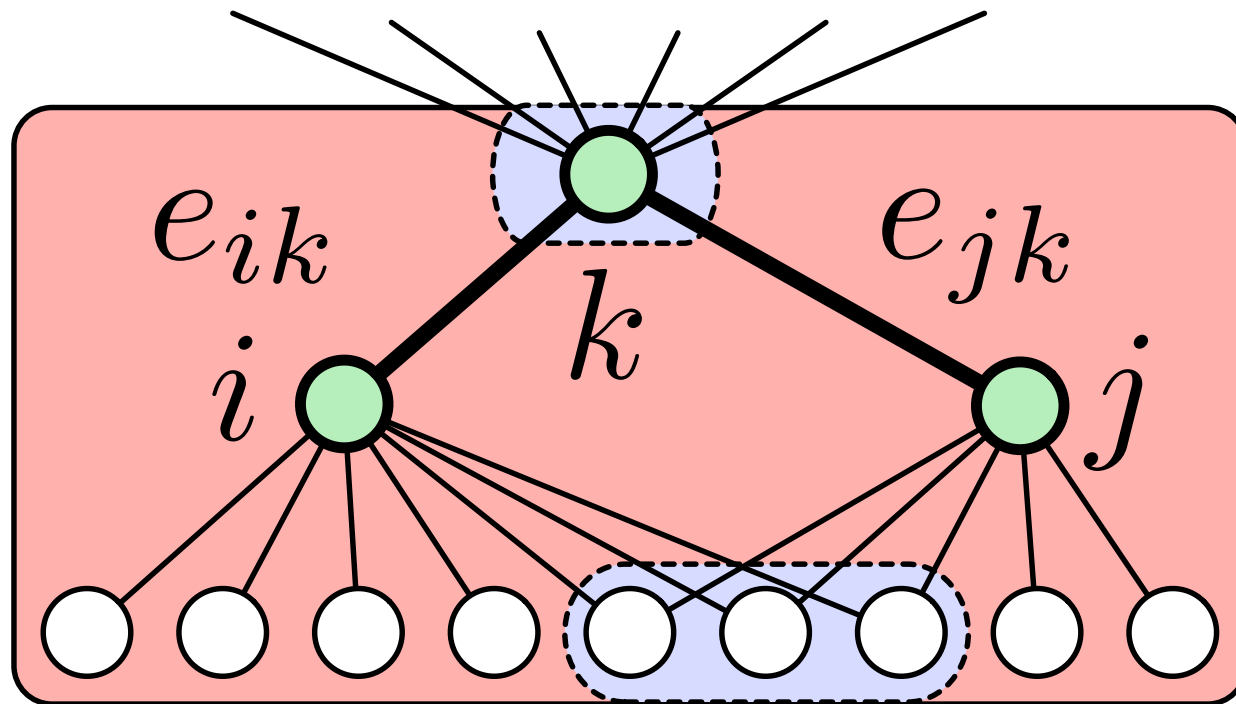
Nodes: multiple membership

Links: unique membership

Similarity between links

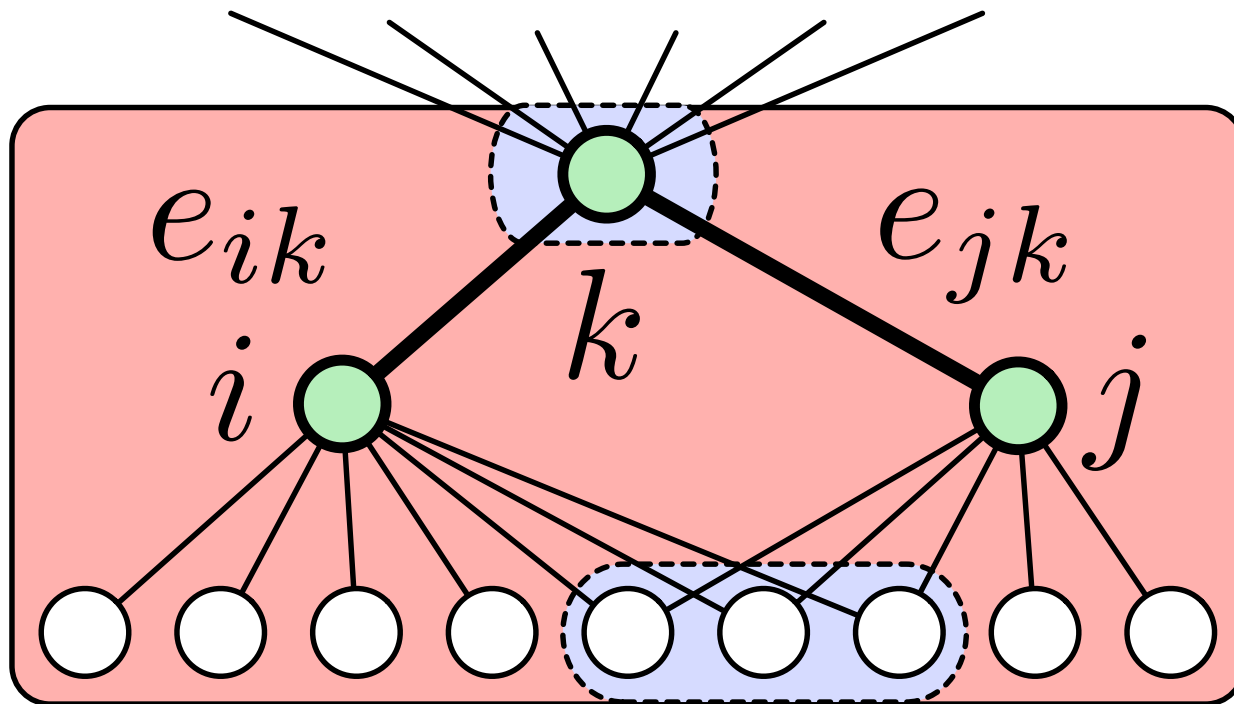


Hierarchical Clustering



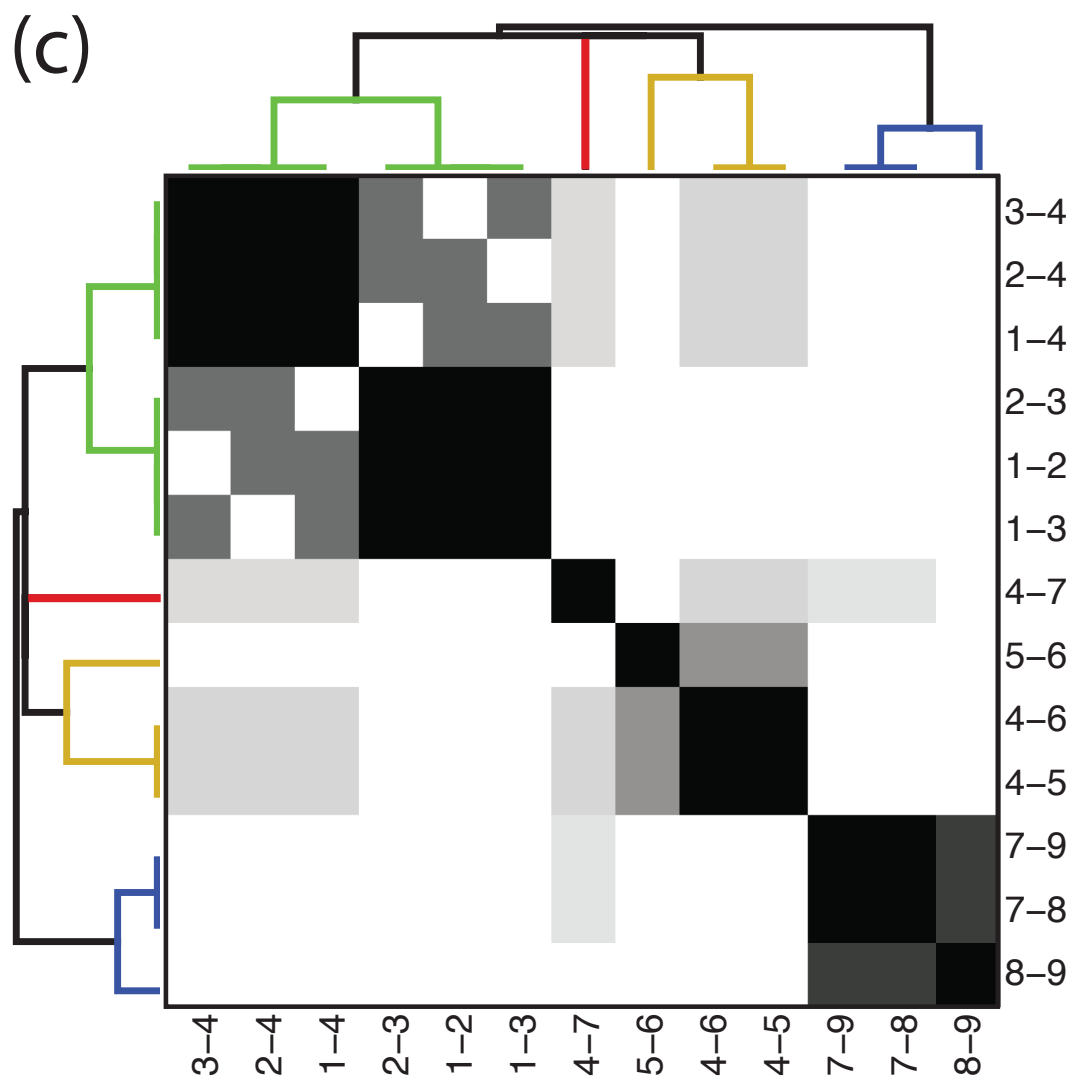
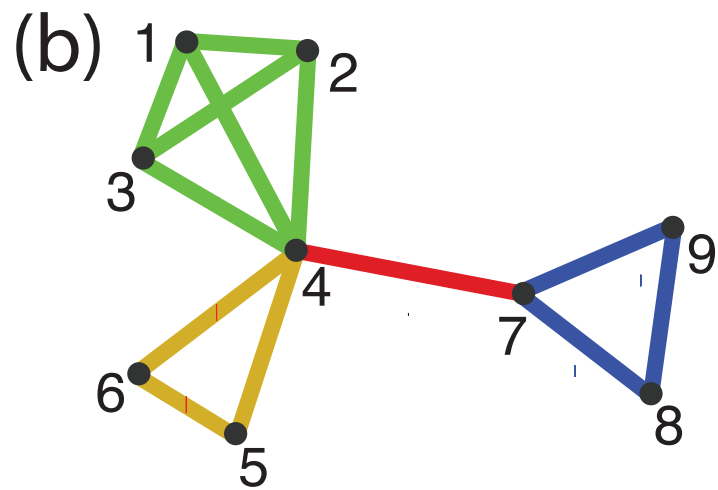
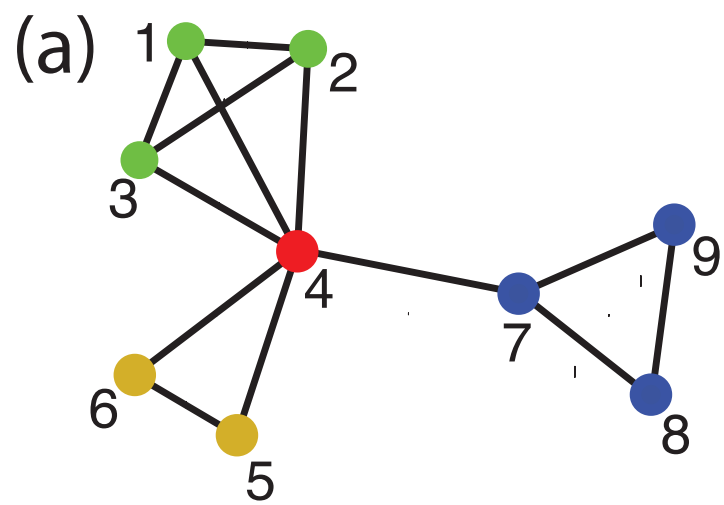
$$n_+(i) \equiv \{x \mid d(i, x) \leq 1\}$$

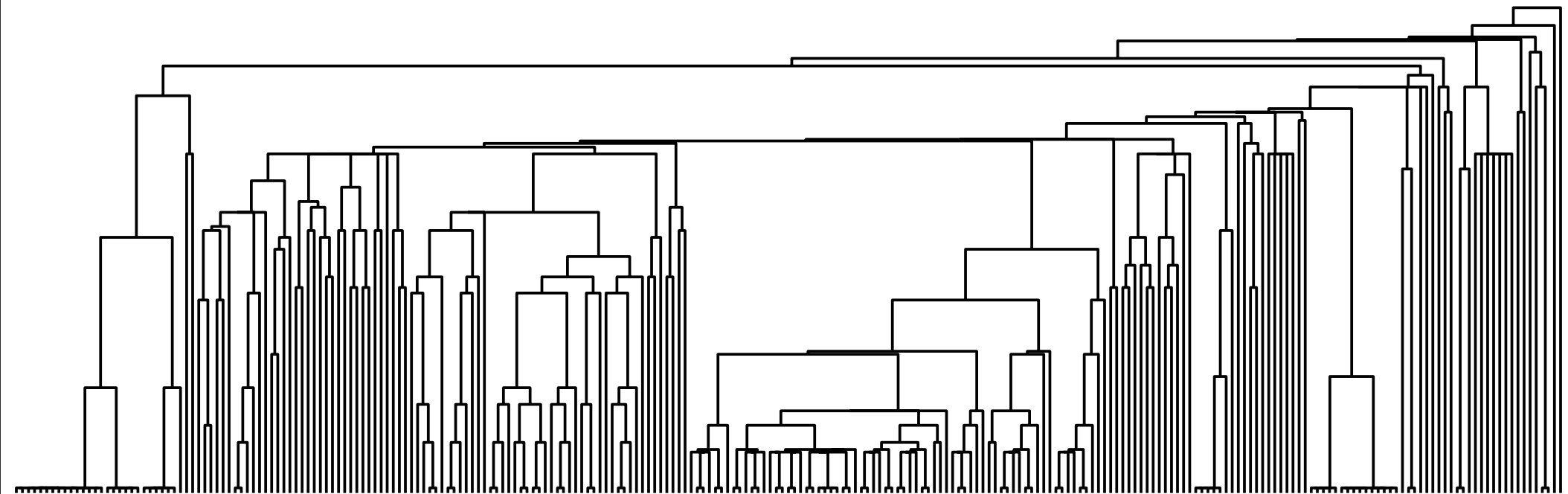
$$S(e_{ik}, e_{jk}) = \frac{|n_+(i) \cap n_+(j)|}{|n_+(i) \cup n_+(j)|}$$



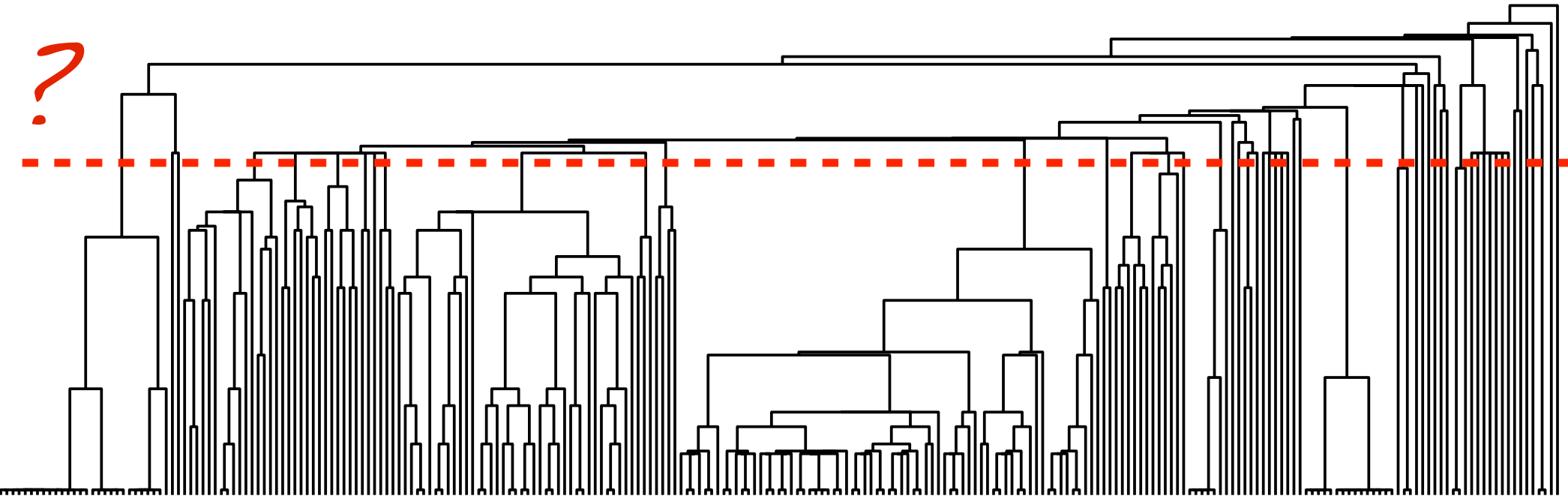
$$n_+(i) \equiv \{x \mid d(i, x) \leq 1\}$$

$$S(e_{ik}, e_{jk}) = \frac{|n_+(i) \cap n_+(j)|}{|n_+(i) \cup n_+(j)|} = \frac{4}{12}$$





?

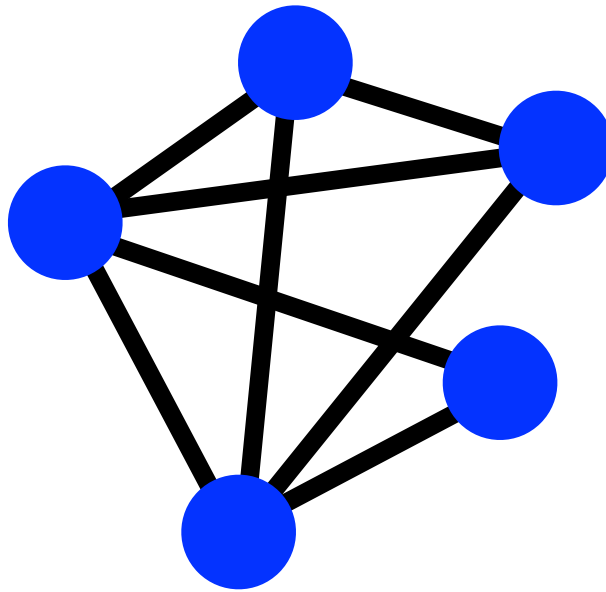


Partition Density

Community c has m_c edges and n_c induced nodes

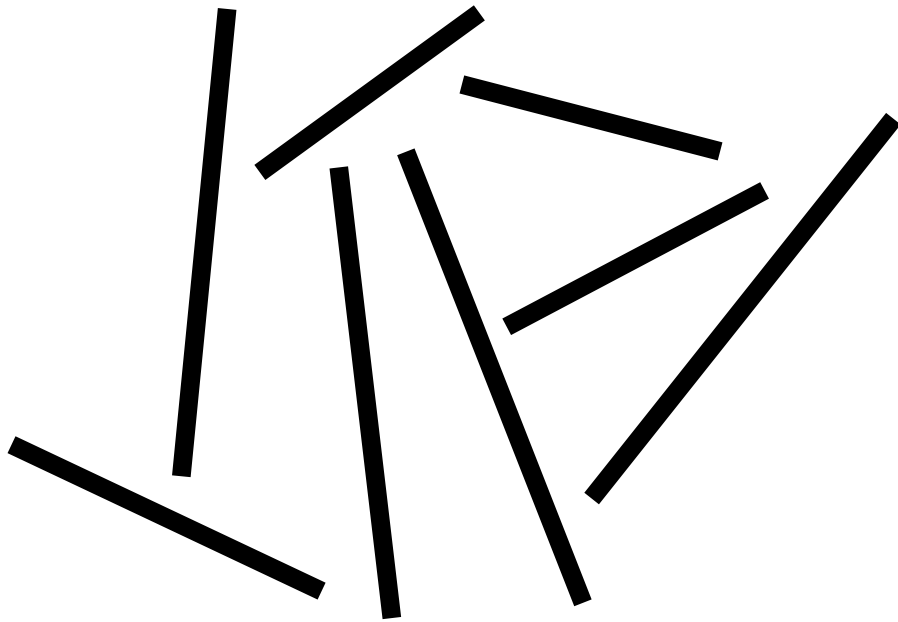
Partition Density

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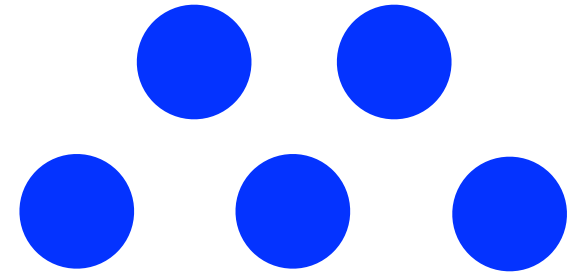


Partition Density

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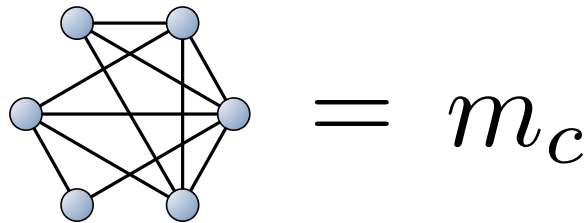
$$m_c = 8$$



$$n_c = 5$$

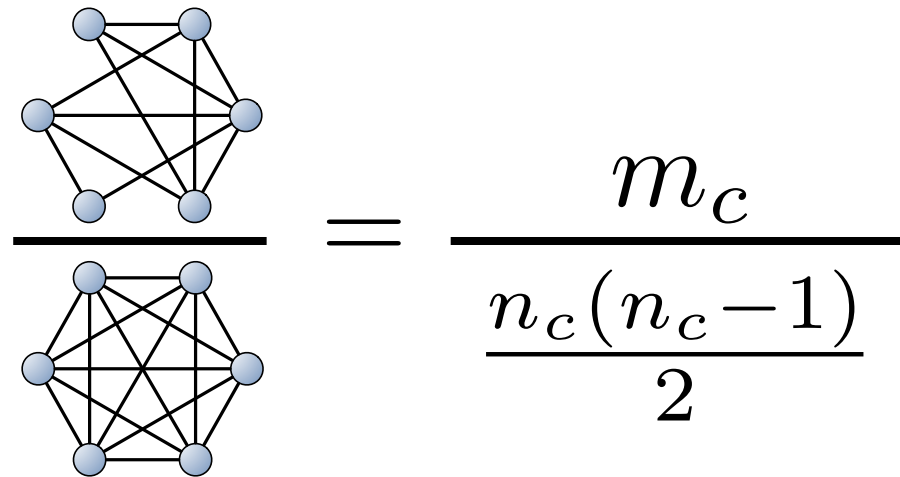
Partition Density

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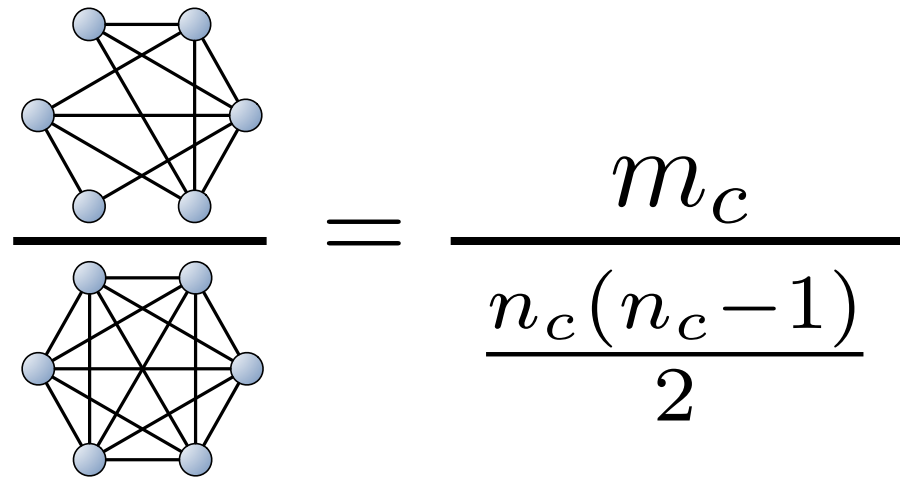
Partition Density

Community c has m_c edges and n_c induced nodes


$$\frac{\text{Graph with } m_c \text{ edges and } n_c \text{ nodes}}{\text{Complete graph } K_{n_c}} = \frac{m_c}{\frac{n_c(n_c-1)}{2}}$$

Partition Density

Community c has m_c edges and n_c induced nodes

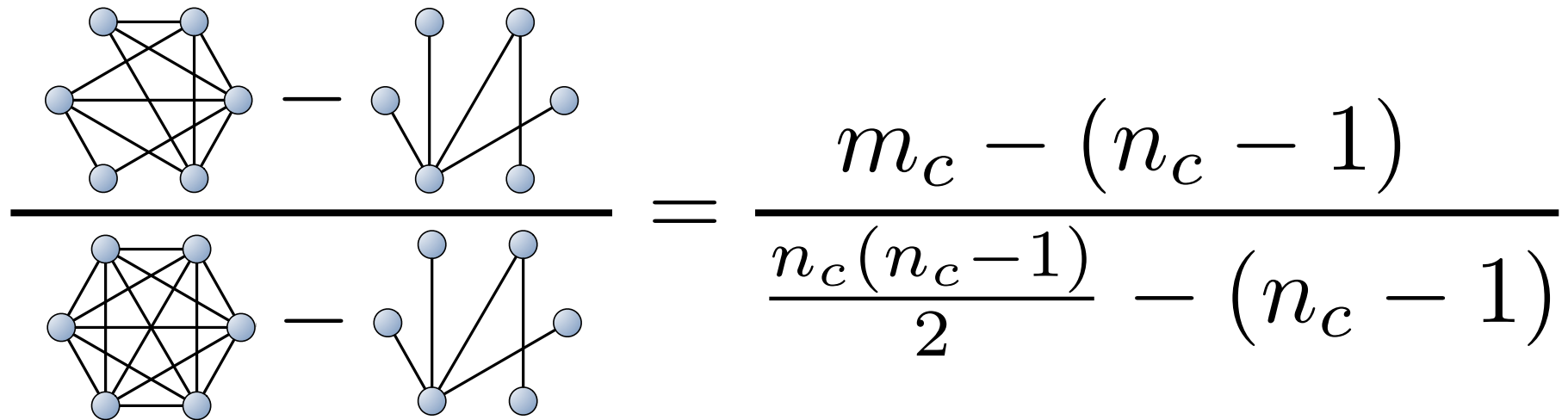

$$\frac{\text{Graph with } m_c \text{ edges}}{\text{Graph with } \frac{n_c(n_c-1)}{2} \text{ edges}} = \frac{m_c}{\frac{n_c(n_c-1)}{2}}$$



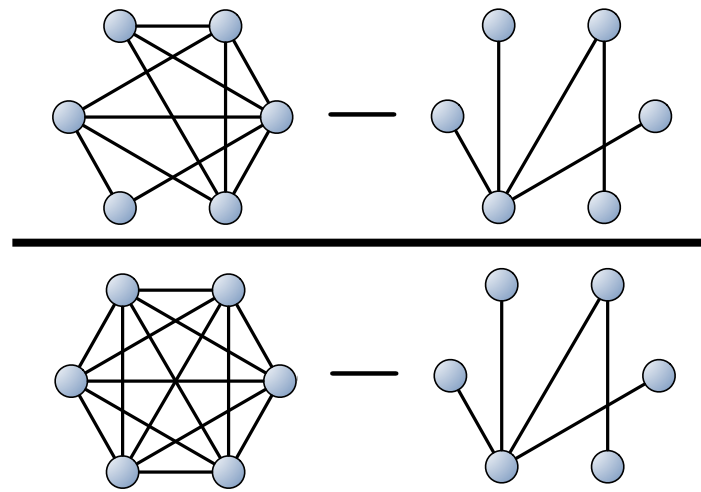
A **single** link is **maximally dense**

Partition Density

Community c has m_c edges and n_c induced nodes


$$\frac{\text{Complete Graph } K_5 - \text{Star Graph}}{\text{Complete Graph } K_5 - \text{Star Graph}} = \frac{m_c - (n_c - 1)}{\frac{n_c(n_c - 1)}{2} - (n_c - 1)}$$

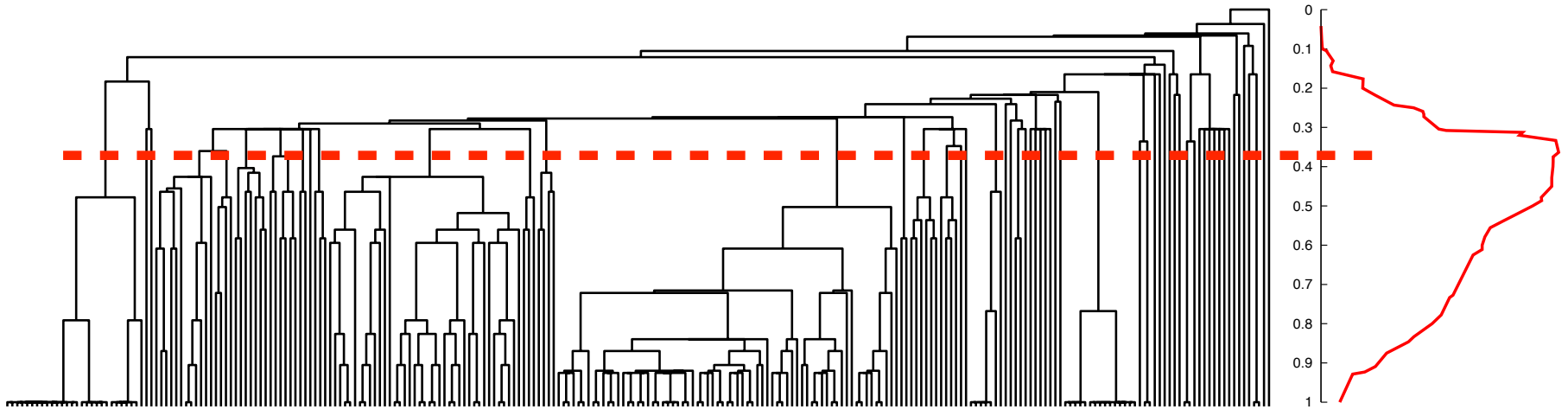
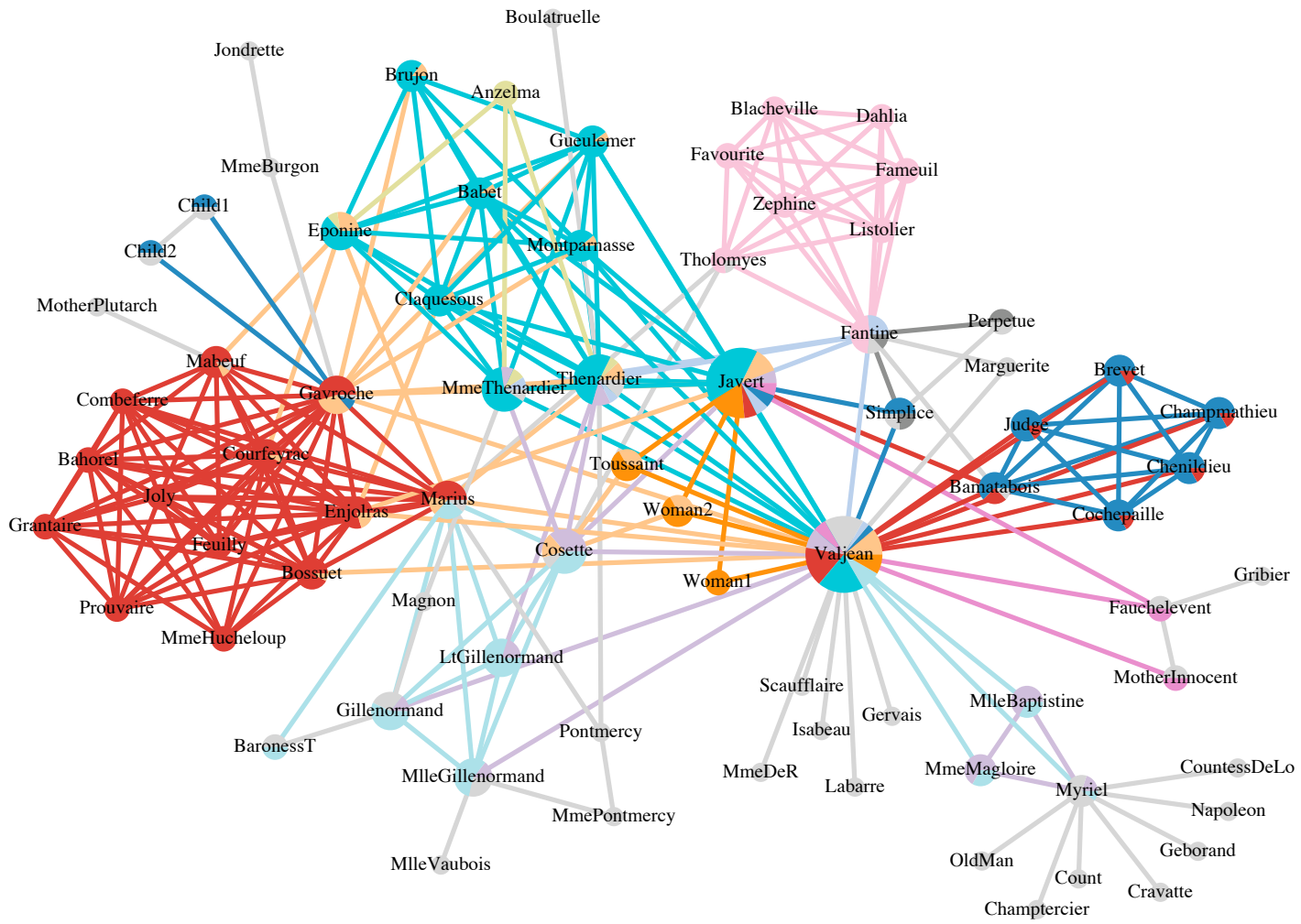
Partition Density

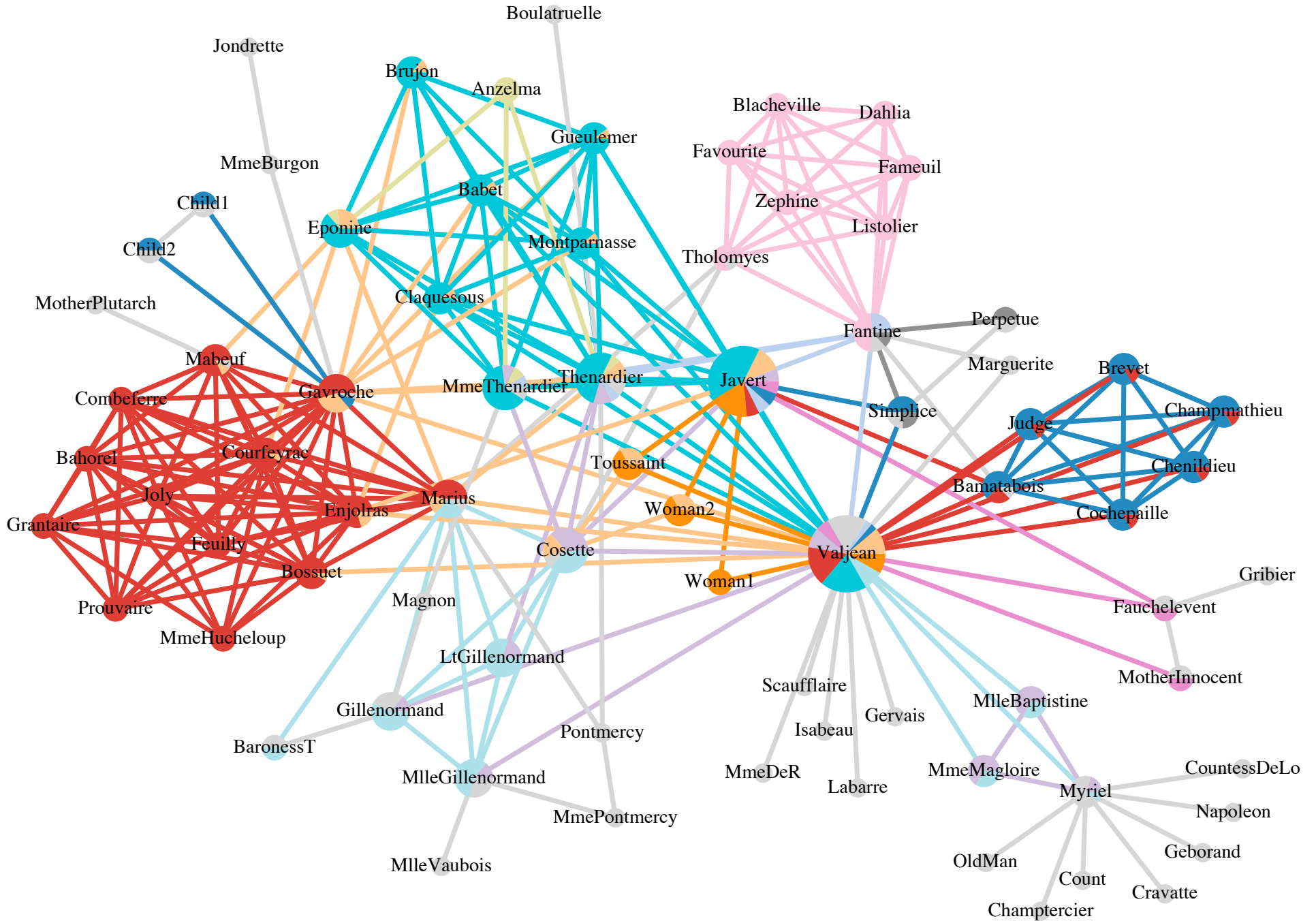

$$\frac{m_c - (n_c - 1)}{\frac{n_c(n_c - 1)}{2} - (n_c - 1)} = 2 \frac{m_c - (n_c - 1)}{(n_c - 2)(n_c - 1)}$$

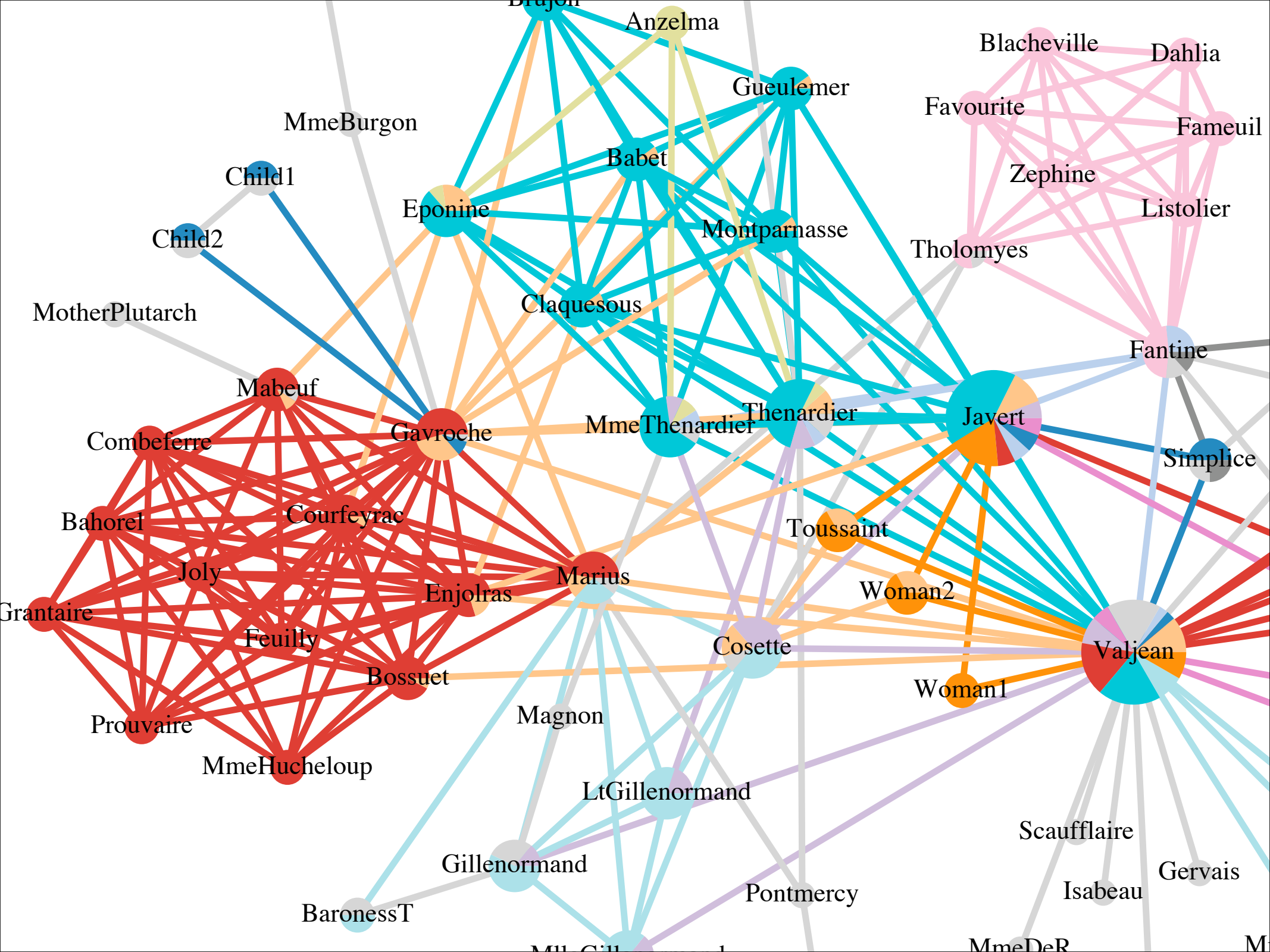
Partition Density

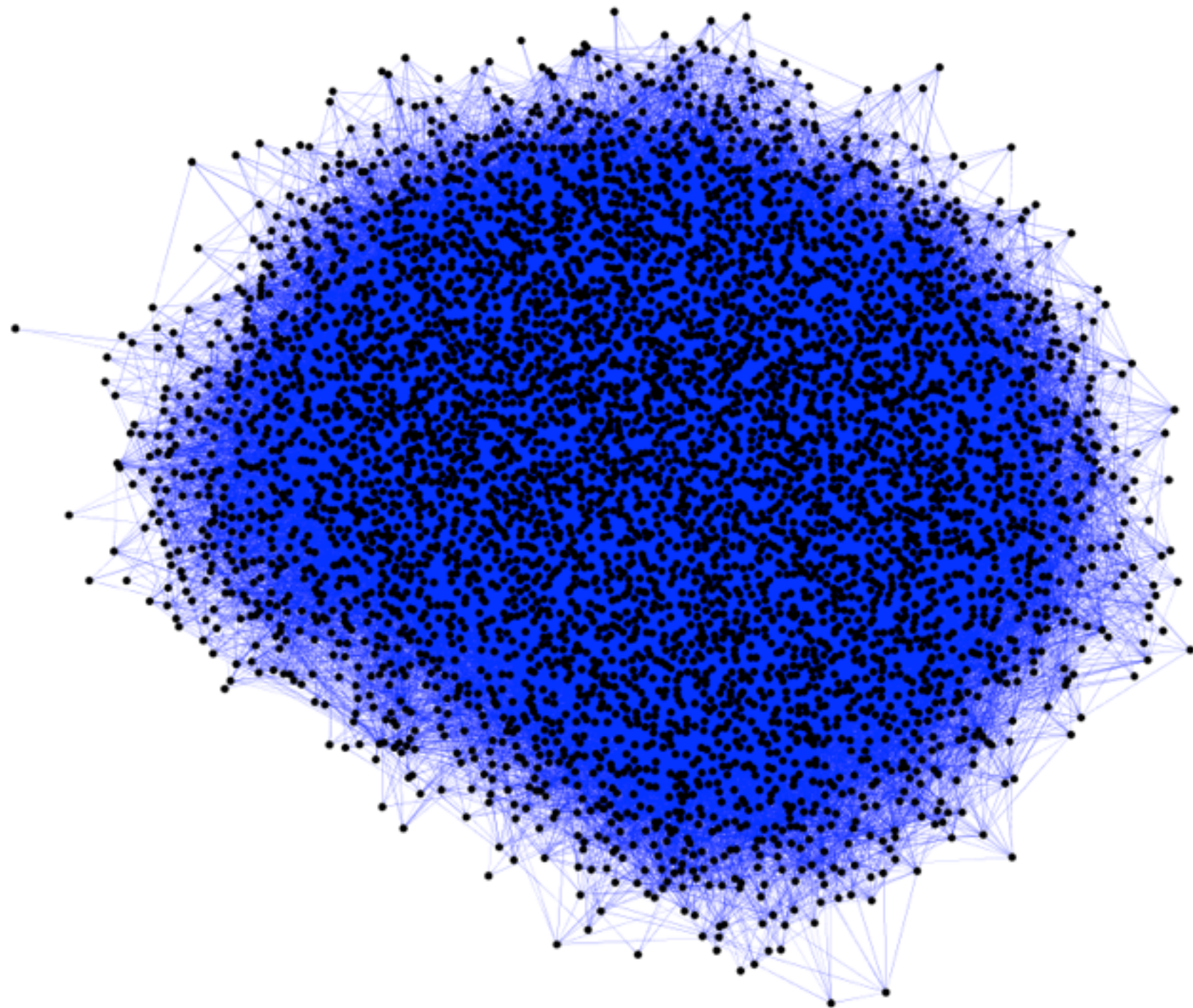
$$\begin{aligned}
 & \frac{\text{[Diagram: 6 nodes, 12 edges] - [Diagram: 6 nodes, 5 edges]}}{\text{[Diagram: 6 nodes, 15 edges] - [Diagram: 6 nodes, 5 edges]}} = \frac{m_c - (n_c - 1)}{\frac{n_c(n_c - 1)}{2} - (n_c - 1)} \\
 & = 2 \frac{m_c - (n_c - 1)}{(n_c - 2)(n_c - 1)}
 \end{aligned}$$

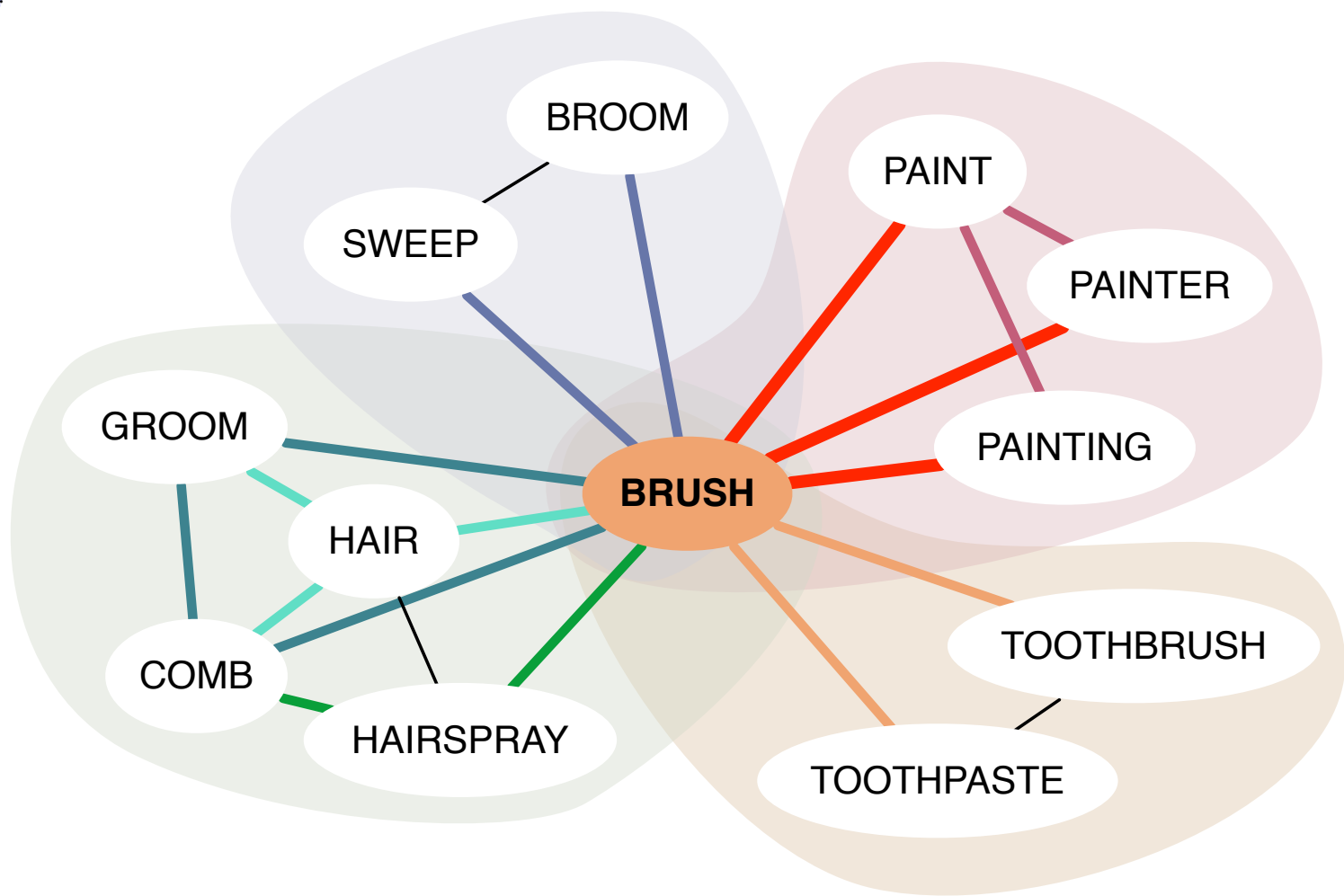
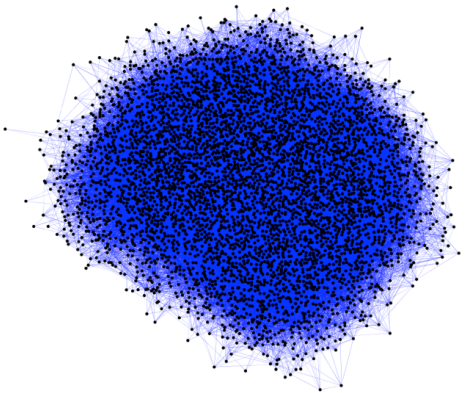
$$D \equiv \frac{2}{M} \sum_c m_c \frac{m_c - (n_c - 1)}{(n_c - 2)(n_c - 1)}$$

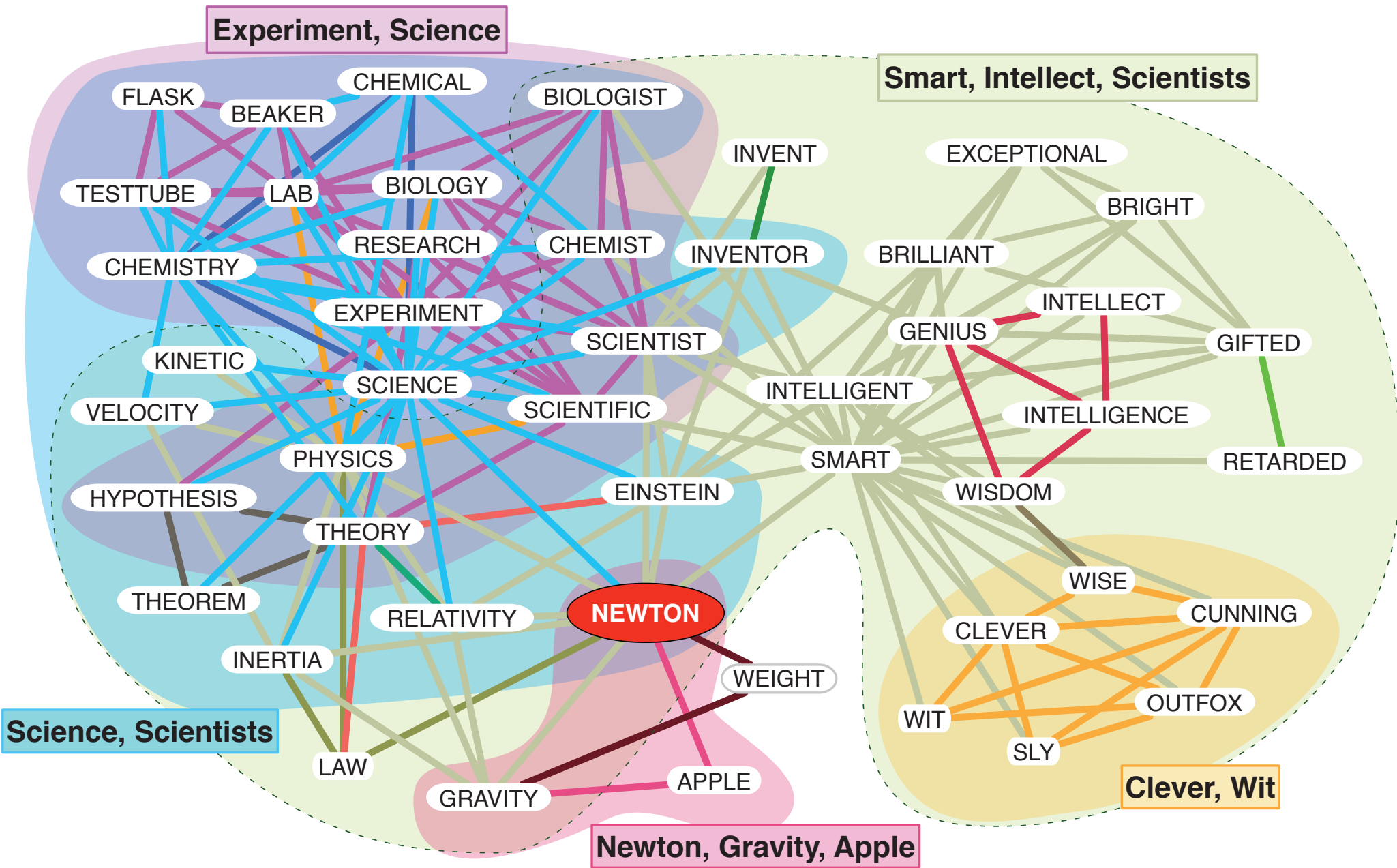












Experiment, Science

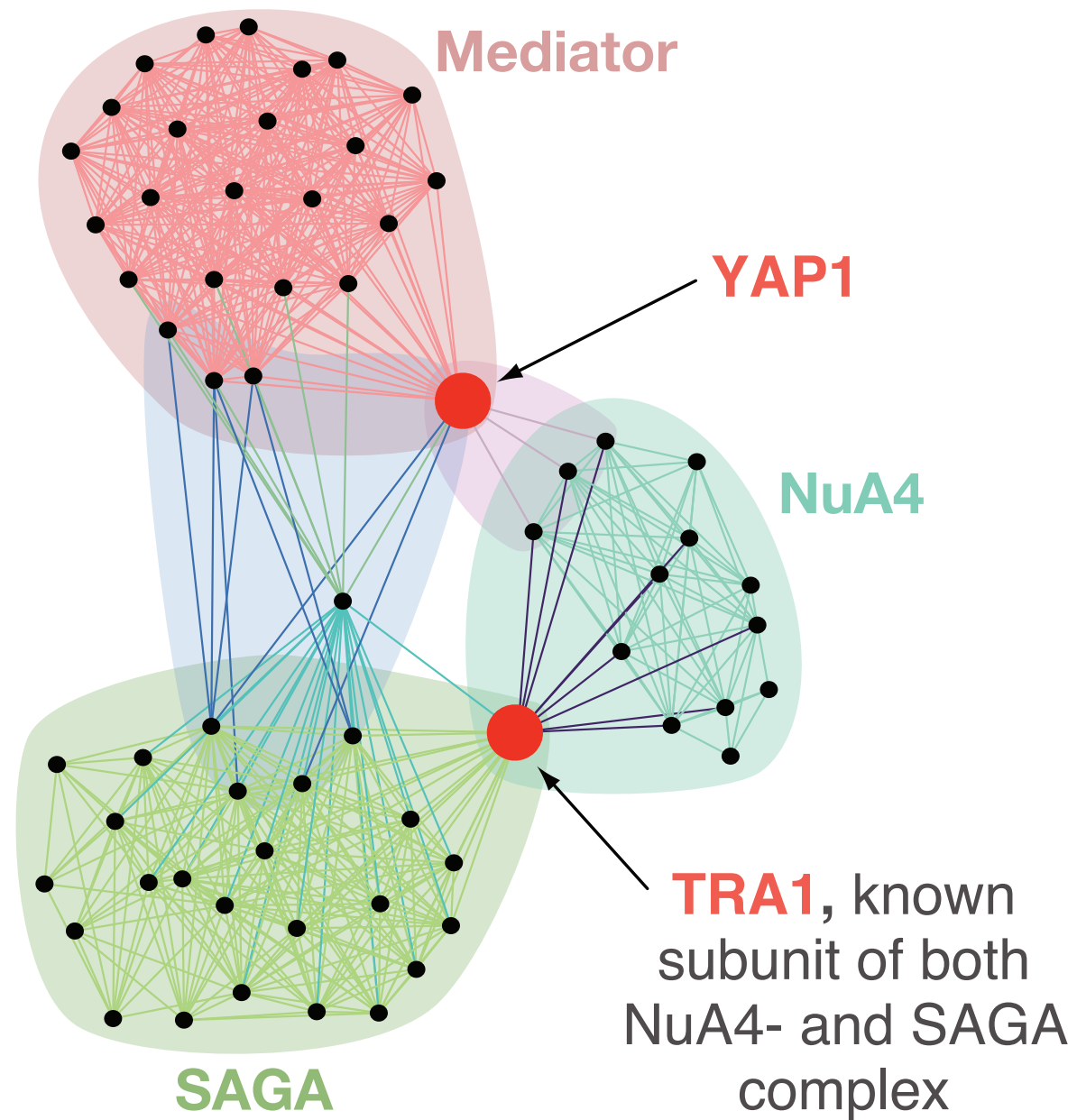
Smart, Intellect, Scientists

Science, Scientists

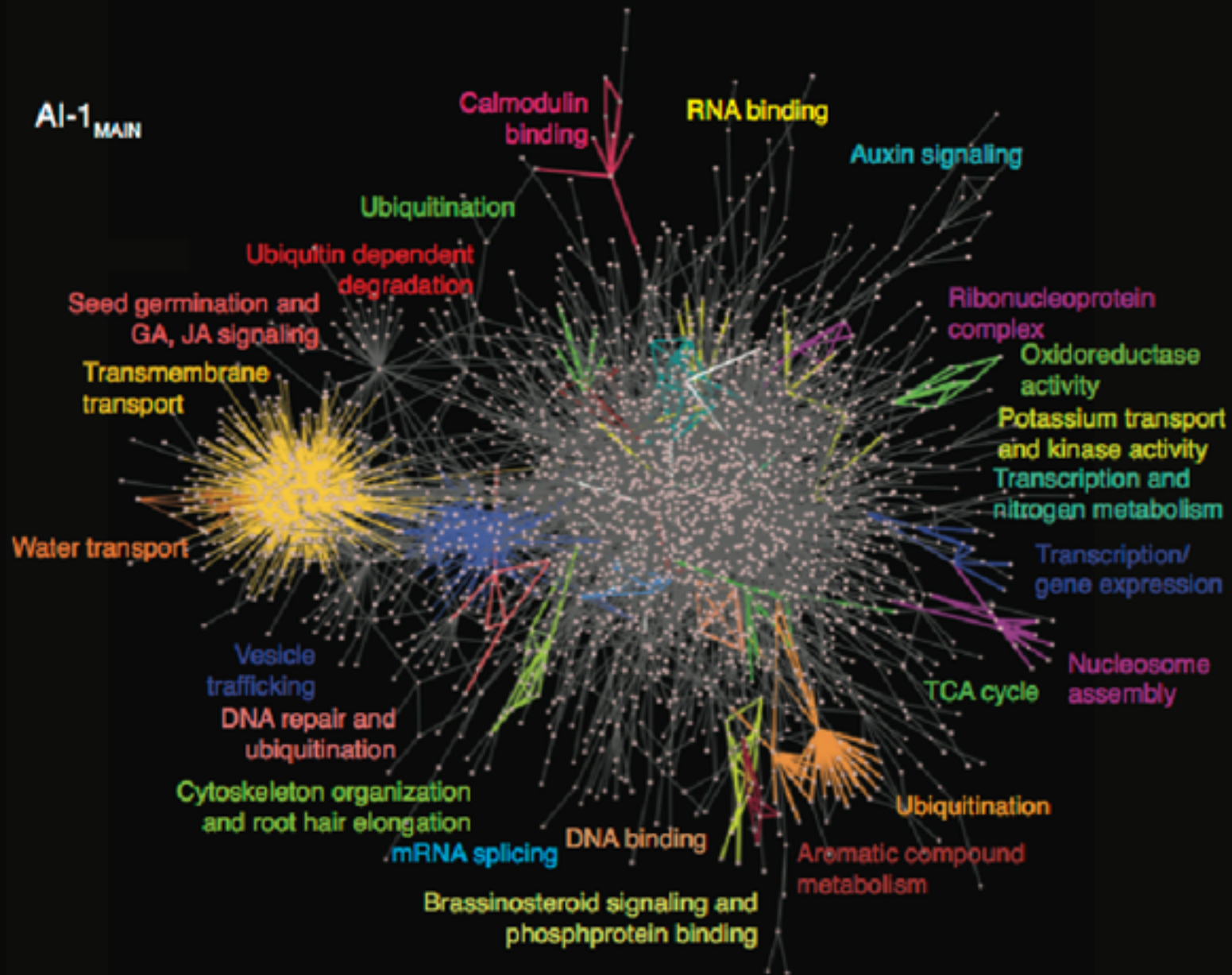
Newton, Gravity, Apple

Clever, Wit

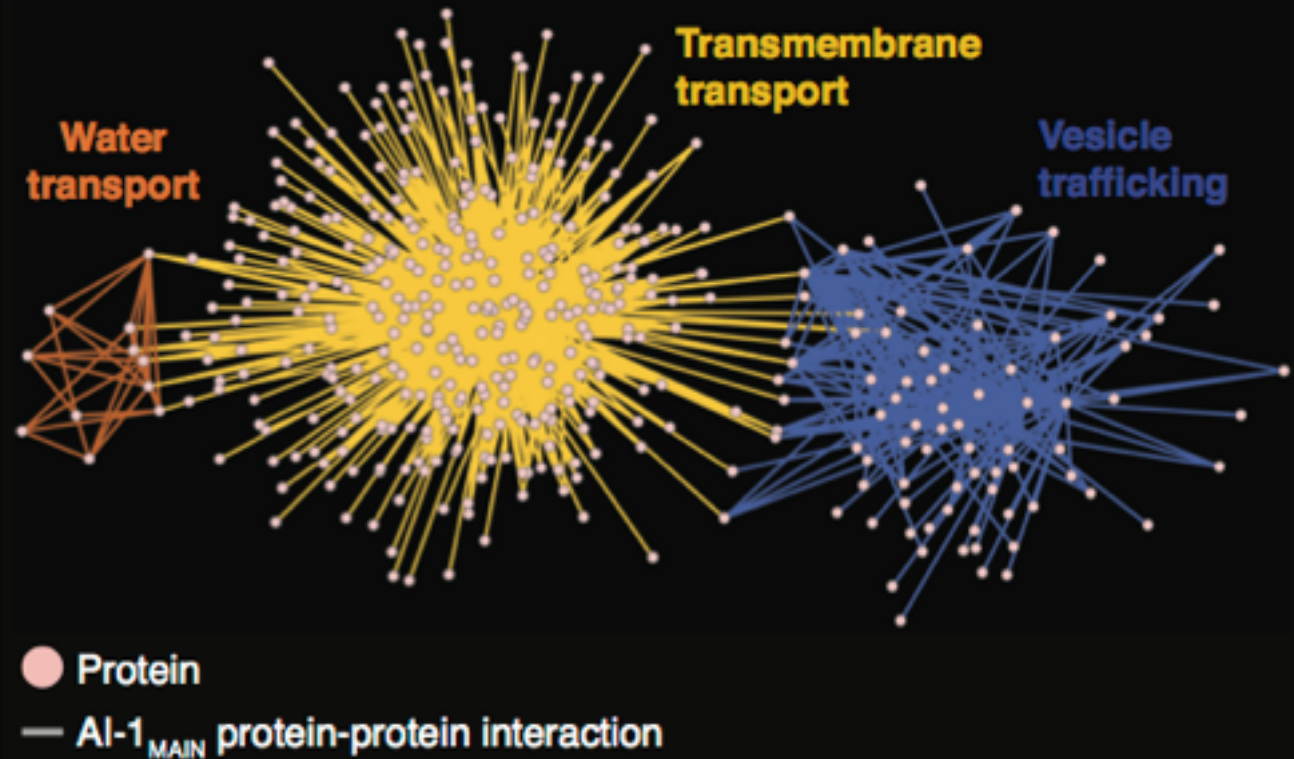
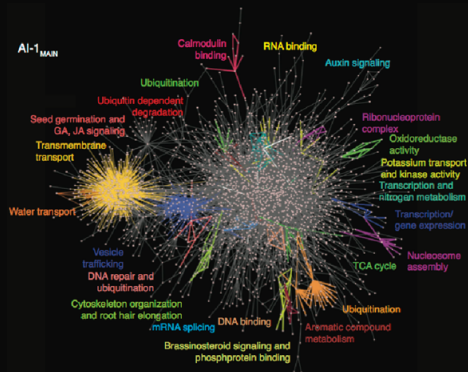
NEWTON



The first plant (genomic scale) interactome



The first plant (genomic scale) interactome



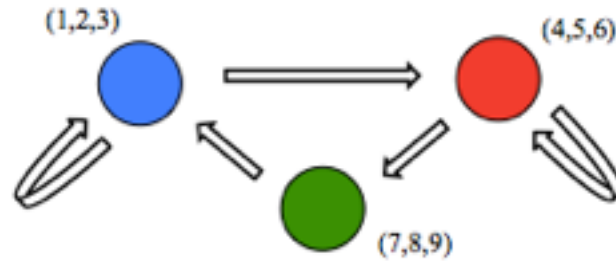
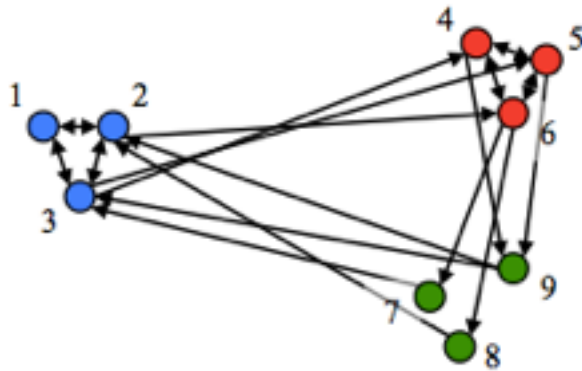
Statistical inference

Given a graph G , and a generative model with parameters θ

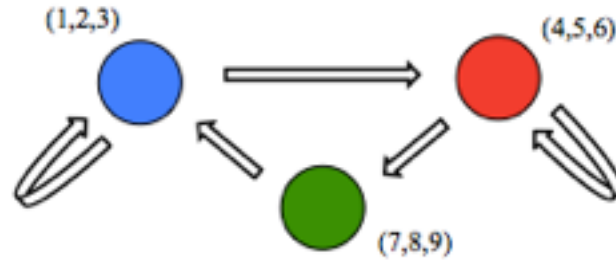
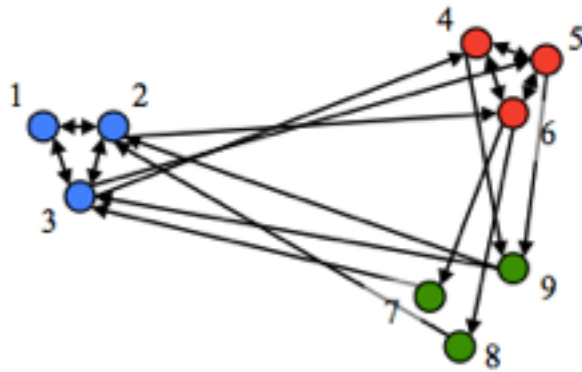
Likelihood

$$P(\theta|G) = \frac{P(G|\theta)P(\theta)}{P(G)}$$

Stochastic Block Model

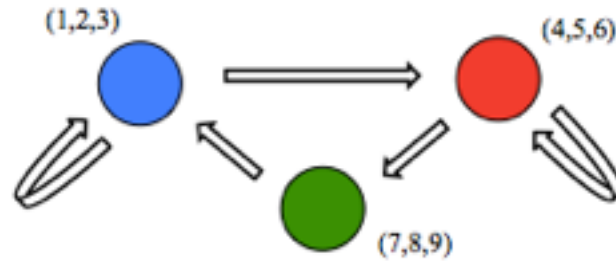
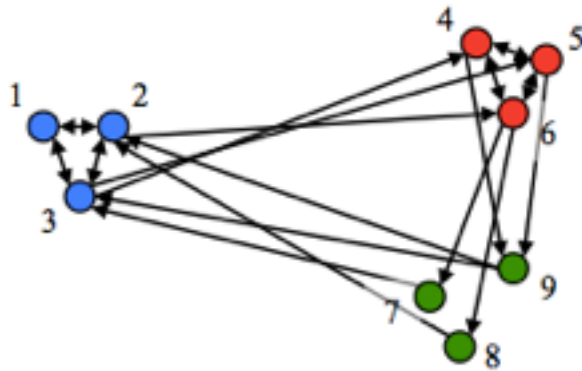


Stochastic Block Model



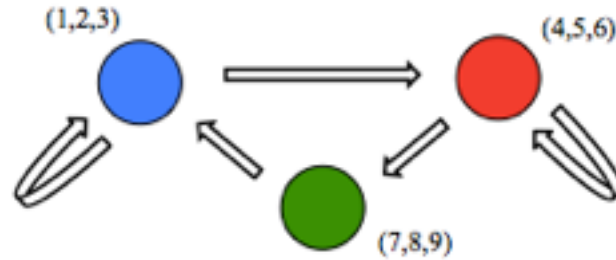
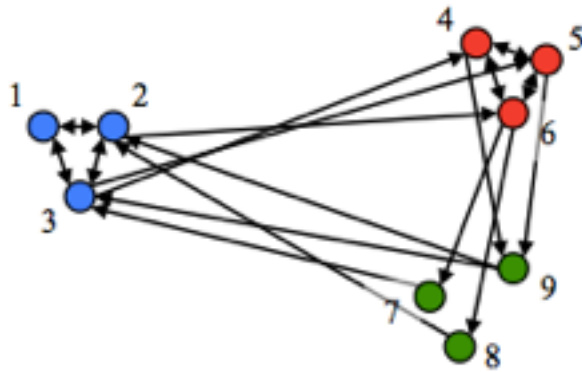
C_i

Stochastic Block Model



$$C_i \quad p_{C_i C_j}$$

Stochastic Block Model



$$C_i \quad p_{C_i C_j}$$

$$\prod_{i < j} p_{C_i C_j}^{A_{ij}} (1 - p_{C_i C_j})^{1 - A_{ij}}$$

So, what should I
use?

1. No silver bullet.

2. Hard to know beforehand.

Accuracy

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- **Infomap** and **Louvain** method are the best in these benchmarks.
- However, the performance depends on what kinds of community structure the benchmark networks assume.
- Good performance in the benchmarks *does not guarantee* good performance in real cases.

Computational complexity

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- Current version of **infomap** also uses louvain-type multilevel optimization and very fast.
- **Link clustering** can also handle large graphs (but it becomes slow with large hubs).

Overlap

Overlap

- If you expect pervasive overlap of communities, you should use overlapping community detection methods.

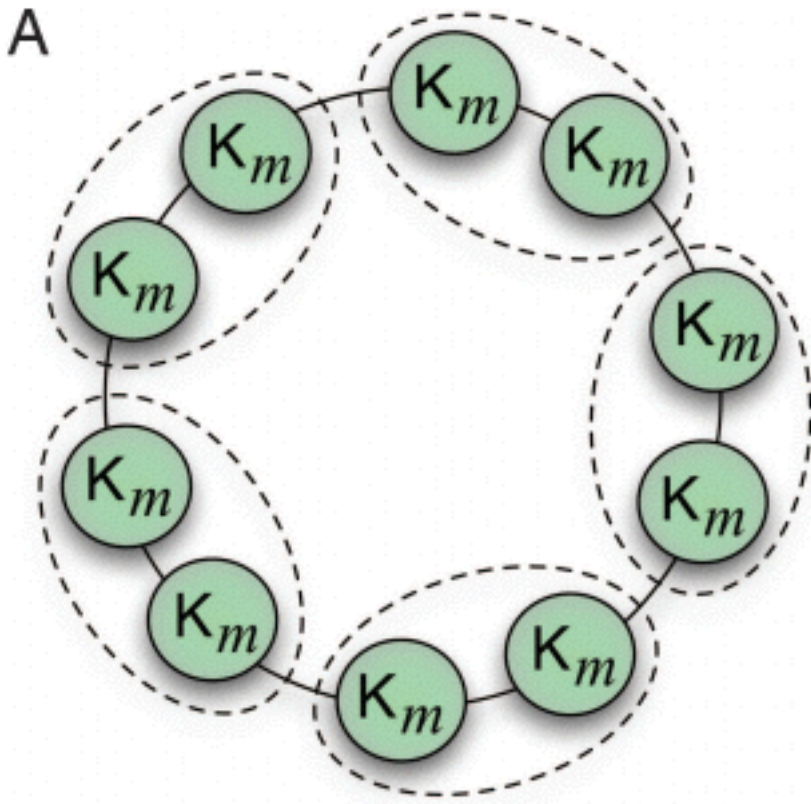
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Overlap

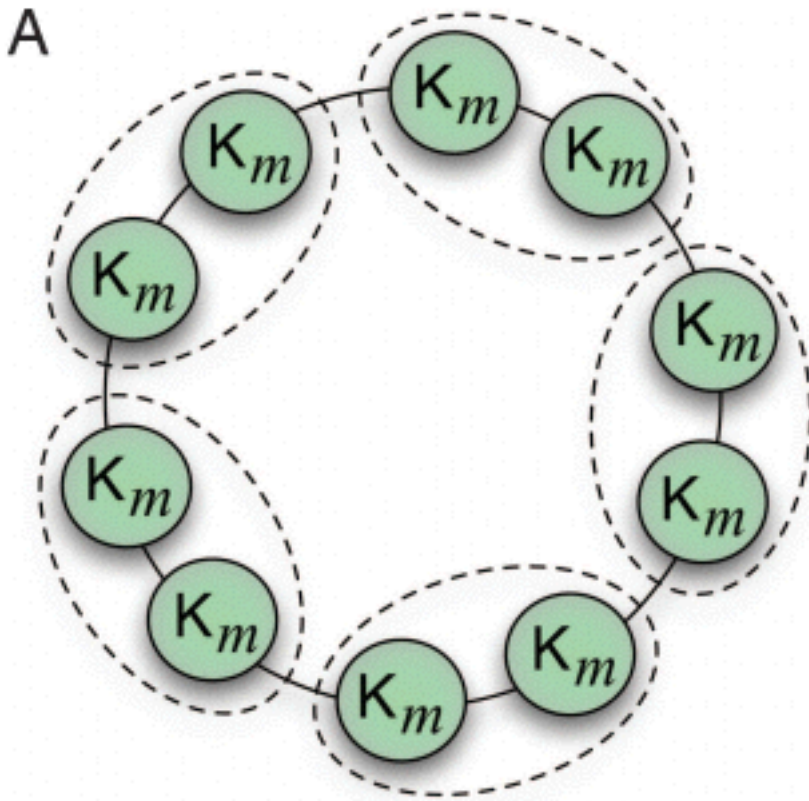
- If you expect pervasive overlap of communities, you should use overlapping community detection methods.
- **Link clustering** and **clique percolation** methods are common choices.
- These methods can detect highly overlapping communities. There are many other methods but most methods only deal with '**fuzzy**' overlaps.

Resolution limit

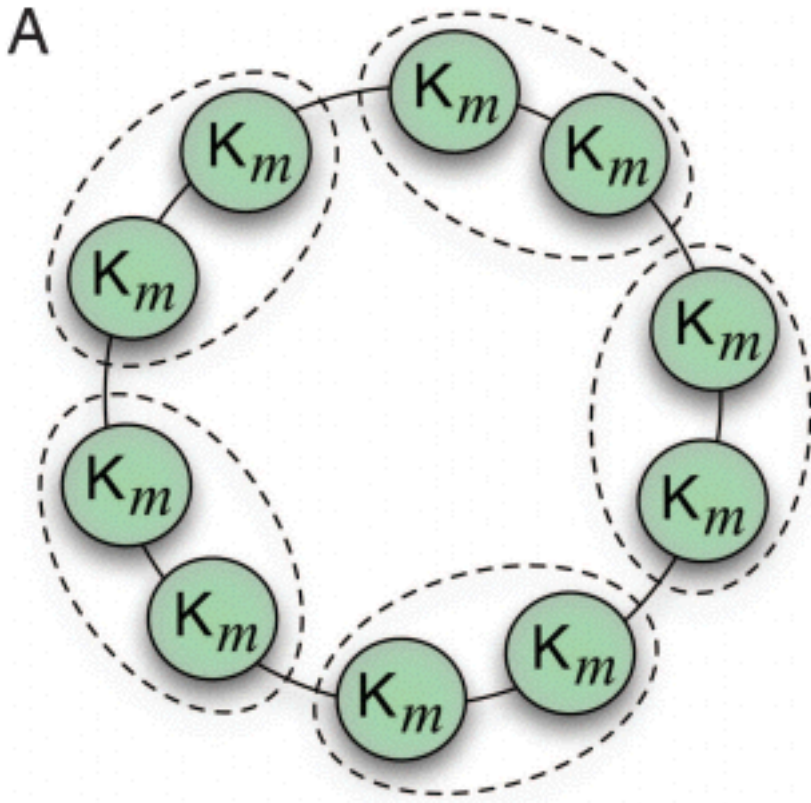


Resolution limit

- Modularity has a resolution limit that depends on the system size.



Resolution limit



- Modularity has a resolution limit that depends on the system size.
- If a community is smaller than this limit, modularity-based optimization cannot find the communities, even though they are cliques.

My heuristic

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- I don't care too much and I just want to get rough clusters in my network — **Infomap** or **Louvain**

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- My network is HUGE and has lots of super-hubs — **Louvain (Infomap)**
- I'd like to see the detailed hierarchical structure — **Link clustering**

THANK YOU!

@yy

yyahn@indiana.edu